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5.10 THE ENTROPY POWER INEQUALITY AND THE BRUNN-MINKOWSKI INEQUALITY

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The Brunn-Minkowski inequality states that the nth root of the volume of the set sum of two sets in Euclidean n-space is greater than or equal to the sum of the nth roots of the volumes of the individual sets. The entropy power inequality states that the effective variance of the sum of two independent random variables with densities in n-space is greater than or equal to the sums of their effective variances. Formally, the inequalities can be seen to be similar. We are interested in determining whether this occurs by chance or whether there is a fundamental idea underlying both inequalities.

Brunn-Minkowski: Let $V(A)$ be the volume of A. If $A, B \subseteq \mathbf{R}^n$, then $V(A+B) \geq V(A'+B')$, where A', B' are n-spheres such that $V(A') = V(A)$ and $V(B') = V(B)$.

Entropy Power: Let $H(X) = -\int f(x) \ln f(x) dx$, where f is the probability density of X. If X and Y are independent n-vectors with probability densities, then $H(X+Y) \geq H(X'+Y')$, where X' and Y' are independent spherical normal with $H(X') = H(X)$ and $H(Y') = H(Y)$.

REFERENCE

[1] H.M. Costa and T.M. Cover, "On the Similarities Between the Entropy Power and Brunn-Minkowski Inequalities," *IEEE Trans. Inf. Theory,* 30, pp. 837-839 (Nov. 1984).