Information efficiency in investment

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Abstract — **We answer the question, what should we say about V when we want to gamble on X, and what is it worth? If $V = X$, we show that every bit of description at rate R is worth a bit of increase $\Delta(R)$ in the doubling rate. Thus the efficiency $\Delta(R)/R$ is equal to 1. For general V, we provide a single letter characterization for $\Delta(R)$. When applied specifically to jointly normal (V, X) with correlation ρ, we find the initial efficiency $\Delta^{'}(0)$ is ρ^2. If V and X are Bernoulli random variables connected by a binary symmetric channel with parameter p, the initial efficiency is $(1-2p)^2$.**

We finally show how much increase in doubling rate is possible when the sender can provide R bits of information about V and side information S is available only to the investor.

Summary

Suppose we are interested in gambling on the outcome of a random variable X. The gambling consists of betting a proportion of wealth $b(x)$ on the outcome x. We would like to maximize the doubling rate, which is the growth rate of wealth when the gambler uses a fixed betting strategy on independent realizations of X. It is well known that Kelly gambling, which is to bet in proportion to the probability mass function of X, is optimal.

Now suppose we allow a description of X at rate R bits per symbol. Let $\Delta(R)$ be the maximum increase in the doubling rate of wealth for transmission rate of R. We prove that $\Delta(R) = R$. Any bit of information one sends about X is worth a bit of increase in the doubling rate.

We next consider the effectiveness of sending information when side information S is available to the investor but not to the encoder. The gambler combines this side information with the partial description of X to form the bet.

Theorem 1 *If X is described at rate R, and side information S is available to the gambler, then,*

$$\Delta(R) = R.$$

We ask what information should be given about a correlated random variable V if we want to help the investor gamble on X. This problem shows some similarities to source coding with side information [4, 1]. The encoder sends R bits about V and the investor uses this information to gamble on X. Here maximal efficiency is not generally possible.

Theorem 2 *When the encoder observes V correlated with X,*

$$\Delta(R) = \max_{p(\tilde{v}|v,x):\ I(\tilde{V};V)\leq R,\ \tilde{V}\to V\to X} I(\tilde{V};X).$$

We establish certain properties of $\Delta(R)$ using entropy maximization results from Witsenhausen and Wyner [3].

Next, we find the increase in the doubling rate when the encoder sends information at rate R about a correlated random variable V with side information S present only at the investor. The investor uses these R bits together with the side information S to invest in the outcome of X.

Theorem 3 *When the encoder observes V, and side information S is available at the investor,*

$$\Delta(R) = \max_{p(\tilde{v}|v,x,s):\ I(V;\tilde{V}|S)\leq R,\ \tilde{V}\to V\to(X,S)} I(\tilde{V};X|S)$$

Finally, we investigate the efficiency of descriptions based on correlated variables. If X and V are both Bernoulli($\frac{1}{2}$) and are associated by a binary symmetric channel with crossover probability p, it can be shown that $\Delta(R)$ has a derivative of $(1-2p)^2$ at $R=0$. Thus, even the most effective description of V relative to the investment in X pays off at the rate of only $(1-2p)^2$ bits of doubling per bit of description.

Now suppose that V and X are jointly Gaussian with correlation ρ. In this case the initial efficiency, $\Delta^{'}(0)$, is equal to ρ^2.

The functional form of $\Delta(R)$ for binary and Gaussian random variables will be developed in [2]. Also, the relationship between the derivative of $\Delta(R)$ at $R=0$ and the Renyi maximal correlation of V and X will be investigated.

References

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