Performance of Universal Portfolios in the Stock Market[1]

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Abstract — **We compare the theoretical and empirical performance of horizon-free universal portfolios for a large number of stock pairs using real stock market data in two scenarios: with and without side information, and with and without short selling.**

I. Summary

The horizon-free μ-weighted universal portfolio is a sequential investment algorithm that has been shown to perform as well as the best constant rebalanced portfolio to first order in the exponent (cf. [1]). Additionally, a number of theoretical properties of the universal portfolio have been derived. We are interested in the performance of the universal portfolio in the actual stock market and how this performance compares with the theory. To this end, we determine the performance of horizon-free universal portfolios for a large number of stock pairs using real stock market data in two scenarios: with and without side information, and with and without short selling.

First, we observe for a large number of stock pairs that the n-day universal portfolio return $\hat{S}_n$ consistently performs near the best achievable constant rebalanced portfolio return S_n^*, and a factor of 28 better than the minimax lower bound of $V_n S_n^*$ established in [3], where $V_n = \left[\sum \binom{n}{n_1,\ldots,n_m} 2^{-nH(n_1/n,\ldots,n_m/n)}\right]^{-1}$, thus indicating that the market is not maximally hostile. We also compute the ratio $S_n^*/\hat{S}_n$ for real data and compare it to the theoretical asymptote $\frac{|J_n|^{(1/2)}}{(m-1)!(2\pi/n)^{(m-1)/2}}$, where m is the number of stocks and J_n is the sensitivity matrix (cf. [1].).

We then extend the universal portfolio by using side information to assign days to certain states, and utilize state constant rebalanced portfolios as in [2]. For a state constant rebalanced portfolio the trading days are divided up into subsequences based on the state information. A constant rebalanced portfolio is then used independently on each subsequence of days. One example of side information y_i for a pair of stocks is to assign each day i to one of two states, 1 or 2, corresponding to the stock with the larger windowed moving average of price relatives for the last k days. The best constant rebalanced portfolio b_i^* and the universal portfolio $\hat{b}_i$ are based on the current and past state information y^i and past price relatives $\mathbf{x}^{i-1}$. The best constant rebalanced portfolio return $S_n^*(\mathbf{x}^n|y^n)$ is the product of the best constant rebalanced portfolio returns for the subsequence of days associated with each state:

$$S_n^*(\mathbf{x}^n|y^n) = \left(\max_b \prod_{\substack{i=1\\ i:y_i=1}}^{n} b^t X_i\right)\left(\max_b \prod_{\substack{i=1\\ i:y_i=2}}^{n} b^t X_i\right).$$

Similarly, $\hat{S}_n$ is the product of the wealth factors associated with the independent running of the universal portfolio on the subsequences of trading days associated with each of the states. For actual stock market data we observe that this simple nonanticipative algorithm achieves factors as large as 10^5 for some stock pairs over a twenty year period. When the side information of the windowed moving average is used for two portfolios without short selling, the log optimal portfolio b^* for each state often exhibits a "bang-bang" effect, where all the wealth is allocated to a single stock. This "bang-bang" effect often has all the wealth pouring into the stock which has been underperforming. Additionally, the "bang-bang" effect indicates that even more wealth can be generated by selling one stock short and buying more of the other. Consequently, we analyze the effect of short selling and the tradeoffs between return and amount of leverage. We also compare the performance of the μ-weighted universal portfolio with the exponentiated gradient universal portfolio as in [4]. Next, we look at the performance of the universal portfolio with and without side information, and with and without short selling for a portfolio of fifty stocks.

Finally, we explore the use of several heuristic methods for increasing the rate at which the universal portfolio learns the stock market. These methods include several ways of creating a fake market associated with the actual market, computing portfolios in the fake market, and mapping portfolios from the fake market back to the actual market.

References

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