Relativistic Information Flow and the Twin Paradox

by Thomas M. Cover

In this year, the 100th anniversary of Einstein's birth, it seems appropriate to reconsider some of the fundamental ideas in relativity theory. I will be discussing some work done with Keith Jarett which shows that there are asymmetries in information flow in the universe which parallel the puzzling asymmetries in the flow of time.

The universe is a computer engaged in computing its next state. In a more usual sense, the universe is a collection of small computing entities, both organic and inorganic, on the surfaces of planets like Earth, which together form distributed computers on the surface of the planet and, in turn, cooperate at the galactic and intergalactic levels. Unfortunately, this computer is cooling as the universe expands, and the computational capacity is therefore being reduced. This sad story is not the focus of our attention.

A distributed computer requires the exchange of information among its elements. In order to understand this information flow, we will investigate the problem of information flow between relativistically moving spaceships. To do this, I would first like to discuss the twin paradox, then the notions of information theory for electromagnetic communication, and finally put these ideas together to show that the communication of information is every bit as asymmetric as the flow of time.

The Twin Paradox

The twin paradox is well known in relativity theory. A traveler in spaceship B leaves spaceship A, travels to a distant star at velocity v, turns around, comes back and finds that he is significantly younger than A. This asymmetry at first appears to be paradoxical. Why couldn't we consider B to be at rest, while A makes the roundtrip? Then A would be younger than B. The answer is that there is a physical experiment that B can perform that will result in a different answer for A. In fact, B feels an initial acceleration and final deceleration that he can measure with physical apparatus. Thus there is not complete symmetry. Nonetheless, the aging does not take place during this acceleration phase, since the acceleration phase can be made a small portion of the time of the entire journey.

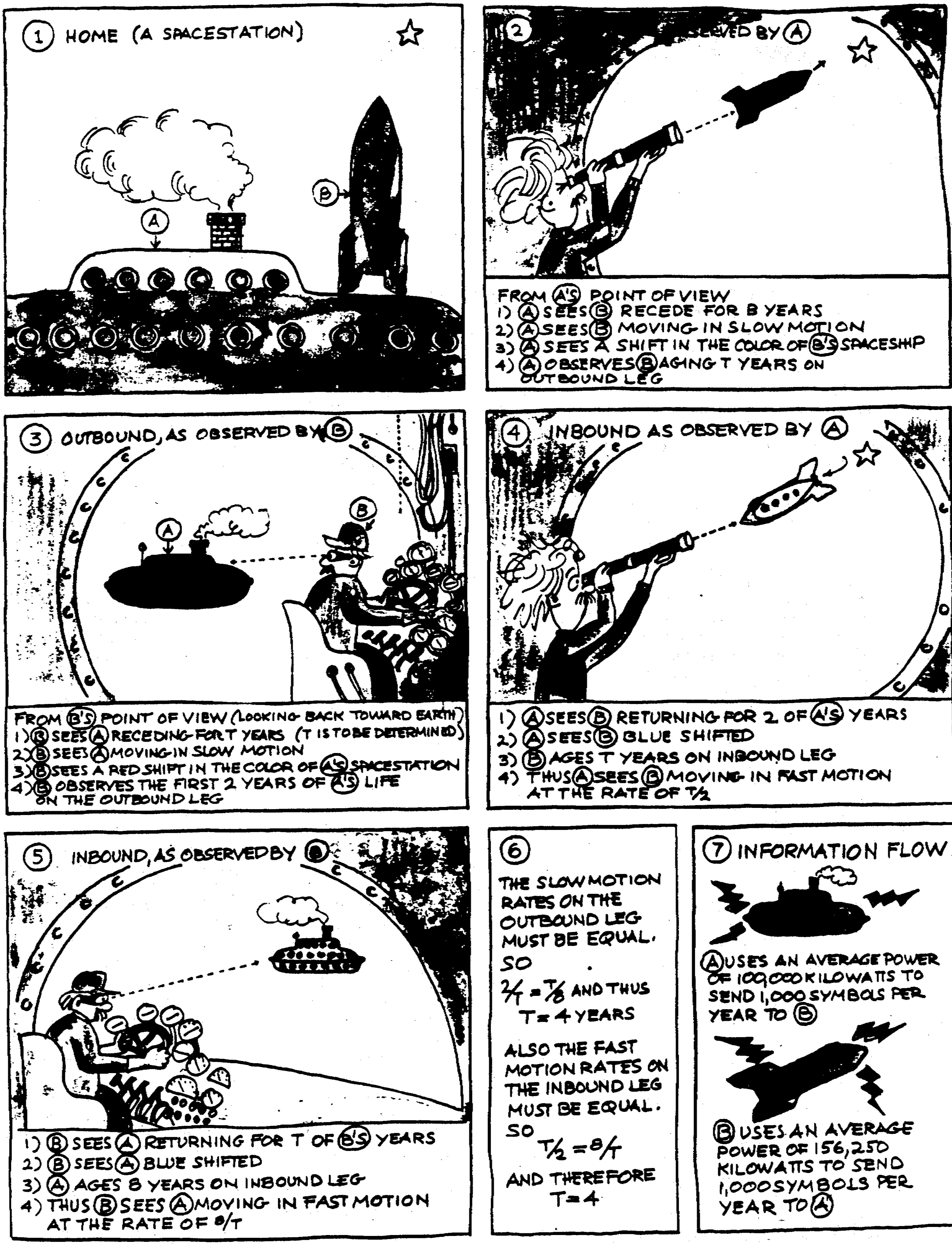
Now I would like to look at the twin problem from first principles and argue that the asymmetry in aging is a necessity of common sense. Or at the very least, I will try to show that the asymmetry in aging is more plausible than other hypotheses that the reader might entertain.

To fix ideas, let A and B be at rest with respect to a star 3 light years away. B then travels from A to the star at 3/5ths the velocity of light. It will therefore take B five of A's years to reach the star. For the moment, let us make the incorrect assumption that A and B both age 5 years on the outgoing leg, and 5 years on the incoming leg, thus making both A and B 10 years older at the end of the roundtrip journey. Now A will see B recede and reach the star after 8 years (5 years of travel necessary to get to the star plus another 3 years for the light at the turning point to reach A.) If A is watching B through a telescope, he will see B moving in slow motion. This slow motion would be the case in both Newtonian and Einsteinian universes simply because B is receding from A. For the moment, assuming that B does in fact age 5 years on the outgoing leg, it follows that A would perceive B moving at the slow motion rate of 5 years of B's life per 8 years of A's observation for an overall slow motion rate of 5/8 of a year per year.

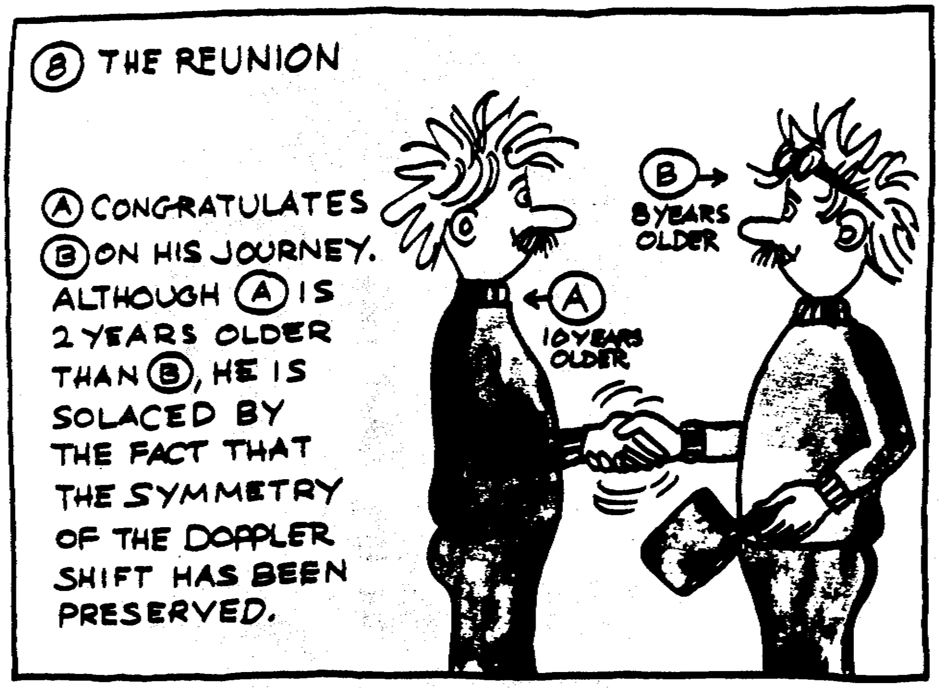
Now let us consider B's observation of A while B is coasting to the star at 3/5ths the velocity of light. When B gets to the star, he will find, since he has been traveling so fast, that only the first 2 years of A's transmission could have reached the star by the time B gets there. This follows because it takes 3 years for light to reach the star and the total travel time in A's frame has been 5 years. Assuming again (incorrectly) that B has aged 5 years on the outbound leg, we see that B will observe 2 years of A's life during 5 years of B's travel. We conclude that B sees A moving in slow motion at rate 2/5.

Now for the problem. Except for the brief initial acceleration of B, we see that both A and B are uniformly moving with respect to one another at 3/5ths the velocity of light. Nonetheless, we see that A perceives B in slow motion at rate 5/8 while B perceives A in slow motion at rate 2/5. What could account for this spectacular asymmetry in the slow motion rates of two uniformly moving rocketships? What could possibly cause one rocketship to be preferred to the other as the ship with the slower perceived slow motion rate? A reader who is disturbed by this asymmetry and wishes to state that there should be none is agreeing with Einstein's principle postulate in relativity and is forced to change some other assumption in order to bring about the intuitively desired symmetry.

Checking back, we see that certain assumptions are physically verifiable. They are B's velocity of 3/5ths the velocity of light, A's age of 5 years, the observation time for A of 8 years, and the 2 years of A's life as observed by B. This leaves only the age of B that we can play with. If we now take the position that the age of B is a derived quantity, we can redo the calculations.



1 HOME (A SPACESTATION)
A
B
2 ERVED BY A
FROM A'S POINT OF VIEW
1) A SEES B RECEDE FOR 8 YEARS
2) A SEES B MOVING IN SLOW MOTION
3) A SEES A SHIFT IN THE COLOR OF B'S SPACESHIP
4) A OBSERVES B AGING T YEARS ON OUTBOUND LEG
3 OUTBOUND, AS OBSERVED BY B
A
B
FROM B'S POINT OF VIEW (LOOKING BACK TOWARD EARTH)
1) B SEES A RECEDING FOR T YEARS (T IS TO BE DETERMINED)
2) B SEES A MOVING IN SLOW MOTION
3) B SEES A RED SHIFT IN THE COLOR OF A'S SPACESTATION
4) B OBSERVES THE FIRST 2 YEARS OF A'S LIFE ON THE OUTBOUND LEG
4 INBOUND AS OBSERVED BY A
1) A SEES B RETURNING FOR 2 OF A'S YEARS
2) A SEES B BLUE SHIFTED
3) B AGES T YEARS ON INBOUND LEG
4) THUS A SEES B MOVING IN FAST MOTION AT THE RATE OF T/2
5 INBOUND, AS OBSERVED BY B
1) B SEES A RETURNING FOR T OF B'S YEARS
2) B SEES A BLUE SHIFTED
3) A AGES 8 YEARS ON INBOUND LEG
4) THUS B SEES A MOVING IN FAST MOTION AT THE RATE OF 8/T
6
THE SLOW MOTION RATES ON THE OUTBOUND LEG MUST BE EQUAL.
SO
2/T = T/8 AND THUS
T = 4 YEARS
ALSO THE FAST MOTION RATES ON THE INBOUND LEG MUST BE EQUAL.
SO
T/2 = 8/T
AND THEREFORE
T = 4
7 INFORMATION FLOW
A USES AN AVERAGE POWER OF 100,000 KILOWATTS TO SEND 1,000 SYMBOLS PER YEAR TO B
B USES AN AVERAGE POWER OF 156,250 KILOWATTS TO SEND 1,000 SYMBOLS PER YEAR TO A



Suppose that B ages T years on the outbound leg. Then A observes T years of B's life in 8 years of A's observation, for a rate of T/8. Similarly, B observes 2 years of A's life in T years of observation for a rate of 2/T. Equating rates, we have T/8 = 2/T, or T = 4 years. Thus we conclude that B has aged 4 years on the outgoing leg, and this preserves the symmetry of slow motion observations of each rocketship on the part of the other.

A similar calculation on the inbound leg shows again that B ages 4 years. Thus A has aged 10 years and B has aged 8 years on the roundtrip. The same calculation, where B travels at the velocity v throughout, yields an age ratio of

$$\gamma = 1/\sqrt{1-v^2/c^2},$$

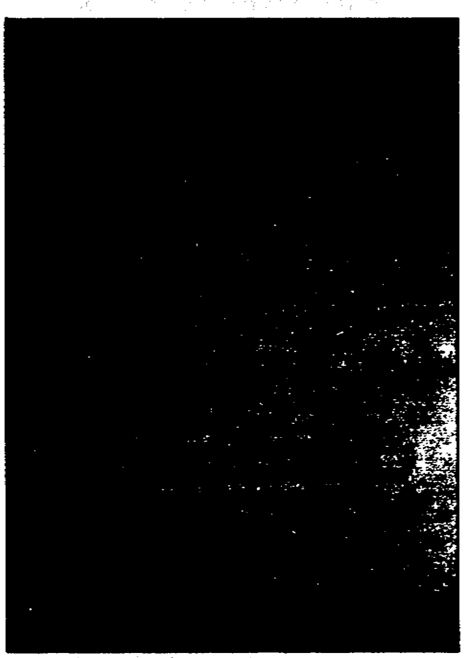
which is the general Lorentz factor for time dilation. In fact, the age of B is the geometric mean of the transmission time and reception time of an impulse of light from A that would strike a mirror at the star at the same time B reaches the star.

Evidently, the assumption of the postulate of relativity in this development seems more plausible than the assumption of equal aging. Indeed, the asymmetry in aging has been demonstrated many times in physical experiments.

Relativistic Information Flow

Now for a new ingredient. Let us suppose that A and B are attempting to communicate with each other. If A and B are at rest and communicating electromagnetically, the so-called Shannon channel capacity for their communication is $C = W \log(1 + P/NW)$ where W is the signalling bandwidth in cycles per second, P is the signalling power, and N is the noise spectral density. The assumption is that any electromagnetic information received by the receiver has added to it some white Gaussian noise with spectral density N watts per cycle per second. The channel capacity C is the number of distinguishable bits per second with arbitrarily small probability of error.

Another way to look at it is this. There are 2TW independent signals



of bandwidth W in time duration T. It can be shown that one can put 2^{CT} signals in a sphere of radius $\sqrt{PT}$ in 2TW-dimensional space in such a manner that the Gaussian additive noise is extremely unlikely to cause one received signal to be confused with another. On the initial Mars spaceprobes, for example, the communication rates were in the 100's of bits per second. Advancing technology (more power and bandwidth, and less receiver noise) has increased this rate to the tens of thousands of bits per second on the recent Jupiter spaceprobe.

Now what happens to all of this at relativistic velocities? First, even the notion of channel capacity, since it is in bits per unit time, has no absolute meaning because of the time ambiguity. Suppose for a moment that the relativistic Doppler factor is α. (Incidentally, in the previous example, $\alpha = 1/2$ on the outgoing leg and $\alpha = 2$ on the incoming leg.) Then the receiver bandwidth becomes $W' = \alpha W$. The capacity becomes $C' = \alpha C$. The received power becomes $P' = \alpha^2 P$ and the additive receiver noise spectral density N remains the same. Thus we find that the receiver channel capacity is $C' = W' \ln(1 + P'/NW') = \alpha W \ln(1 + \alpha P/NW)$. Correcting for the transmitter's clock, we see that the transmitter can send at rate

$$C = W \ln(1 + \alpha P/NW) \text{ bits/second.}$$

Proceeding with this analysis, we find that if the transmitter sends at constant rate C, then he must change his power by the factor $1/\alpha$. Now if A sends pulses to B at A's rate $1/\alpha$, then the pulses arrive with clocklike regularity at B. But the power varies like $1/\alpha$. Thus the energy sent by the stay-at-home A to the traveler B is proportional to B's age T_B. On the other hand, the number of bits sent from A to B is proportional to T_A. Similarly, B's energy and number of bits sent are proportional to T_A and T_B respectively. Thus the energy per bit sent for A relative to the energy per bit sent by B is $\gamma^2 = (T_B/T_A)^2$. For example, if B travels at such a velocity that he ages only one tenth as fast as A, then he requires 10 times as much energy to send 1/10 as much information for a total inefficiency ratio of 100. In fact, it can be shown that the above inefficiency ratio holds for arbitrary

Albert Einstein presented his special theory of relativity in a 1905 paper appearing in the Annalen der Physik, *under the title "Zur Elektrodynamik bewegter Körper" (On the Electrodynamics of Moving Bodies). From this, the translation of Einstein's most famous paper, we reprint here the section explaining what later became famous as the "twin paradox." Inspection of Einstein's original 1905 article shows, contrary to popular opinion, that the twin paradox can be developed using only the notion of special relativity without recourse to the general theory.*

Further, we imagine one of the clocks which are qualified to mark the time t when at rest relatively to the stationary system, and the time τ when at rest relatively to the moving system, to be located at the origin of the coordinates of k, and so adjusted that it marks the time τ. What is the rate of this clock, when viewed from the stationary system?

Between the quantities x, t, and τ, which refer to the position of the clock, we have, evidently, $x = vt$ and

$$\tau = \frac{1}{\sqrt{(1 - v^2/c^2)}}(t - vx/c^2)$$

Therefore,

$$\tau = t\sqrt{(1-v^2/c^2)} = t-(1-\sqrt{(1-v^2/c^2)})t$$

Whence it follows that the time marked by the clock (viewed in the stationary system) is slow by $1-\sqrt{(1-v^2/c^2)}$ seconds per second, or—neglecting magnitudes of fourth and higher order—by $\frac{1}{2}v^2/c^2$.

From this there ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity v along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $\frac{1}{2}tv^2/c^2$ (up to magnitudes of fourth and higher order), t being the time occupied in the journey from A to B.

It is at once apparent that this result still holds good if the clock moves from A to B in any polygonal line, and also when the points A and B coincide.

If we assume that the result proved for a polygonal line is also valid for a continuously curved line, we arrive at this result: If one of two synchronous clocks at A is moved in a closed curve with constant speed until it returns to A, the journey lasting t seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be $\frac{1}{2}tv^2/c^2$ seconds slow. Thence we conclude that a balance-clock* at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions.

*Not a pendulum-clock, which is physically a system to which the Earth belongs. This case had to be excluded.

roundtrips even in the presence of accelerations and gravitational effects.

In fact, we can go a little bit farther. In some poetic sense, we can consider the twin who ages more slowly to be traveling into his brother's future. Then the result we just mentioned shows that it is less efficient to send information into the future than it is for the future to send information into the past.

Summary

In Joseph Haldeman's science fiction novel, "The Forever War," spaceships from Earth engage in a battle in a distant galaxy. When the spaceships resume the battle after a two-year roundtrip to Earth, they are dismayed to find that the enemy technology is much advanced. In fact, the enemy is 100 years older, having stayed at home the entire time!

This discussion of the inefficiencies of communication between traveler and stay-at-home is merely the first page in the future space traveler's communication handbook.

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