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3.14 CONJECTURE: FEEDBACK DOESN'T HELP MUCH

Thomas M. Cover

Departments of Electrical Engineering
and Statistics
Stanford University
Stanford, CA 94305

Consider the additive Gaussian noise channel with stationary time-dependent noise

$$Y(k) = X(k) + Z(k) ,$$

where $\{ Z(k) \}$ has power spectral density $N(f)$. A $(2^{nR}, n)$ feedback code for such a channel is given by a collection of functions

$$x_k^{(n)}(W, Y_1, Y_2, \ldots, Y_{k-1}) ,$$

$$k = 1, 2, \ldots, n , \quad W \in \{ 1, 2, \ldots, 2^{nR} \}$$

and a decoding function

$$g^{(n)} : \mathbf{R}^n \to \{ 1, 2, \ldots, 2^{nR} \} .$$

Throughout we have a power constraint

$$\mathop{E}_{Y} \frac{1}{n} \sum_{k=1}^{n} (x_k^{(n)} (W, \mathbf{Y}^{k-1}))^2 \leq P , \quad \text{for all } W .$$

Let

$$Y_k = x_k(W, Y^{k-1}) + Z_k ,$$

and let $W^{(n)}$ be uniformly distributed over $\{ 1, 2, \ldots, 2^{nR} \}$. We say that R is an *achievable rate* if there exists a sequence of $(2^{nR}, n)$ codes such that

$$P\{ g^{(n)}(\mathbf{Y}^n) \neq W^{(n)} \} \to 0 ,$$

as $n \to \infty$. The *feedback capacity* C_{FB} is defined to be the supremum of the achievable rates. The *nonfeedback capacity* C_{NFB} is defined to be the supremum of achievable rates over all codes $x_k^{(n)}(W)$ not depending on $\mathbf{Y}$.

Clearly, $C_{FB} \geq C_{NFB}$, with equality if $\{ Z_k \}$ is white noise. In general, I hope that a relation like

$$C_{FB}(P) \leq C_{NFB}(2P) \tag{1}$$

is true.

In particular, the above inequality would imply

$$C_{FB} \leq 2C_{NFB} \tag{2}$$

and

$$C_{FB} \leq C_{NFB} + 1/2 . \tag{3}$$

The first inequality is interesting at low powers; the last at high powers. Inequality (2) was stated by Pinsker and proved by Ebert [1], while (3) has been proved by Pombra and Cover [2]. But is (1) true?

The investigation hinges on maximization of

$$\frac{1}{n} I(W;\mathbf{Y}) = \frac{1}{n} (h(Y_1, \ldots, Y_n) - h(Z_1, \ldots, Z_n))$$

with and without feedback.

REFERENCES

1. P.M. Ebert, "The Capacity of the Gaussian Channel with Feedback," *BSTJ*, pp. 1705-1712 (Oct. 1970).

2. S. Pombra and T. Cover, "Gaussian Feedback Capacity," to be submitted to *IEEE Trans. Inf. Theory.*