is obtained. The analogy of (3) is that the matrix S is not stochastic but is an array of known complex numbers. Differentiation with respect to variables multiplied by elements of this matrix is completely valid. Morgera's (2) is valid. However, in this equation, S is not a variable, so that

$$\frac{\partial}{\partial d_i} \ln [\det (S)] = 0 \tag{5}$$

and my (7) follows.

Morgera's statement that his (2) is obtained if and only if the elements of the eigenspectrum are distinct is a mystery to me. It may be that he means that the circumstance that the extremum is obtained when all d_i are equal negates the applicability of partial differentiation. If this is what is meant, this is clearly erroneous. As an example, the function

$$Q = \sum_{i=1}^{P} (z_i - 1)^2$$

attains the minimum value of zero for each $z_i = 1$. The correct solution is easily found by computing $\partial Q/\partial z_i$; $i = 1, 2, \cdots P$ and setting each partial equal to zero.

The last major point of Morgera's communication is the lack of rigor in progressing from (7) to (8). The validity of the diagonalizability of A by a similarity transformation is based upon recognizing that for this problem there is a unique ML estimator of the covariance matrix. Note that when A is diagonalizable, the unique matrix with equal eigenvalues is a scalar multiple of the identity matrix. So that if A is diagonalizable it must equal N times the identity matrix as in (9). If A is not diagonalizable, there is not a unique matrix with the eigenvalues of N. For this situation there is not a unique ML estimator.

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Adaptive Line Enhancement and Spectrum Analysis

D. W. TUFTS

Abstract—The notion that adaptive filters may estimate and track the frequency of a phase-modulated sinusoid in noise better than a spectrum analyzer is examined. It is argued, using both theoretical and experimental results, that spectrum analysis performs better than adaptive filtering.

In addition, the method of spectrum analysis can be improved for such applications by the use of frequency-slope processing, or by the generation of a partially coherent reference waveform by maximum posterior probability estimation of the phase function.

Adaptive line enhancement has been proposed as an improved method for a) estimating and tracking the frequency of a phase-modulated, sinusoidal signal in noise and b) detecting the presence of such a signal in noise. Widrow *et al.* and Griffiths argue that adaptive filtering may have better performance than spectrum analysis in such applications [1], [2]. In this letter I argue that, on the contrary, spectrum analysis appears to have advantages in performance which can be demonstrated both theoretically and experimentally.

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Consider the case in which the frequency of the sinusoid is almost constant over the delay range of the adaptive filter. This is the condition given by Griffiths for effective operation of the adaptive filter [2, p. 214]. In Griffiths' example E [2, p. 219] in which the frequency is varying sinusoidally with a period of 21 samples, the delay range of the adaptive filter was taken to be $L = 4$ samples. We assume that no *a priori* knowledge is available about the form of frequency variation. The range within which the frequency lies—which could be the whole Nyquist frequency range—is assumed to be known.

Under these circumstances Viterbi has derived the maximum-likelihood estimate of frequency within each of the intervals over which it is almost constant [3, ch. 10]. in the presence of additive Gaussian noise. Let us assume that each such interval has duration $L = 4$ samples, as in Griffiths' example. The maximum-likelihood estimate is obtained by spectrum analysis (SA), which could be realized using the fast Fourier transform algorithm (FFT). However, the size of the array to be transformed would not be 4 samples, but more likely 256 or 512 samples. That is, the data array must be heavily padded with zeros to attain the desired frequency resolution.

The frequency resolution is defined in this note to be the accuracy of the measurement of the instantaneous frequency (i.e., the derivative of the phase function) of a phase-modulated sinusoid. *For a fixed nonzero window length* over which the instantaneous frequency is almost constant, the frequency resolution of a zero-padded discrete Fourier transform (DFT) depends on 1) the signal-to-noise ratio (SNR) of the input data, 2) the residual mismatch between the signal and the closest constant-frequency sinusoid, and 3) the number of frequency bins which are formed (and, hence, the amount of zero padding). If there is no noise with the signal, the frequency resolution monotonically improves, as more uniformly spaced frequency bins are added over the same frequency range, until the residual mismatch of the signal, if any, limits the accuracy of the frequency measurement.

In practical applications the attainable frequency resolution naturally depends on the type and precision of *a priori* knowledge of the variation of signal frequency or phase, and how that *a priori* knowledge is used in the measurement of frequency. Here, following Griffiths [2, p. 219], we assume only that the frequency is almost constant over the window of nonzero data.

The spectrum analysis method can be improved by testing a small number of frequency-slope bins associated with each center-frequency bin. This enables a frequency-coherent reference to be used over a longer span of data, because the frequency rate is being matched as well as the frequency. Here this is called sloped spectrum analysis (SSA).

A sinusoidally varying frequency was simulated with additive noise in order to track the frequency by the SA method, based on maximum-likelihood estimation of frequency and by the SSA method. The parameters were chosen to match those chosen by Griffiths in his example E [2, p. 219]. Good tracking of frequency (to an accuracy of about 0.005 Hz) was obtained down to an SNR of 10 dB.

Examples of these experimental results are presented in Tables I, II, and III below. The same signal and noise values are used in each of the three cases, but the noise is scaled to obtain the desired SNR. Only three values of frequency slope were used for the SSA method, namely, zero and ±0.00374 Hz/s. These values cover the range of frequency-slope values of the signal. The frequency estimates appear to be more accurate than those obtained by adaptive filtering.

It is difficult to compare the complexity of the SA method with that of adaptive filtering. Although the former was realized by a zero-padded FFT algorithm to obtain the results of Tables I, II, and III, this is not the most efficient realization.

Widrow *et al.* suggest that it may be possible to detect low-level sinusoids in noise more effectively by use of adaptive line enhancement [1, pp. 1713–1716. The spectrum analyzer with which they compare adaptive filtering is not allowed to coherently integrate over the full set of input samples over which the frequency is constant. This would improve its performance.

If a phase-modulated sinusoid is to be detected, then results of detection theory can be applied [3, ch. 8], [4, ch. 4]. The modeled phase function, any *a priori* information, and the input data should then be used to estimate the phase function [5], [6] and form a partially coherent reference waveform [3, ch. 8].

TABLE I
FREQUENCY IN Hz

F((IN)	F(SSA)	F(SA)
0.2103	0.2113	0.2090
0.2078	0.2074	0.2070
0.2046	0.2054	0.2031
0.2009	0.2015	0.1992
0.1972	0.1976	0.1953
0.1937	0.1937	0.1934
0.1908	0.1917	0.1895
0.1887	0.1875	0.1875
0.1876	0.1875	0.1875
0.1876	0.1875	0.1875
0.1887	0.1895	0.1895
0.1908	0.1911	0.1934
0.1937	0.1930	0.1953
0.1972	0.1969	0.1992
0.2009	0.2008	0.2031
0.2046	0.2047	0.2070
0.2078	0.2067	0.2090
0.2103	0.2109	0.2109
0.2119	0.2129	0.2129
0.2125	0.2129	0.2129
0.2119	0.2109	0.2109

Comparison of input frequency, F(IN), frequency estimate using sloped spectrum analysis, F(SSA), and frequency estimate using spectrum analysis F(SA). The signal samples are those specified by Griffiths [2, p. 219] The number of frequency bins over a 1 Hz band is 512. (the bin separation is 0.00195 Hz). The frequency slope for the SSA method is ±0.00374 Hz/s. SNR is 70 dB.

TABLE II
FREQUENCY IN Hz

F((IN)	F(SSA)	F(SA)
0.2103	0.2074	0.2051
0.2078	0.2054	0.2031
0.2046	0.2008	0.2031
0.2009	0.1996	0.1973
0.1972	0.1976	0.1953
0.1937	0.1895	0.1895
0.1908	0.1930	0.1953
0.1887	0.1878	0.1855
0.1876	0.1859	0.1855
0.1876	0.1852	0.1875
0.1887	0.1956	0.1934
0.1908	0.1937	0.1914
0.1937	0.1872	0.1895
0.1972	0.1969	0.1992
0.2009	0.2012	0.2012
0.2046	0.2087	0.2109
0.2078	0.2106	0.2129
0.2103	0.2207	0.2207
0.2119	0.2113	0.2090
0.2125	0.2132	0.2129
0.2119	0.2028	0.2051

Comparison of values of input frequency and frequency estimates as in Table I, except SNR = 20 dB.

TABLE III
FREQUENCY IN Hz

F((IN)	F(SSA)	F(SA)
0.2103	0.2031	0.2031
0.2078	0.2035	0.2031
0.2046	0.2028	0.2031
0.2009	0.1996	0.1973
0.1972	0.1973	0.1973
0.1937	0.1875	0.1875
0.1908	0.1969	0.1992
0.1887	0.1859	0.1836
0.1876	0.1839	0.1836
0.1876	0.1833	0.1855
0.1887	0.1996	0.1973
0.1908	0.1937	0.1914
0.1937	0.1833	0.1855
0.1972	0.1969	0.1992
0.2009	0.2015	0.2012
0.2046	0.2126	0.2148
0.2078	0.2126	0.2148
0.2103	0.2308	0.2285
0.2119	0.2093	0.2070
0.2125	0.2132	0.2129
0.2119	0.1989	0.2012

Comparison of values of input frequency and frequency estimates as in Table I, except SNR = 15 dB.

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Reply[1] *by L. J. Griffiths*[2]

Tufts' discussion correctly points out that conventional spectrum analysis can be used to accurately determine the instantaneous frequency of a sinusoid which is observed in the presence of additive white noise. The advantage of using adaptive filters for estimating and tracking instantaneous frequencies is not that such filters perform better than optimal detectors for a specific problem. They do not. Rather, the advantage lies in the fact that adaptive estimators are extremely robust. That is, they perform remarkably well over a wide range of input signal parameters and statistics with no *a priori* knowledge regarding the precise nature of these parameters. In addition, the performance of the adaptive filter has been shown to be relatively insensitive to the two available filter parameters—filter length L and the normalized adaptive proportionality constant α. Increasing or decreasing either of these parameters by a factor of two causes little change in adaptive performance. Examples of this behavior are presented in [1].

Of course, once the precise statistical description of an input waveform is available, optimal filters can be derived and will undoubtedly outperform adaptive filters. A note of caution for this approach, however, must be injected. The performance of optimal processors is not always robust. If a mistake is made in characterizing the input and the wrong "optimal" processor is employed, the results can often be dramatically in error. As an example, if the spectrum analysis method (SA) described in the above correspondence is applied to an input containing

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two closely spaced sinusoids, and a four-sample analysis interval is used, the spectrum obtained after adding zeros to achieve the desired frequency resolution will not contain peaks at the appropriate instantaneous frequency points. This is true even for the case of infinite-input signal-to-noise ratio. Thus searching for maxima in the output could easily lead to frequency estimation errors greater than 50 percent for the two-input sinusoid case. In fact, a derivation of the multiple-input optimal estimator shows that it differs significantly from the direct SA method used by Tufts. In contrast, the same form of the adaptive frequency estimator works well for either single-or several-input frequency-modulated sinusoids, as shown in [1]. More recent results have also shown that the adaptive method provides good estimates for the case of burst-type inputs in which the sinusoids are present only during short random time intervals.

In summary, I agree completely with Tufts' observation that one should always apply all available *a priori* information when designing detection and estimation signal processing filters. Care must be taken, however, to ensure that the performance of these filters does not rapidly deteriorate when small errors exist between the actual and assumed input statistics. For those cases which seem to occur all too frequently in practice—that is, when one has absolutely no reliable *a priori* knowledge regarding the structure of the desired signals—I know of no better overall procedure than to use a simple adaptive filter based on the LMS algorithm.

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Reply[3] by B. Widrow,[4] J. Glover,[5] J. McCool,[6] and J. Treichler

We thank Tufts for his comments on the discussion of adaptive line enhancing that appears in Appendix D of our paper "Adaptive Noise Cancelling: Principles and Applications" [1]. It is clear to us that our suggestion was in error that given equal input data, the line enhancer might outperform the DFT in detecting a single sine wave in white Gaussian noise. In this classic case, the DFT implements a finite-length approximation to the matched filter and is therefore close to optimal. We believe that our simulations are correct, but that our analysis and interpretation of the results are subject to question. It would have been more meaningful to compare D_{DFT} with $\sqrt{D_{\mathrm{ALE}}}$ rather than with D_{ALE} as we did in Appendix D. On the basis of this comparison, analysis shows that the adaptive line enhancer and the DFT are equally effective as a single-sine-wave detector.

Tufts suggests that in restricting the length L of the DFT window to less than the length of the total data block, the DFT was in effect disadvantaged in our study. This issue is not clear in Appendix D. We should note here that the adaptive filter length was restricted in the same way, in that the number of weights was made equal to the number of DFT points, precluding any comparative advantage or disadvantage. Such length restrictions are useful in the presence of an input signal with finite bandwidth. For best results, the DFT length and the adaptive-filter length should both be set to correspond to the reciprocal of the signal bandwidth, so that the frequency resolution of each method is matched to the signal linewidth.

We regret the error in Appendix D and wish to point out that this section is independent of the rest of the paper. We did caution the reader that "the concepts . . . have not been in existence long enough to provide an adequate perspective."

Since the line enhancer is a new and unusual filtering system, it is sometimes a difficult or complex matter to resolve questions regarding its behavior with input signals of finite bandwidth, colored noise, multiple signals in noise, etc. People who have observed the performance of the line enhancer are very enthusiastic about its potential and are anxious to understand its behavior from a theoretical standpoint. Experiments are easy to perform but are sometimes difficult to interpret. In the following experiments, adaptive-filtering and adaptive-line-enhancing techniques have been applied to some interesting problems in spectral analysis. The same kinds of problems can also be solved by conventional methods based on the DFT.

The experiments presented in Fig. 1 are similar to the experiments described in Appendix D of [1] with the following exception: in the experiments presented here, we use the transfer function magnitude of the line-enhancer adaptive filter, rather than magnitude squared as we did in Appendix D, for comparison with the power density spectrum of the primary input, consisting of a sine-wave signal in noise. For the experiment depicted in Fig. 1(a), the noise was white; for Fig. 1(b) and (c), the noise was 50-percent white, 50-percent colored. The colored noise had 25-percent bandwidth and was generated by passing white noise through a two-conjugate-pole filter. In each case, the SNR was 0.01562. The adaptive filter had 128 weights; the DFT had 128 points. The sampling frequency was 1. The total amount of data used was 32 768 samples in each case.

The DFT takes in blocks and uniformly weights the samples, whereas the line enhancer works on a steady-flow basis and exponentially weights the data over time. In comparing the "data consumption" of the two techniques, the data used by the line enhancer were defined as the number of samples inputted during four time constants of the adaptive process.

Inspection of the plots of Fig. 1, all drawn to the same linear scale, shows that in each case, using either the DFT or the line enhancer, the amplitudes of the signal components present (the spikes) were approximately the same and the background noise levels were similar. In one case, however, where the signal frequency was close to the peak of the colored-noise component (Fig. 1(c)), the noise output of the line enhancer was about 3 dB higher than that of the DFT power spectrum. The line enhancer in each case was implemented with its delay set at 256 samples, which was adequate to decorrelate and eliminate the colored-noise components.

Fig. 2 illustrates a different kind of experiment, where a large-amplitude signal summed with small-amplitude signals in noise causes difficulties in detecting and/or resolving the small signals.

Fig. 2(a) shows the formation of "input A" as the sum of white noise of unit power with three sinusoidal signals: the first of power equal to 125 at frequency $f_1 = 0.1796875$, the second of power 0.125 at frequency $f_2 = 0.15625$, and the third of power 0.5 at frequency $f_3 = 0.421875$. The sampling frequency is 1. Notice that signal 1 was one thousand times more powerful than signal 2, and that they both were close in frequency. Fig. 2(b) is a block diagram of the system used in this experiment, its primary input indicated by A. Its outputs are B and C, which represent, respectively, the "error" of the adaptive process and the adaptive-filter output. An additional output is the weight vector of the adaptive filter, comprising its impulse response, which can be Fourier transformed to provide a transfer function as discussed above.

Fig. 2(c) is a linear plot of the DFT power spectrum of input A. The frequencies of the three signals are indicated by the arrows. Notice that signal 2 is not resolvable; it is buried in the second sidelobe of signal 1. Even when this spectrum is taken through a Hanning window and plotted on a log scale, signal 3 is visible but smaller in amplitude than many of the sidelobes of signal 1, and signal 2 is undetectable.

When the DFT spectrum of the error B is taken instead, a power spectrum is obtained that clearly shows the weak signal at frequency f_2. This spectrum, plotted on a linear scale in Fig. 2(d), also shows the weak signal at frequency f_3 and the broad-band background noise. The strong signal 1 was totally cancelled as a result of the adaptive process.

The plots of Figs. 2(c) and 2(d) were normalized so that full scale corresponds to the largest amplitude point of each plot. The spectra of both A and B were taken from 128 data points. There was no ensemble averaging.

The 64-weight adaptive filter cancelled the strong signal within about five cycles of frequency f_1, i.e., within about 30 sample periods. The DFT spectrum of error B was then taken so that the amount of data used in forming it was only slightly greater than the quantity of data used in forming the DFT spectrum of input A. The line enhancer used as a strong line canceller appears to have improved the capability of the DFT to resolve and detect weak signals when they are close in frequency to a strong interference. Although an equivalent result could have been obtained by the DFT alone using a wider window, it probably would require substantially more data.

The line enhancer can be used as an alternative to the DFT as a detector and estimator of weak signals in noise. It also provides useful output signals and can function as a self-tuning filter, to tune in some and tune out other signals automatically. It is a very promising methodology for spectral analysis and is related to the maximum

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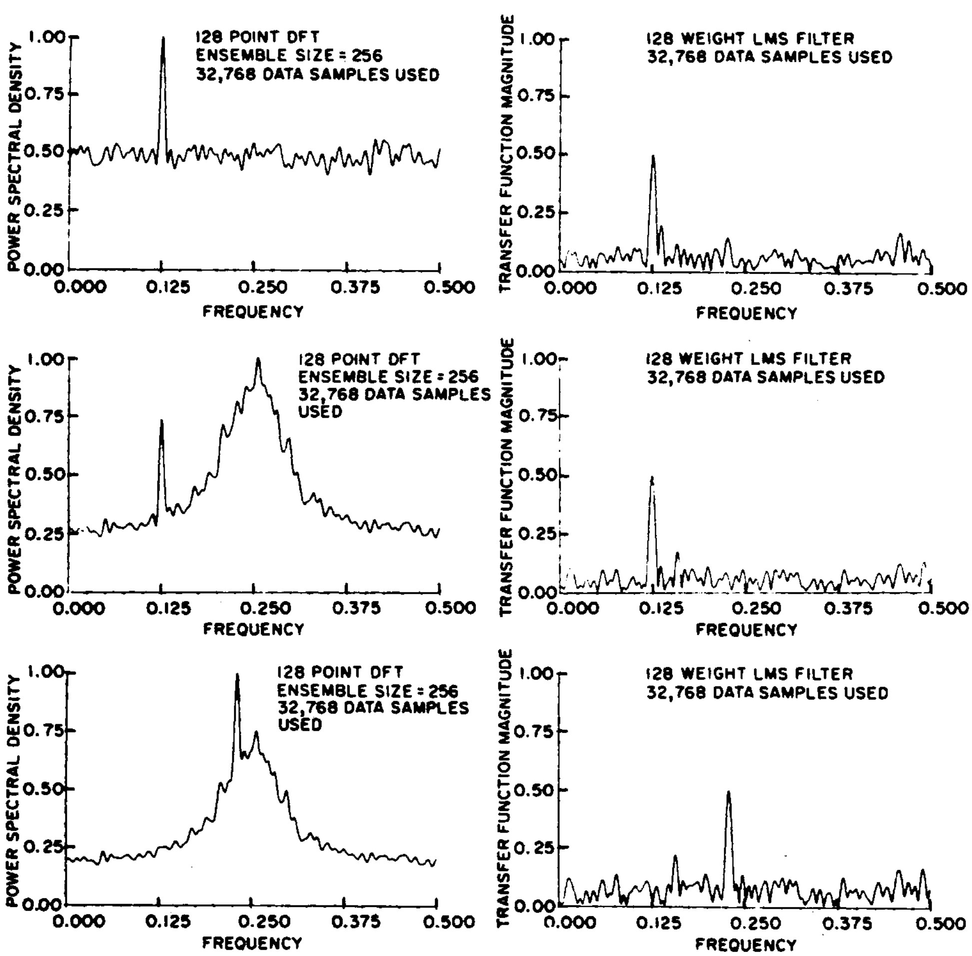

Fig. 1. Classical spectral analysis (DFT)—left, compared with adaptive line enhancing—right. (a) Single line in white noise. (b) Single line in 50-percent white, 50-percent colored noise. (c) Single line in 50-percent white, 50-percent colored noise.

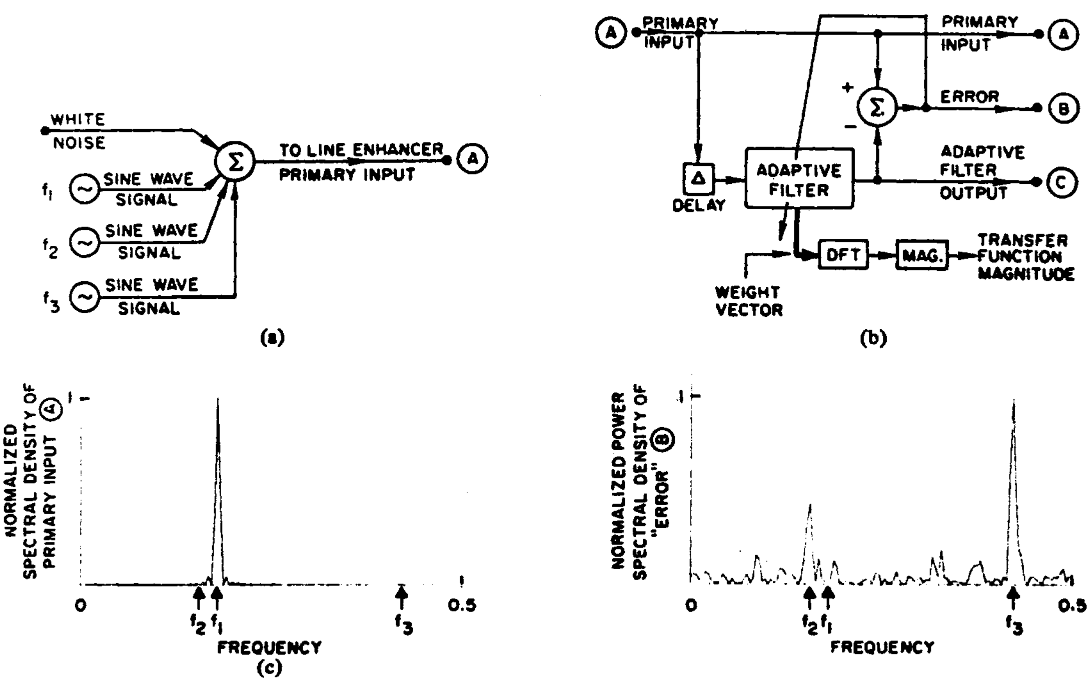

Fig. 2. Adaptive cancelling of a strong signal to enhance the detection and resolution of weak signals in noise. (a) Formation of input A. (b) Adaptive line enhancer. (c) Power density spectrum of input A. (d) Power density spectrum of output B.

entropy technique [2], [3]. Since the line enhancer is entirely different in structure from the DFT, it may be more easily implemented in some cases. We hope to publish a comprehensive work on adaptive spectral analysis in the future, contingent on more extensive understanding of the behavior of the line enhancer with inputs of the types described above.

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Realization of a Discrete Fourier Transform (DFT) Module for Incorporation in FFT Processors

A. POMERLEAU, M. FOURNIER, AND H. L. BUIJS

Abstract—**For applications requiring high-speed and in-place treatment, it is often advantageous to realize special-purpose computers. This paper describes a discrete Fourier transform (DFT) module for incorporation in fast Fourier transform (FFT) processors. The module is highly suitable for real input applications requiring high-speed transformations. It attributes one point to all frequency channels in one clock cycle. This treatment is not only well suited for the present technology, but appears to be more attractive in view of recent trends in digital circuitry.**

I. Formulation

In the evaluation of a N point discrete Fourier transform for high-speed applications with FFT algorithms where $N = r^n$, each of the n passes may be computed in a manner such that each input is processed sequentially and its contribution is attributed to all frequency channels in one clock cycle [1].

In such a case, the basic equation to evaluate each r point transform without recourse to the FFT algorithm is

$$F(j) = \frac{1}{r} \sum_{k=0}^{r-1} f(k) \exp - 2\pi i \frac{kj}{r}, \qquad j = 0, 1, \cdots, r-1.$$

In this expression, $f(k)$ is the input function and $F(j)$ are the Fourier transform coefficients. For the general case, the evaluation of $F(j)$ requires r^2 complex operations[1] to uniquely determine the spectrum. For a serial-input parallel-output circuit, it would then require r complex multipliers and r complex adders. However, this number can be greatly reduced when r is a multiple of 4, since there are only $(r-4)/4$ different absolute values of the real and imaginary parts of the r roots of 1, neglecting 0 and 1. Furthermore, when the input function is real and only the nonredondant terms are evaluated, an additional reduction in the number of mathematical operations is possible. The implementation of a module in which the data are treated in magnitude, while the sign and that of the trigonometric coefficients are taken into account in the adders, then requires the following.

1) r real adders with their associate memories.
2) $(r-4)/4$ multipliers operating in parallel on real numbers. This quantity is required since there are only $(r-4)/4$ absolute different values in the r roots of 1, neglecting 0 and 1, if r is a multiple of 4.
3) A relatively small quantity of multiplexers, since some of the adders have always access to the same weighted values.
4) Control units having three main functions:
   a) to direct the weighted data toward the proper accumulator;
   b) to add or subtract the weighted data;
   c) to apply the inhibit function (multiplication by zero).

Table I shows the number of adders, multipliers, and multiplexers for different values of r.

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[1] Complex operation: A complex operation is defined by a complex multiplication followed by a complex addition.

TABLE I
NUMBER OF COMPONENTS REQUIRED FOR r-POINT TRANSFORM

r	accumulators	multipliers	multiplexers quantity	multiplexers no. of inputs
4	4	0	0	
8	8	1	1	2
12	12	2	2	3
			1	2
16	16	3	2	4
			1	2
20	20	4	4	5
			2	3
			2	2
24	24	5	4	6
			2	3
			3	2
r	r	$\frac{r-4}{4}$	4	r/4
			.	.
			.	.
			.	.
			.	.

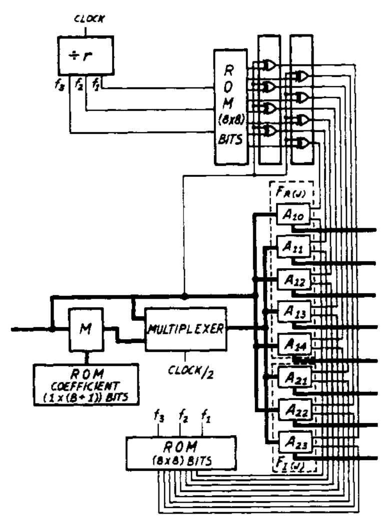

Fig. 1. Eight-point Fourier transform module.

II. Realization

Figure 1 shows the realization of an 8-point Fourier transform. It is a serial-input parallel-output circuit where all the bits forming a word are propagated in parallel.

A. Coding

In each module, the input data words are represented by B bits expressed in magnitude and sign. The multiplications are done in magnitude only and the adders operate in 2's complement code. Therefore, a code conversion circuit is not required between the multiplier and the accumulators.

B. Multipliers

As shown in Table I, it requires only one multiplier for a radix 8 Fourier transform. Here the data are represented by B bits, while the