A REVIEW OF ADAPTIVE ANTENNAS

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ABSTRACT

Most adaptive receiving arrays permit beam steering in a selected "look" direction while rejecting or nulling strong interferences which arrive at angles other than the look direction. Nulling is accomplished by adjusting parameters of a signal processor connected to the array sensing elements (whether at rf, audio, or seismic frequencies) to minimize total output power. A signal arriving in the look direction would not be nulled because the adaptive process is constrained to maintain a predetermined sensitivity in the look direction.

A substantial literature exists in the field. The September 1976 issue of the IEEE Transactions on Antennas and Propagation was dedicated to the subject of adaptive antennas. It is the purpose of this presentation to compare the works of Howells and Applebaum, Widrow, and Frost.

I. INTRODUCTION

There are several forms of adaptive antennas that have appeared in the literature, each having its own performance goals. The most widely developed and discussed form is a receiving adaptive beamformer capable of "looking" in any user specified direction while nulling incident interference signals without knowing *a priori* their angles of incidence. The main lobe of the antenna array is thus steered in any desired look direction, whether the sought-after signal is present or not. Interferring or jamming signals cause nulls to develop in the side lobe structure. Generally, the stronger the interference, the deeper the null.

Adaptive antennas consist of sensor arrays connected to signal processors having the capability of variable phasing and/or variable weighting. Adaptive algorithms, which may be hardware or software implemented, are used to control the variable parameters which in turn control the antenna pattern.

Hundreds of papers on adaptive antennas have appeared in the Proceedings of the IEEE, in the IEEE Transactions on Antennas and Propagation, the IEEE Transactions on Aerospace and Electronics Systems, and in the geophysical literature. The September, 1976 issue of IEEE Transactions on Antennas and Propagation was a special issue on adaptive antennas containing a set of papers and references which by and large represented the state of art at that time. This issue would be an excellent starting point for a student of the subject.

In historical order, this paper highlights the adaptive nulling schemes which have been proposed by Howells and Applebaum [1,2], Widrow et al. [3], and Frost [4]. An in depth study including the works of Griffiths, Compton, Zahm, Gabriel, White, Mermoz, Capon, Brennan, Reed, and many others can only be obtained from the literature.

We begin with a discussion of adaptive filtering and adaptive noise cancelling [5]. This is essential to an understanding of the Howells and Applebaum approach.

II. ADAPTIVE NOISE CANCELLING

Figure 1 shows an adaptive noise cancelling system. A signal s is transmitted over a channel to a sensor that also receives a noise n_0 uncorrelated with the signal. The combined signal and noise $s + n_0$ form the *primary input* to the canceler. A second sensor receives a noise n_1 uncorrelated with the signal but correlated in some unknown way with the noise n_0. This sensor provides the *reference input* to the canceller. The noise n_1 is filtered to produce an output y that is as close a replica as possible of n_0. This output is subtracted from the primary input $s + n_0$ to produce the system output $z = s + n_0 - y$. The characteristics of the signal and noise transmission paths are as a rule unknown.

In the system shown in Fig. 1 the reference input is processed by an adaptive filter. An adaptive filter differs from a fixed filter in that it automatically adjusts its own impulse response. Adjustment is accomplished through an algorithm that responds to an error signal dependent, among other things, on the filter's output. Thus with the proper algorithm, the filter can

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operate under changing conditions and can readjust itself continuously to minimize the error signal.

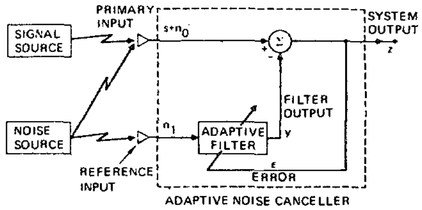


Fig. 1. The adaptive noise cancelling concept.

The error signal used in an adaptive process depends on the nature of the application. In noise cancelling systems the practical objective is to produce a system output $z = s + n_0 - y$ that is a best fit in the least squares sense to the signal s. This objective is accomplished by feeding the system output back to the adaptive filter and adjusting the filter through an LMS adaptive algorithm [5,6] to minimize total system output power. In an adaptive noise cancelling system, in other words, the system output serves as the error signal for the adaptive process.

Assume that s, n_0, n_1, and y are statistically stationary and have zero means. Assume that s is uncorrelated with n_0 and n_1, and suppose that n_1 is correlated with n_0. The output z is

$$z = s + n_0 - y \quad . \qquad (1)$$

Squaring, one obtains

$$z^2 = s^2 + (n_0-y)^2 + 2s(n_0-y) \quad . \qquad (2)$$

Taking expectations of both sides of (2), and realizing that s is uncorrelated with n_0 and with y, yields

$$E[z^2] = E[s^2] + E[(n_0-y)^2] + 2E[s(n_0-y)]$$

$$= E[s^2] + E[(n_0-y)^2] \quad . \qquad (3)$$

The signal power $E[s^2]$ will be unaffected as the filter is adjusted to minimize $E[z^2]$. Accordingly, the minimum output power is

$$\min E[z^2] = E[s^2] + \min E[(n_0 - y)^2] \quad . \qquad (4)$$

When the filter is adjusted so that $E[z^2]$ is minimized, $E[(n_0-y)^2]$ is, therefore, also minimized. The filter output y is then a best least squares estimate of the primary noise n_0. Moreover, when $E[(n_0-y)^2]$ is minimized, $E[(z-s)^2]$ is also minimized, since, from (1),

$$(z-s) = (n_0-y) \quad . \qquad (5)$$

Adjusting or adapting the filter to minimize the total output power is thus tantamount to causing the output z to be a best least squares estimate of the signal s for the given structure and adjustability of the adaptive filter and for the given reference input.

In certain instances the available reference input to an adaptive noise canceller may contain low-level signal components in addition to noise. These signal components will cause some cancellation of the primary input signal.

Figure 2 shows an adaptive noise canceller whose reference input contains signal components and whose primary and reference inputs contain additive correlated noises. Additive uncorrelated noises have been omitted to simplify the analysis. The signal components in the reference input are assumed to be propagated via the transfer function $\mathcal{J}(z)$.

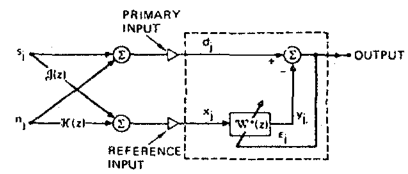


Fig. 2. Adaptive noise canceller with components in the reference input.

It is shown in ref. [5] that the ratio of the output signal power density to the output noise power density, designated as $\rho_{out}(j\omega)$ for the system of Fig. 2, is equal to the reciprocal of the corresponding ratio $\rho_{ref}(j\omega)$ at the reference input to the adaptive filter. Thus

$$\rho_{out}(j\omega) = \frac{1}{\rho_{in}(j\omega)} \quad . \qquad (6)$$

This signal-to-noise inversion concept was used to advantage in the Howells-Applebaum scheme.

III. HOWELLS-APPLEBAUM SIDELOBE CANCELLER

The original Howells-Applebaum system [1,2] had the capability of receiving a signal while nulling a jammer, assuming the jammer was very strong compared to the signal. The array consisted of two separated omnidirectional sensors, one to provide the primary input, another to provide the reference input. Signal-to-noise inversion caused the output to contain strong signal and weak jammer components.

Using a conventional time delay and sum beamformer to provide the primary input and a single array element to provide the reference input, Fig. 3 illustrates the Howells-Applebaum approach. It shows an adaptive noise cancelling system designed to pass a plane-wave signal received in the main beam of an antenna array and to discriminate against strong interference in the near field or in a minor lobe of the array. If one assumes that the signal and interference have overlapping and similar power spectra and that the interference power density is twenty times greater than the signal power density at the individual array element, then the signal-to-noise ratio at the reference input ρ_{ref} is 1/20. If one further assumes that, because of array gain, the signal power equals the interference power at the array output, then the signal-to-noise ratio at the primary input ρ_{pri} is 1. After convergence of the adaptive filter the signal-to-noise ratio at the system output will thus be

$$\rho_{out} = 1/\rho_{ref} = 20 .$$

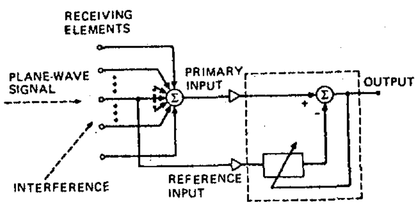


Fig. 3. Adaptive noise cancelling applied to a receiving array.

To further demonstrate reduction of sidelobe pickup with adaptive noise cancelling, the array of Fig. 4 was computer simulated. The array consisted of a circular pattern of 16 equally spaced omnidirectional elements. The outputs of the elements were delayed and summed to form a main beam steered at a relative angle of 0^o. A simulated "white" signal consisting of uncorrelated samples of unit power was assumed to be incident on this beam. Simulated interference with the same bandwidth and with a power of 100 was incident on the main beam at a relative angle of 58^o. The output of the beamformer served as the canceller's primary input, and the output of element 4 was arbitrarily chosen as the reference input. The canceller included an adaptive filter with 14 weights.

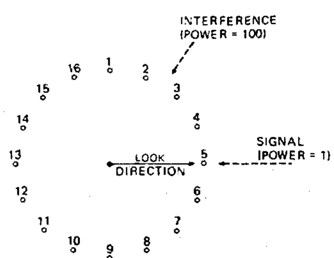


Fig. 4. Array configuration for adaptive sidelobe cancelling experiment.

Figure 5 shows two series of computed directivity patterns, a single frequency pattern at $\frac{1}{4}$ the sampling frequency, the other an average of eight frequencies, from $\frac{1}{8}$ to $\frac{3}{8}$ of the sampling frequency. These patterns indicate the evolution of the main beam and sidelobes as observed by stopping the adaptive process after the specified number of iterations. Note the deep nulls that develop in the direction of the interference. At the start of adaptation all canceller weights were set at zero, providing a conventional 16-element circular beam pattern.

The signal-to-noise ratio at the system output, averaged over the eight frequencies, was found after convergence to be +20 dB. The signal-to-noise ratio at the single array element was -20 dB. This result bears out the expectation that the signal-to-noise ratio at the system output would be the reciprocal of the ratio at the reference input, which is derived from a single element.

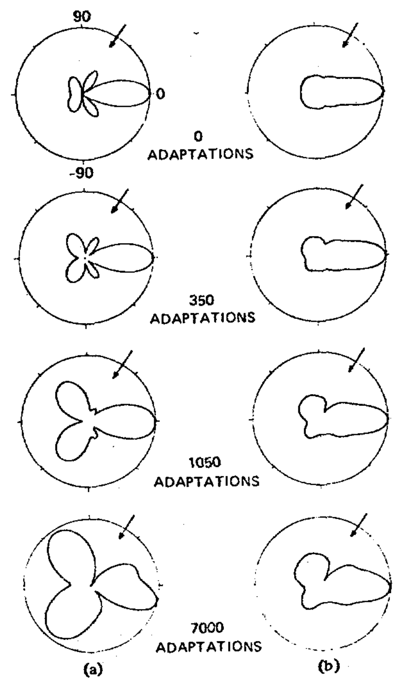


Fig. 5. Results of adaptive sidelobe cancelling experiment. (a) Single frequency pattern (0.5 relative to folding frequency). (b) Average of eight frequencies (0.25 to 0.75 relative to folding frequency).

IV. WIDROW ET AL ADAPTIVE BEAMFORMER

Another approach reported by Widrow, Mantey, Griffiths, and Goode [3] makes use of adaptivity both in forming and steering the receiving beam and in nulling external sources of interference.

This scheme incorporates an adaptive filter for each array element. The filters normally contain two weights for narrowband systems, many weights for broadband systems. The broadband configuration is shown in Fig. 6. Adjusting the weights permits control of gain and phase on the signal flow paths from each antenna element to the system output at many points in the frequency domain. This is a most general scheme.

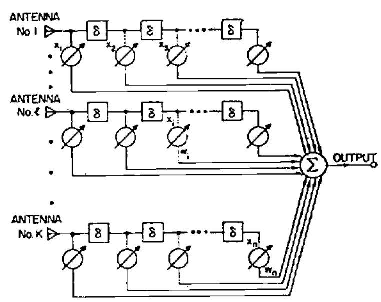


Fig. 6. Adaptive array configuration for receiving broadband signals.

The weights of the system of Fig. 6 have been adjusted by the LMS algorithm although the filters act independently in controlling the individual signal flow paths, all of the weights are adapted simultaneously in a single process. A block diagram is shown in Fig. 7.

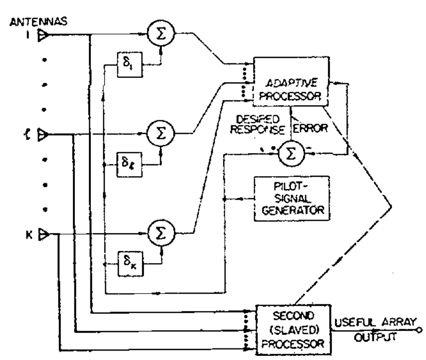


Fig. 7. Adaptation with a pilot signal.

The objective is to cause the array to have a predetermined sensitivity in the look direction; for example, let the gain be specified to be unity and the phase zero in the passband of interest. In addition, the objective is to have low gain to incoming signals or noises outside the look direction. The system of Fig. 7 partially achieves these objectives. A local signal generator synthesizes a "pilot signal" used as a desired response for the adaptive process and with special conditioning, as a portion of the input signals to the adaptive filters. The conditioning involves variously delaying the pilot signal to simulate the effects of the reception of a plane wave phase front passing over the array, arriving from the look direction. Thus, additive input components are applied locally which appear as if they were due to a pilot signal external to the receiving array, arriving in the look direction. The adaptive processor is thus trained to receive the pilot signal. Any other external signal would be uncorrelated with the pilot signal and would tend to be eliminated. *Attenuation of inputs by an array is associated* with nulls. Reception of the pilot signal on the other hand causes the formation of the main beam in the look direction. Note that a second processor whose weights are copied from the adaptive processor is used to obtain the useful output signal. This output does not contain pilot signal components which were utilized strictly for purposes of adaptation.

To demonstrate the performance of the system of Fig. 7, a series of simulation experiments have been undergone involving a wide variety of array geometries and signal and noise configurations. For simplicity of presentation, the example outlined here is restricted to a narrowband circular planar array composed of 12 ideal isotropic radiators. The LMS adaptation algorithm was used. All experiments were begun with the initial condition that all weight values were equal. The simulated array is shown in Fig. 8.

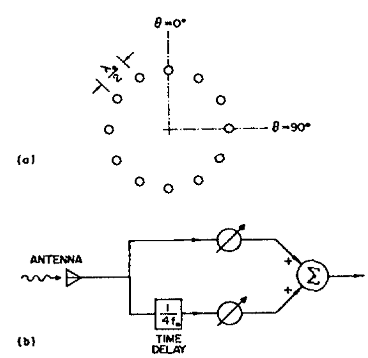


Fig. 8. Array configuration and processing for narrowband experiments.

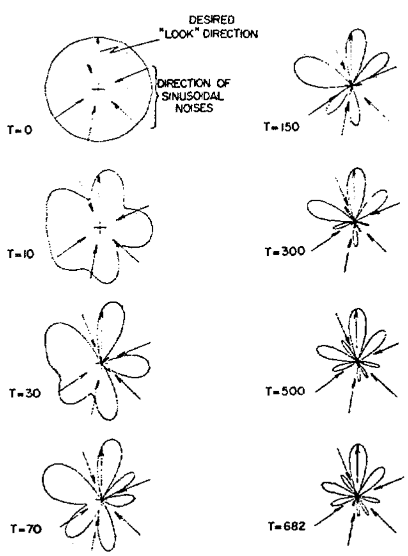
In this example, the noise field was composed of five directional sinusoidal interferers, each of amplitude 0.5 and power 0.125, acting simultaneously. Their frequencies are shown in Table I. In addition, superposed uncorrelated "white" Gaussian noises of power 0.5 were present at each of the antenna elements.

TABLE I

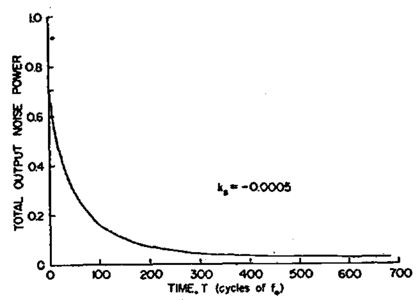
The Directional Interferences

Noise Direction (degrees)	Noise Frequency (times f_0)
67	1.10
134	0.95
191	1.00
236	0.90
338	1.05

Figure 9(a) shows the evolution of the directivity pattern, plotted at frequency f_0, from the initial conditions to the finally converged (adapted) state. The latter was achieved after 682 cycles of the frequency f_0. The learning curve for this experiment is shown in Fig. 9(b). The final array sensitivities in the five noise directions relative to the array sensitivity in the desired look direction varied between -26 dB to -38 dB, averaging about -30 dB. The signal-to-noise ratio was improved by a factor of about 15 over that of a single isotropic radiator. The learning curve shows how the mean square error of the adaptive process decreases as adaptation progresses. The curve is theoretically a sum of exponentials whose time constants are predictable.



a.



b.

Fig. 9. Evolution of the directivity pattern while learning to eliminate five directional noises and uncorrelated noises. (Array configuration of Fig. 8.) (a) Sequence of directivity patterns during adaptation. (b) Learning curve (total number of adaptations = 20T).

V. THE FROST ADAPTIVE BEAMFORMER

The Howells-Applebaum beamformer loses sensitivity to strong signals as a result of the adaptive process. Likewise, the main beam of the Widrow et al. beamformer is supported only by a "soft constraint." A strong signal on boresight could cause loss of sensitivity in the look direction. The Frost beamformer [4] has the same functional objectives as the Widrow beamformer except that the look direction constraint is "hard," i.e., maintained absolutely. In addition, like the Griffiths beamformer [7] that preceded it, the Frost beamformer requires no pilot signal.

The Frost beamformer can be structured like the adaptive array of Fig. 6. The question is, how are the weights to be adapted? This is well described in Frost's paper. We can discuss the process only briefly here. Refer to Fig. 10. The phase front of an incoming signal from the look direction is assumed to arrive simultaneously at all the sensors. Bulk delays, not shown, are presumed for beamsteering. Consider the array of transversal filters. If the tap weights were summed by columns, and if the sum of the weights of the first column were unity and the sums of all the other columns of weights were zero, it is clear that the signal arriving from the look direction would propagate to the output with a gain of unity.

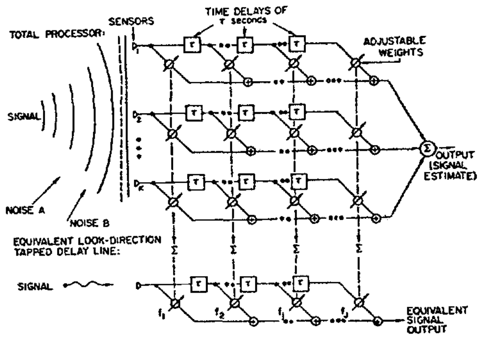


Fig. 10. Broadband antenna array and equivalent processor for signals coming from the look direction.

If the weights were varied but the column sums were constrained as above, unit gain in the look direction would always be assured. The Frost algorithm varies the weights to minimize total output power subject to this linear constraint on the weights. The result is an output which is a minimum variance estimate of all signal components arriving exactly from the look direction. The Frost algorithm creates a hard-constrained gain of unity in the look direction, while nulling strong interferences arriving from other directions. It is the most highly developed and sophisticated algorithm of them all.

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