5.4 GAMBLER'S RUIN:
A RANDOM WALK ON THE SIMPLEX[†]

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It is known that if two gamblers with capitals p and $1 - p$, respec-
tively, engage in a fair game (we can model it by Brownian motion on the
unit interval starting at p) until one of the gamblers goes broke, then the
gambler with initial capital p will win the game with probability p.
Now suppose that there are m gamblers with capitals corresponding to a
point $\mathbf{p}$ in the simplex $p_i \geq 0$, $\sum p_i = 1$. A random walk in the sim-
plex occurs, and the gamblers go broke one by one. Once a gambler goes
broke, he stays broke. What is the induced probability distribution on the
order in which the gamblers go broke?
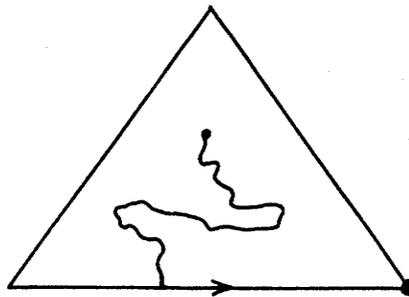
[†] Hajek has exhibited a solution to this problem for $m = 3$ gamblers. See
Chapter VI.