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OPEN PROBLEMS IN INFORMATION THEORY

Thomas M. Cover
Stanford University
Stanford, California

1. A Question on the Prediction of Ergodic Processes

The statement that "we can learn the statistics of an ergodic process from a sample function with probability 1" is being investigated for operational significance.

Let $\{X_n\}_{n=-\infty}^{\infty}$ be a stationary binary ergodic process with conditional probability distributions $p(x_{n+1}|x_n,\ldots,x_1)$, $n=1,2,\ldots$. We know that we can learn the statistics with probability 1, but can we learn p fast enough? In other words, does there exist an estimate $\hat{p} : X \times X^* \to [0,1]$, X^* = collection of all finite strings, for which

$$\hat{p}(X_{n+1}|X_n,\ldots,X_1) - p(X_{n+1}|X_n,\ldots,X_1) \to 0$$

with probability 1? Does there also exist a predictor $\hat{p}$ yielding the convergence of

$$\hat{p}(X_0|X_{-1},X_{-2},\ldots,X_{-n}) \to p(X_0|X_{-1},X_{-2},\ldots) .$$

Since the statement of this problem, Bailey and Ornstein have obtained some as yet unpublished results on this question that indicate a negative answer to the first question and a positive answer to the second.

2. The Simplex Conjecture

Prove that the maximum content of intersection of an n-dimensional spherical simplex of content Ω with an n-dimensional spherical cap is achieved by a regular spherical simplex centered on the axis of the cap. This will yield the optimality of the simplex coding conjecture.

3. Hypothesis Testing with Finite Memory

What is the structure of the ε-optimal finite state machine for testing k hypotheses? The solution is known for $k = 2$, [1].

4. Broadcast Channel Capacity

The capacity region for the broadcast channel remains unknown [2,3,4]. It seems very likely that the answer will be "information theoretic" and pretty.

5. Multiple Access Channels with Feedback

Gaarder and Wolf [5] have shown that feedback increases channel capacity for discrete memoryless multiple access channels. (See also Cover, Leung [6]). This may be seen from the fact that feedback gives the two senders knowledge of each other's messages before the receiver obtains total knowledge, thus allowing cooperative signalling for subsequent transmissions. What is the capacity region?

6. Non Increase of Capacity for Broadcast Channels with Feedback

Shannon has shown that the capacity of an ordinary discrete memoryless channel is not increased by feedback. The same result should hold for discrete memoryless broadcast channels.

7. A Converse for Parallel Degraded Broadcast Channels

The capacity region for a degraded broadcast channel has been established by Bergmans and Gallager. It is known that the capacity C of two parallel ordinary discrete memoryless channels of capacities C_1 and C_2 is given by

$$2^C = 2^{C_1} + 2^{C_2} .$$

(That is, the number of effectively noise free letters $M_1 = 2^{C_1}$ and $M_2 = 2^{C_2}$ add together.)

It is quite easy to construct a counterpart achievable rate region for independent messages for two parallel degraded broadcast channels,

$$(X, Y_1 \times Y_2, p(y_1,y_2|x)) ,$$

$$(X', Y_1' \times Y_2', p(y_1',y_2'|x)) .$$

Let $C(\underline{\alpha}) = \sup_{R_1,R_2} (\alpha_1 R_1 + \alpha_2 R_2)$ over all achievable rates (R_1,R_2). Then $2^{C(\underline{\alpha})} = 2^{C_1(\underline{\alpha})} + 2^{C_2(\underline{\alpha})}$ is achievable.

If both channels are degraded in the same direction (i.e. $y_1 > y_2$, $y_1' > y_2'$), then the parallel channel is degraded and the Gallager converse establishes $C(\underline{\alpha})$ to be the capacity region.

A question remains: If the channels are degraded in opposite direction, is $C(\underline{\alpha})$ the capacity region?

8. A Converse for Parallel Multiple Access Channels

Let $C(\underline{\alpha})$ denote the parametric form of the capacity region for a discrete memoryless multiple access channel. Does

$$2^{C(\underline{\alpha})} = 2^{C_1(\underline{\alpha})} + 2^{C_2(\underline{\alpha})}$$

define the capacity region for a parallel discrete memoryless multiple access channel? Clearly $C(\underline{\alpha})$ is achievable. This question should be easier than the previous question.

References

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