An Outer Bound to the Capacity Region of the Broadcast Channel

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***Abstract*— An outer bound to the capacity region of the two-receiver discrete memoryless broadcast channel is given. The outer bound is tight for all cases where the capacity region is known. When specialized to the case of no common information, this outer bound is shown to be contained in the Körner-Marton outer bound. This containment is shown to be strict for the *binary skew-symmetric* broadcast channel. Thus, this outer bound is in general tighter than all other known outer bounds on the discrete memoryless broadcast channel.**

1. Introduction

We consider a discrete memoryless (DM) broadcast channel where the sender wishes to communicate common as well as separate messages to two receivers [2]. Formally, the channel consists of an input alphabet $\mathcal{X}$, output alphabets $\mathcal{Y}$ and $\mathcal{Z}$, and a probability transition function $p(y,z|x)$. A $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ code for this channel consists of (i) three messages (M_0, M_1, M_2) uniformly distributed over $[1, 2^{nR_0}] \times [1, 2^{nR_1}] \times [1, 2^{nR_2}]$, (ii) an encoder that assigns a codeword $x^n(m_0, m_1, m_2)$, for each message triplet $(m_0, m_1, m_2) \in [1, 2^{nR_0}] \times [1, 2^{nR_1}] \times [1, 2^{nR_2}]$, and (iii) two decoders, one that maps each received y^n sequence into an estimate $(\hat{m}_0, \hat{m}_1) \in [1, 2^{nR_0}] \times [1, 2^{nR_1}]$ and another that maps each received z^n sequence into an estimate $(\hat{\hat{m}}_0, \hat{m}_2) \in [1, 2^{nR_0}] \times [1, 2^{nR_2}]$.

The probability of error is defined as

$$P_e^{(n)} = \mathrm{P}(\hat{M}_0 \neq M_0 \text{ or } \hat{\hat{M}}_0 \neq M_0 \text{ or } \hat{M}_1 \neq M_1 \text{ or } \hat{M}_2 \neq M_2).$$

A rate tuple (R_0, R_1, R_2) is said to be achievable if there exists a sequence of $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n)$ codes with $P_e^{(n)} \to 0$. The capacity region of the broadcast channel is the closure of the set of achievable rates.

The capacity region for this channel is known only for some classes, including the degraded, less noisy, more capable, and semi-deterministic channels (see [4] for a survey of these results). Additionally, general inner bounds by Cover [3], van der Meulen [9] and Marton [7] and outer bounds by Körner and Marton [7] and Sato [8] have been established. Furthermore, the Körner and Marton [7] outer bound is known to be tight for all cases where capacity is known.

In this paper we introduce an outer bound on the capacity region of the DM broadcast channel based on results in [5] and show that it is strictly tighter than existing outer bounds. The outer bound is presented in the next section. In Section 3, the outer bound is specialized to the case of no common information. In Section 3-C, we show that when there is no common information, our outer bound is contained in the Körner-Marton bound and in Section 4 we show that this containment is strict.

2. Outer bound

The following is an outer bound to the capacity region of the two-receiver DM broadcast channel.

Theorem 2.1: The set of rate triples (R_0, R_1, R_2) satisfying

$$\begin{aligned} R_0 &\leq \min\{I(W;Y), I(W;Z)\} \\ R_0 + R_1 &\leq I(U,W;Y) \\ R_0 + R_2 &\leq I(V,W;Z) \\ R_0 + R_1 + R_2 &\leq \min\{I(U,W;Y) + I(V;Z|U,W), \\ &\qquad I(V,W;Z) + I(U;Y|V,W)\}. \end{aligned}$$

for some joint distribution of the form

$$p(u,v,w,x) = p(u)p(v)p(w|u,v)p(x|u,v,w),$$

constitutes an outer bound to the capacity region for the DM broadcast channel.

Proof: The arguments are essentially the same as those used in the converse for the more capable broadcast channel class [5]. We make the identification $U_i = M_1, V_i = M_2$ and $W_i = M_0, Y^{i-1}, Z_{i+1}^n$. The

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independence of the messages M_1 and M_2 implies the independence of the auxiliary random variables U and V as specified. ■

3. Outer Bound with No Common Information

Note that the outer bound given in Theorem 2.1 immediately leads to the following outer bound for the case when there is no common information, i.e., $R_0 = 0$.

The set of all rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq I(U, W; Y) \\ R_2 &\leq I(V, W; Z) \\ R_1 + R_2 &\leq \min\{I(U, W; Y) + I(V; Z|U, W), \\ &\qquad I(V, W; Z) + I(U; Y|V, W)\}, \end{aligned} \tag{3.1}$$

for some joint distribution of the form $p(u, v, w, x) = p(u)p(v)p(w|u, v)p(x|u, v, w)$, constitutes an outer bound on the capacity of the DM broadcast channel with no common information.

The following theorem gives a *possibly weaker* outer bound that we will use in the rest of the paper.

Theorem 3.1: Consider the DM broadcast channel with no common information. The set of rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq I(U; Y) \\ R_2 &\leq I(V; Z) \\ R_1 + R_2 &\leq \min\{I(U; Y) + I(V; Z|U), \\ &\qquad I(V; Z) + I(U; Y|V)\}, \end{aligned}$$

for some choice of joint distributions $p(u, v, x) = p(u, v)p(x|u, v)$, constitutes an outer bound to the capacity region for the DM broadcast channel with no common information.

Proof: This follows by redefining U as (U, W) and V as (V, W) in equation (3.1). ■

In the following subsections we prove results that aid in the evaluation of the above outer bound.

A. X - Deterministic function of U, V suffices

Denote by $\mathcal{C}$ the outer bound in Theorem 3.1 and let $\mathcal{C}_d$ be the same bound but with X restricted to be a deterministic function of U, V, i.e. $\mathrm{P}(X = x|U = u, V = v) = 0$ or 1. We now show that these two bounds are identical.

Lemma 3.2: $\mathcal{C} = \mathcal{C}_d$.

Note that it suffices to show that $\mathcal{C} \subset \mathcal{C}_d$. Let $\mathrm{P}(U = u, V = v) = p_{uv}$ and let $\mathrm{P}(X = x|U = u, V = v) = \delta^x_{uv}$. W.l.o.g., let $\mathcal{X} = \{0, 1, .., m-1\}$.

Remark 3.3: We shall use the following notation. If the random variables X, Y, Z are generated using auxiliary random variables U', V' according to $p(u', v')$ and $p(x|u', v')$, then we denote them by X', Y', Z'.

Now, construct random variables U^*, V^* having cardinalities $m\|U\|, m\|V\|$ as follows: Split each value u taken by U into m-values $u_0, ..., u_{m-1}$ and similarly split each value v taken by V into m-values $v_0, ..., v_{m-1}$. Let[1]

$$\begin{aligned} \mathrm{P}(U^* = u_i, V^* = v_j) &= \frac{1}{m}\mathrm{P}(U = u, V = v, X = (i-j)_m) \\ \mathrm{P}(X = k|U^* = u_i, V^* = v_j) &= \begin{cases} 1 & \text{if } k = (i-j)_m \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{3.2}$$

Lemma 3.4: The following hold:

(i) $\mathrm{P}(U^* = u_i) = \frac{1}{m}\mathrm{P}(U = u)$ for $0 \leq i \leq m-1$.
(ii) $\mathrm{P}(V^* = v_i) = \frac{1}{m}\mathrm{P}(V = v)$ for $0 \leq i \leq m-1$.
(iii) $\mathrm{P}(X^* = k|U^* = u_i) = \mathrm{P}(X = k|U = u)$ for $0 \leq k, i \leq m-1$.
(iv) $\mathrm{P}(X^* = k|V^* = v_i) = \mathrm{P}(X = k|V = v)$ for $0 \leq k, i \leq m-1$.

Proof: Observe that,

$$\begin{aligned} \mathrm{P}(U^* = u_i) &= \sum_{v \in \mathcal{V}} \sum_{j=1}^{m} \mathrm{P}(U^* = u_i, V^* = v_j) \\ &= \sum_{v \in \mathcal{V}} \sum_{j=1}^{m} \frac{1}{m}\mathrm{P}(U = u, V = v, X = (i-j)_m) \\ &= \sum_{v \in \mathcal{V}} \frac{1}{m}\mathrm{P}(U = u, V = v) \\ &= \frac{1}{m}\mathrm{P}(U = u). \end{aligned}$$

Proof of (ii) follows similarly. To show (iii), consider

$$\begin{aligned} \mathrm{P}(X^* = k|U^* = u_i) &= \sum_{v \in \mathcal{V}} \sum_{j=1}^{m} \mathrm{P}(X^* = k, V^* = v_j|U^* = u_i) \\ &\overset{(a)}{=} \sum_{v \in \mathcal{V}} \mathrm{P}(X^* = k, V^* = v_{(i-k)_m}|U^* = u_i) \\ &= \sum_{v \in \mathcal{V}} \mathrm{P}(V^* = v_{(i-k)_m}|U^* = u_i) \\ &= \sum_{v \in \mathcal{V}} \frac{1}{m}\frac{\mathrm{P}(X = k, V = v, U = u)}{P(U^* = u_i)} \\ &\overset{(b)}{=} \sum_{v \in \mathcal{V}} \mathrm{P}(X = k, V = v|U = u) \\ &= \mathrm{P}(X = k|U = u), \end{aligned}$$

[1]The notation $(l)_m$ is used to denote the remainder of the operation of dividing l by m; also referred to as the $\mod$ function.

where (a) follows from the fact that the rest of the terms are zero by construction and (b) follows from (i) using the fact that $\mathrm{P}(U^* = u_i) = \frac{1}{m}\mathrm{P}(U = u)$. The proof of (iv) follows similarly. ■

The following corollary follows from: the above lemma, the fact that X^* is a deterministic function of (U^*, V^*), and the fact that $(U^*, V^*) \to X^* \to (Y^*, Z^*), (U, V) \to X \to (Y, Z)$ form Markov chains with $p(y^*, z^*|x^*) = p(y, z|x)$. The proofs are straightforward and omitted.

Corollary 3.5: The following hold:

(i) $P(X^* = i) = P(X = i) \;\text{ for }\; 0 \le i \le m-1$
(ii) $H(Y^*|U^*) = H(Y|U)$
(iii) $H(Z^*|U^*) = H(Z|U)$
(iv) $H(Y^*|V^*) = H(Y|V)$
(v) $H(Z^*|V^*) = H(Z|V)$
(vi) $H(Y^*|U^*, V^*) = H(Y^*|X^*) = H(Y|X) \le H(Y|U, V)$
(vii) $H(Z^*|U^*, V^*) = H(Z^*|X^*) = H(Z|X) \le H(Z|U, V)$.

We are now ready to prove Lemma 3.2

Proof of Lemma 3.2: Corollary 3.5 implies that

$$\begin{aligned}
I(U; Y) &= I(U^*; Y^*), \\
I(V; Z) &= I(V^*; Z^*), \\
I(U; Y|V) &\le I(U^*; Y^*|V^*), \\
I(V; Z|U) &\le I(V^*; Z^*|U^*), \\
I(X; Y|V) &= I(X^*; Y^*|V^*), \text{ and} \\
I(X; Z|U) &= I(X^*; Z^*|U^*).
\end{aligned} \tag{3.3}$$

Thus $\mathcal{C} \subset \mathcal{C}_d$, which completes the proof of Lemma 3.2. ■

Thus the outer bound in Theorem 3.1 can be re-expressed as follows.

Lemma 3.6: The set of rate pairs satisfying

$$\begin{aligned}
R_1 &\le I(U; Y) \\
R_2 &\le I(V; Z) \\
R_1 + R_2 &\le \min\{I(U; Y) + I(X; Z|U), \\
&\qquad I(V; Z) + I(X; Y|V)\},
\end{aligned}$$

for some distribution $p(u, v, x) = p(u, v)p(x|u, v)$, where $p(x|u, v) = 0$ or 1, constitute an outer bound on the DM broadcast channel with no common information.

Remark 3.7: Note that the constraint $p(x|u, v) = 0$ or 1 is useful for evaluating the region but can be removed from the definition, since, as before, for any (U, V, X) one can construct random variables (U^*, V^*, X^*) according to equation (3.2); and from equation (3.3) it follows that the region (R_1, R_2) evaluated by (U, V, X) is identical to that evaluated by (U^*, V^*, X^*).

B. Cardinality bounds on U,V

We now establish bounds on the cardinality of U, V. From Remark 3.7, we know that $p(x|u, v)$ can be arbitrary.

Fact 3.8: Given $p(u), p(x|u), p(v), p(x|v)$, if $p(x)$ is consistent, i.e.,

$$\sum_{u \in \mathcal{U}} p(X = x|u)p(u) = \sum_{v \in \mathcal{V}} p(X = x|v)p(v)$$

for every $x \in \mathcal{X}$, then there exist $p(u, v)$ and $p(x|u, v)$ that are consistent with $p(u), p(x|u), p(v), p(x|v)$.

Remark 3.9: A canonical way to generate such a joint triple is to generate X according to $p(x)$ and then generate U, V conditionally independent of X according to $p(u|x)$ and $p(v|x)$.

Now for any $U \to X \to (Y, Z)$, using standard arguments from [1], there exists a (U^*, X^*) with $\|U^*\| \le \|X\| + 2$, such that $I(U; Y) = I(U^*; Y^*)$ and $I(X; Z|U) = I(X^*; Z^*|U^*)$. Similarly, there is a V^* with $\|V^*\| \le \|X\| + 2$, such that $I(V; Z) = I(V^*; Z^*)$ and $I(X; Y|V) = I(X^*; Y^*|V^*)$. From Fact 3.8, it follows that there exists a triple (U^*, V^*, X^*) consistent with the pairs (U^*, X^*) and (V^*, X^*). Thus we can assume that $\|U\| \le \|X\| + 2, \quad \|V\| \le \|X\| + 2$.

C. Comparison to Körner-Marton outer bound

The outer bound of Körner and Marton [7] is given by $\mathcal{O} = \mathcal{O}_y \cap \mathcal{O}_z$, where $\mathcal{O}_y$ is the set of rate pairs (R_1, R_2) satisfying

$$\begin{aligned}
R_1 &\le I(X; Y) \\
R_2 &\le I(V; Z) \\
R_1 + R_2 &\le I(V; Z) + I(X; Y|V),
\end{aligned}$$

for some distribution $p(v)p(x|v)$ and $\mathcal{O}_z$ is the set of rate pairs (R_1, R_2) satisfying

$$\begin{aligned}
R_2 &\le I(X; Z) \\
R_1 &\le I(U; Y) \\
R_1 + R_2 &\le I(U; Y) + I(X; Z|V),
\end{aligned}$$

for some distribution $p(u)p(x|u)$.

From Lemma 3.6, it is clear that $\mathcal{C} \subset \mathcal{O}_y$ and $\mathcal{C} \subset \mathcal{O}_z$. Hence

$$\mathcal{C} \subset \mathcal{O} = \mathcal{O}_y \cap \mathcal{O}_z$$

and $\mathcal{C}$ is in general contained in the Körner-Marton outer bound. In the following section, we show that the containment is strict for the *binary skew-symmetric* broadcast channel.

4. Binary Skew-Symmetric Channel

Consider the Binary Skew-Symmetric Channel (BSSC) shown in Figure 1, which was introduced by Hajek and Pursley [6]. For the rest of this paper we assume that $p = \frac{1}{2}$, though a similar analysis can be carried out for any other choice of p.

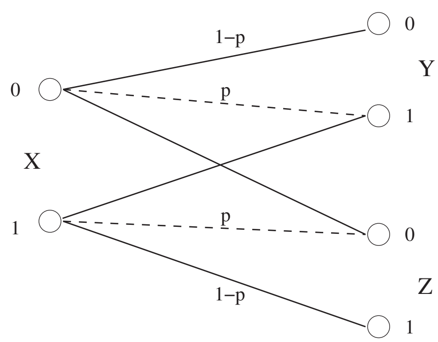

Fig. 1. Binary Skew Symmetric Channel

In [6] the Cover-Van der Meulen achievability region, $\mathcal{D}$, for BSSC was evaluated. In particular it was shown that the line segment joining $(R_1, R_2) = (0.2411.., 0.1204..)$ to $(R_1, R_2) = (0.1204.., 0.2411..)$ is on the boundary of $\mathcal{D}$. Note that on this line segment $R_1 + R_2 = 0.3616....$

Claim 4.1: The line segment connecting $(R_1, R_2) = (0.2280.., 0.1431..)$ to $(R_1, R_2) = (0.1431.., 0.2280..)$ lies on the boundary of $\mathcal{C}$.

Proof: Note that the sum rate is bounded by

$$R_1 + R_2 \le \frac{1}{2}(I(U;Y) + I(X;Z|U) + I(V;Z) + I(X;Y|V)).$$

We proceed to maximize the RHS of the above inequality over all $p(u,v,x)$. Assume that (U_o, V_o, X_o) maximizes R and let

$$R_m = \frac{1}{2}(I(U_o;Y_o) + I(X_o;Z_o|U_o) + I(V_o;Z_o) + I(X_o;Y_o|V_o)).$$

Consider a triple (U', V', X) with $\mathcal{U}' = \mathcal{V}, \mathcal{V}' = \mathcal{U}$, such that

$$\begin{aligned} \mathrm{P}(U' = u', V' = v') &= \mathrm{P}(U_o = v', V_o = u') \\ \mathrm{P}(X = x|U' = u', V' = v') &= \mathrm{P}(X = 1 - x|U_o = v', V_o = u'). \end{aligned} \tag{4.1}$$

By the symmetry of the channel,

$$\begin{aligned} I(U';Y') &= I(V_o;Z_o), \\ I(V';Z') &= I(U_o;Y_o), \\ I(X';Z'|U') &= I(X_o;Y_o|V_o), \text{ and} \\ I(X';Y'|V') &= I(X_o;Z_o|U_o). \end{aligned}$$

Therefore,

$$R_m = \frac{1}{2}(I(U';Y') + I(X';Z'|U') + I(V';Z') + I(X';Y'|V')).$$

Let $Q \in \{1, 2\}$ be an independent random variable that takes values 1 or 2 with equal probability and define $U^* = (\tilde{U}, Q)$ and $V^* = (\tilde{V}, Q)$ as $(Q = 1, \tilde{U}, \tilde{V}, X) \sim (U_o, V_o, X)$ and $(Q = 2, \tilde{U}, \tilde{V}, X) \sim (U', V', X)$, respectively. Then

$$\begin{aligned} \mathrm{P}(X = x|\tilde{U} = u, \tilde{V} = v, Q = 1) &= \mathrm{P}(X = x|U_o = u, V_o = v) \\ \mathrm{P}(X = x|\tilde{U} = u, \tilde{V} = v, Q = 2) &= \mathrm{P}(X' = x|U' = u, V' = v). \end{aligned}$$

Observe that

$$\begin{aligned} I(X^*;Y^*|V^*) &= \frac{1}{2}(I(X_o;Y_o|V_o) + I(X';Y'|V')) \\ I(U^*;Y^*) &= H(Y^*) - H(Y^*|U^*) \\ &= H(Y^*) - \frac{1}{2}(H(Y_o|U_o) + H(Y'|U') \\ &\overset{(a)}{\ge} \frac{1}{2}(H(Y_o) + H(Y')) \\ &\quad - \frac{1}{2}(H(Y_o|U_o) + H(Y'|U')) \\ &= \frac{1}{2}(I(U_o;Y_o) + I(U';Y')), \end{aligned}$$

where (a) follows by the concavity of the entropy function.

Similarly

$$\begin{aligned} I(X^*;Z^*|U^*) &= \frac{1}{2}(I(X_o;Z_o|U_o) + I(X';Z'|U')) \text{ and} \\ I(V^*;Z^*) &\ge \frac{1}{2}(I(V_o;Z_o) + I(V';Z')). \end{aligned}$$

Therefore,

$$R_m \le I(U^*;Y) + I(X;Z|U^*) + I(V^*;Z) + I(X;Y|V^*).$$

Now, by the construction of (U^*, V^*, X^*), $\mathrm{P}(X^* = 1) = 0.5$. Thus to compute R_m, it suffices to consider X such that $\mathrm{P}(X = 1) = 0.5$.

Using standard optimization techniques, it is not difficult to see that the following (U, X) and (V, X) maximize the terms $I(U;Y) + I(X;Z|U)$ and $I(V;Z) + I(X;Y|V)$, respectively, subject to $\mathrm{P}(X = 1) = 0.5$. Let $\alpha = 0.5 - \sqrt{105}/30 \approx 0.1584$. Then a set of maximizing pairs $\mathrm{P}(U,X)$ and $\mathrm{P}(V,X)$ can be described by

$$\begin{aligned} \mathrm{P}(U=0) &= \frac{0.5}{1-\alpha}, & \mathrm{P}(U=1) &= \frac{0.5-\alpha}{1-\alpha}, \\ \mathrm{P}(X=1|U=0) &= \alpha, & \mathrm{P}(X=1|U=1) &= 1, \\ \mathrm{P}(V=0) &= \frac{0.5}{1-\alpha}, & \mathrm{P}(V=1) &= \frac{0.5-\alpha}{1-\alpha}, \\ \mathrm{P}(X=0|V=0) &= \alpha, & \mathrm{P}(X=0|V=1) &= 1. \end{aligned}$$

Substituting these values, we obtain

$$\begin{aligned} R_1 + R_2 &\le \frac{1}{2}\left(I(U;Y) + I(X;Z|U)\right. \\ &\qquad \left. + I(V;Z) + I(X;Y|V)\right) \\ &\le 0.3711.., \end{aligned}$$

which implies that the sum rate is upper bounded by 0.3711... . We now show that this value is tight.

As before, let $\alpha = 0.5 - \sqrt{105}/30 \approx 0.1584$. Consider the following (U,V,X)

$$\begin{aligned} \mathrm{P}(U=0,V=0) &= \frac{\alpha}{1-\alpha}, \\ \mathrm{P}(X=1|U=0,V=0) &= 0.5, \\ \mathrm{P}(U=0,V=1) &= \frac{0.5-\alpha}{1-\alpha}, \\ \mathrm{P}(X=1|U=0,V=1) &= 0, \\ \mathrm{P}(U=1,V=0) &= \frac{0.5-\alpha}{1-\alpha}, \\ \mathrm{P}(X=1|U=1,V=0) &= 1. \end{aligned}$$

The region evaluated by this (U,V,X) is given by all rate pairs (R_1,R_2) satisfying

$$\begin{aligned} R_1 &\le I(U;Y) = 0.2280.. \\ R_2 &\le I(V;Z) = 0.2280.. \\ R_1 + R_2 &\le \min\{I(U;Y) + I(X;Z|U), \\ &\qquad I(V;Z) + I(X;Y|V)\} \\ &= 0.3711.. \end{aligned} \tag{4.2}$$

Thus the line segment joining $(R_1,R_2) = (0.2280..,0.1431..)$ to $(R_1,R_2) = (0.1431..,0.2280..)$ lies on the boundary of $\mathcal{C}$. ■

We now show that the region $\mathcal{C}$ is strictly larger than the Cover-van der Meulen region $\mathcal{D}$ and strictly smaller than the Körner-Marton outer bound.

The line segment joining $(R_1,R_2) = (0.2280..,0.1431..)$ to $(R_1,R_2) = (0.1431..,0.2280..)$ that lies on the boundary of $\mathcal{C}$ is strictly outside the line segment joining $(R_1,R_2) = (0.2411..,0.1204..)$ to $(R_1,R_2) = (0.1204..,0.2411..)$ that lies on the boundary of the Cover-van der Meulen region $\mathcal{D}$ [6].

Consider the following random variables (U,X).

$$\begin{aligned} \mathrm{P}(U=0) &= 0.6372 \\ \mathrm{P}(U=1) &= 0.3628 \\ \mathrm{P}(X=1|U=0) &= 0.2465 \\ \mathrm{P}(X=1|U=1) &= 1. \end{aligned}$$

For this pair $I(U;Y) = 0.18616..$ and $I(X;Z|U) = 0.18614...$ Hence the point $(R_1,R_2) = (0.1861, 0.1861)$ lies inside the region $\mathcal{O}_y$. By symmetry, the same point lies inside $\mathcal{O}_z$ and hence it lies inside $\mathcal{O}_y \cap \mathcal{O}_z$, the Körner-Marton outer bound. Note that $R_1 + R_2 = 0.3722 > 0.3711..$ and therefore this point lies outside $\mathcal{C}$.

5. Conclusion

We presented an outer bound on the capacity region of the DM broadcast channel, which is tight for all the special cases where the capacity is known. We showed that the bound is strictly smaller than the Körner-Marton for the BSS channel. The outer bound, however, is strictly larger than the Cover-van der Meulen region for this channel. We believe that the outer bound given by equation (3.1) is strictly tighter than the one given in Theorem 3.1 for the BSSC.

References

[1] R F Ahlswede and J Kőrner. Source coding with side information and a converse for degraded broadcast channels. *IEEE Trans. Info. Theory*, IT-21(6):629–637, November, 1975.

[2] T Cover. Broadcast channels. *IEEE Trans. Info. Theory*, IT-18:2–14, January, 1972.

[3] T Cover. An acheivable rate region for the broadcast channel. *IEEE Trans. Info. Theory*, IT-21:399–404, July, 1975.

[4] T Cover. Comments on broadcast channels. *IEEE Trans. Info. Theory*, IT-44:2524–2530, October, 1998.

[5] A El Gamal. The capacity of a class of broadcast channels. *IEEE Trans. Info. Theory*, IT-25:166–169, March, 1979.

[6] B Hajek and M Pursley. Evaluation of an achievable rate region for the broadcast channel. *IEEE Trans. Info. Theory*, IT-25:36–46, January, 1979.

[7] K Marton. A coding theorem for the discrete memoryless broadcast channel. *IEEE Trans. Info. Theory*, IT-25:306–311, May, 1979.

[8] H Sato. An outer bound to the capacity region of broadcast channels. *IEEE Trans. Info. Theory*, IT-24:374–377, May, 1978.

[9] E van der Meulen. Random coding theorems for the discrete memoryless broadcast channel. *IEEE Trans. Info. Theory*, IT-21:180–190, March, 1975.