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4.16 COMMUNICATION COMPLEXITY OF SHIFTS

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Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, where $X_i \sim$ Bernoulli (1/2). Let $\mathbf{Y} = (X_{T+1}, X_{T+2}, \ldots, X_T)$, where T is uniformly distributed over $\{0, 1, 2, \ldots, n-1\}$. Thus $\mathbf{y}$ is a cyclic T-shift of $\mathbf{x}$. Here $T + k$ is modulo n.

How many bits must $\mathbf{y}$ communicate to $\mathbf{x}$ in order that $\mathbf{x}$ can determine the shift T? We claim that $\log(n+1)$ bits are sufficient. Simply cycle $\mathbf{y}$ until $\sum_{i=1}^{n} y_{i+k} 2^i$ is largest, then transmit k. This works whenever $\mathbf{x},\mathbf{y}$ determine k.

The problem is much harder if $\mathbf{y}' = \mathbf{y} \oplus \mathbf{e}$, where $\mathbf{e} \sim$ Bernoulli (p). The noise in $\mathbf{y}'$ ruins the above approach. Now how many bits are required?