Tradeoff Between Message and State Information Rates

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Abstract — **We consider a communication problem where the sender has access to the channel state information and wishes to send both the message information and the state information across the channel. The novelty in characterizing the tradeoff between the message information rate and state estimation error arises primarily because of the inability to encode and decode the state information. The tradeoff region is typically difficult to obtain even for a simple channel. In this work, we characterize the optimal tradeoff for the binary channel $Y^n = X^n \oplus S^n \oplus Z^n$, where S^n is available at the transmitter. We also prove the optimality of the extreme points of the conjectured tradeoff region for the additive Gaussian channel $Y^n = X^n + S^n + Z^n$, with S^n i.i.d $\sim N(0, Q)$ known at the encoder and unknown noise Z^n i.i.d $\sim N(0, N)$.**

I. Introduction

Consider the communication problem depicted in Figure 1. The sender wishes to send an index $W \in \{1, 2, ..., 2^{nR}\}$ (i.e., message information) as well as a description of the signal S^n (i.e., state information) in n uses of a memoryless channel $p(y|x, s)$. We assume the state sequence $S_1, S_2, ..., S_n$ is independent and identically distributed according to $p(s)$. The transmitter observes S^n and sends information W by transmitting $X^n(W, S^n)$. The receiver receives $Y^n \sim \prod_{i=1}^n p(y_i|x_i, s_i)$ and decodes $\hat{W}(Y^n) \in \{1, 2, ..., 2^{nR}\}$ and forms the estimate $\hat{S}^n(Y^n)$ of the state S^n. The probability of decoding error and state estimation error are given by $P_e^{(n)} = \frac{1}{2^{nR}} \sum_{i=1}^{2^{nR}} Pr\{\hat{W} \neq i | W = i\}$ and $D = E[d(S^n, \hat{S}^n)]$. We are interested in characterizing the optimal tradeoff between information rate and state estimation error.

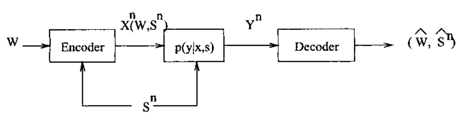

Figure 1: Channel model: $Y^n \sim \prod_{i=1}^n p(y_i|x_i, s_i)$

The following examples consistent with Figure 1 also motivate the search for a general theory for optimal tradeoff between pure information rate and state estimation error.

Digital simulcast: A TV station may simulcast a digital signal (W) over the existing analog transmission (S^n). The digital message may represent information that enhances the analog signal quality, or it may represent some control signals. Due to the interference between the two signals, the TV station must choose the tradeoff between the amount of pure information conveyed and the analog signal quality.

A computer memory with defects: In general, a computer memory may contain some defects stuck at 0 or 1. Instead of discarding these cells or risking faulty readouts, it is possible to increase the memory utilization and reliability when either or both the encoder and the decoder have access to the defect locations [2]. Also, instead of directly informing the decoder of the defect locations, it is possible to simultaneously record the data and help the decoder identify the defect locations.

Various other examples can be found in [3].

II. Main results

Definition 1 *A pair (R, D) is said to be achievable if there exists a sequence of $(2^{nR}, n)$ codes $\{X^n(W, S^n), W \in \{1, ..., 2^{nR}\}\}$ such that $E[d(S^n, \hat{S}^n)] \leq D$ and $P_e^{(n)} = \frac{1}{2^{nR}} \sum_{i=1}^{2^{nR}} Pr\{\hat{W} \neq i | W = i\} \to 0$ as $n \to \infty$.*

For a simple binary additive channel with state information at the encoder, we find the optimal tradeoff region.

Theorem 1 *The optimal (R, D) tradeoff region for the binary additive channel: $Y^n = X^n \oplus S^n \oplus Z^n$, where $S \sim Bern(r)$ and $Z \sim Bern(p)$ with a Hamming distortion measure is given as $(R, D) = (1 - H(p) - \alpha, H^{-1}(H(r) - \alpha))$, where $0 \leq \alpha \leq \min\{1 - H(p), H(r)\}$. Here, $H^{-1}(x) = q$ such that $-q\log(q) - (1-q)\log(1-q) = x$ and $0 \leq q \leq \frac{1}{2}$.*

We prove the optimality of the extreme points of the achievable tradeoff region [4] for an additive Gaussian channel $Y^n = X^n + S^n + Z^n$ with state information S^n known at the transmitter [1] and mean-squared error distortion measure. We believe the optimal (R, D) tradeoff region for this channel is given by $(R, D) = (\frac{1}{2}\log\left(1 + \frac{\gamma P}{N}\right), \frac{(\gamma P + N)Q}{(N+Q+P+2\sqrt{(1-\gamma)PQ}}), 0 \leq \gamma \leq 1$. The optimality of the points corresponding to $\gamma = 0$ and $\gamma = 1$ is proved in the following theorem.

Theorem 2 *The end points of the optimal tradeoff region for an additive Gaussian channel with state information at the transmitter are $(R, D) = (0, \frac{QN}{(\sqrt{Q}+\sqrt{P})^2+N})$ and $(R, D) = (\frac{1}{2}\log(1+\frac{P}{N}), \frac{Q(N+P)}{Q+N+P})$.*

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