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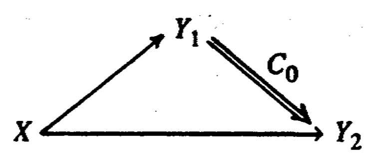
3.15 THE CAPACITY OF THE RELAY CHANNEL

Thomas M. Cover

Departments of Electrical Engineering
and Statistics
Stanford University
Stanford, CA 94305

Consider the following seemingly simple discrete memoryless relay channel:



Here Y_1, Y_2 are conditionally independent and conditionally identically distributed given X, that is, $p(y_1, y_2 \mid x) = p(y_1 \mid x)\, p(y_2 \mid x)$. Also, the channel from Y_1 to Y_2 does not interfere with Y_2. A $(2^{nR}, n)$ code for this channel is a map $x : 2^{nR} \rightarrow X^n$, a relay function $r : Y_1^n \rightarrow 2^{nC_0}$, and a decoding function $g : 2^{nC_0} \times Y_2^n \rightarrow 2^{nR}$. The probability of error is given by

$$P_e^{(n)} = P\{\ g(r(y_1), y_2) \neq W\ \},$$

where W is uniformly distributed over 2^{nR} and

$$p(w, y_1, y_2) = 2^{-nR} \prod_{i=1}^{n} p(y_{1i} \mid x_i(w)) \prod_{i=1}^{n} p(y_{2i} \mid x_i(w)).$$

Let $C(C_0)$ be the supremum of the achievable rates R for a given C_0, that is, the supremum of the rates R for which $P_e^{(n)}$ can be made to tend to zero.

We note the following facts:

1. $C(0) = \sup_{p(x)} I(X; Y_2)$.

2. $C(\infty) = \sup_{p(x)} I(X; Y_1, Y_2)$.

3. $C(C_0)$ is a nondecreasing function of C_0 .

What is the critical value of C_0 such that $C(C_0)$ first equals $C(\infty)$?

REFERENCES

[1] T. Cover and A. El Gamal, "Capacity Theorems for the Relay Channel," *IEEE Trans. Inf. Theory,* IT-25, No. 5, pp. 572-584 (Sept. 1979).