Conflict Between State Information and Intended Information

Thomas M. Cover*
Stanford University, Electrical Engineering and Statistics, Stanford, CA 94305-9510, U.S.A.

Abstract — **A channel with state known at the sender can be used to send information. It can also be used to reveal the underlying state of the channel to the receiver. We investigate the inherent conflict in doing both.**

Summary: The conflict between state information and transmitted information is nicely exemplified by a magic trick. In this trick, five cards are dealt at random to player A. Player A inspects these five cards (the state of the channel) and withholds one of the cards, giving the other four to player B (the receiver). Player B then names the missing card in player A's hand. The question is, how does player A transmit information to B about the missing card?

We specialize this problem to a channel in which the state is given by m iid uniform [0,1] random variables. The action consists of deleting one of them, and the received signal is the unordered set of the remaining $m-1$ points. We ask for the channel capacity of this channel and also for the amount of information that can be sent about the unknown state of the channel.

Finally we examine a channel $Y = X+S+Z$, with signal X of power P, additive Gaussian noise Z, and additive Gaussian noise S, where the state S is known to the sender and not to the receiver. The capacity of this channel is known from Costa's work [1] on writing on dirty paper to be $\frac{1}{2}\log(1+\frac{P}{N})$, where P is the signal power and N is the power of the additive noise Z. Apparently, the power of the state information S is irrelevant. Thus the channel capacity is unaffected by the state of the channel just so long as the sender knows it.

But what if we wish to send the state of the channel to the receiver and we are uninterested in sending any extra independent information? We discuss the extent to which there is a conflict between sending the state of the channel and sending independent information.

Acknowledgements

This analysis has grown out of discussions with Josh Singer, Assaf Zeevi, and Michael Baer.

References

[1] M. Costa, Writing on Dirty Paper, *IEEE Transactions on Information Theory*, IT-29:439-441, May 1983.

*Supported by NSF grant #NCR-9628193 and JSEP contract #DAAG55-97-1-0115