Chapter in *Open Problems in Communication and Computation.*
Springer-Verlag, 1987. T. Cover and B. Gopinath, editors.

5.2. ERGODIC PROCESS SELECTION[†]

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Let $\{ (X_i, Y_i) \}_{i=1}^{\infty}$ be a jointly ergodic stationary stochastic process. Define a selection function $\delta_n : \mathrm{X}^{n-1} \times \mathrm{Y}^{n-1} \rightarrow \{ 0, 1 \}$, $n = 1, 2, \ldots$. We wish to maximize

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} (\delta_i(X_1, \ldots, X_{i-1}, Y_1, Y_2, \ldots, Y_{i-1})\, X_i$$

$$+ (1 - \delta_i(X_1, \ldots, X_{i-1}, Y_1, \ldots, Y_{i-1}))\, Y_i)$$

over all selection functions. Thus δ_i chooses either X_i or Y_i to add to the running average.

It is intuitively clear that

$$\delta_i = \begin{cases} 1, & E\{ X_i \mid \text{Past} \} > E\{ Y_i \mid \text{Past} \} \\ 0, & \qquad\qquad\qquad\quad < \\ \text{arb.}, & \qquad\qquad\qquad\quad = \end{cases}$$

will maximize the above limit of the average return. The proof may be tricky.

[†] See Hajek's solution to this problem under moment constraints in Chapter VI.