# Lecture Notes 7
# Fixed Pattern Noise

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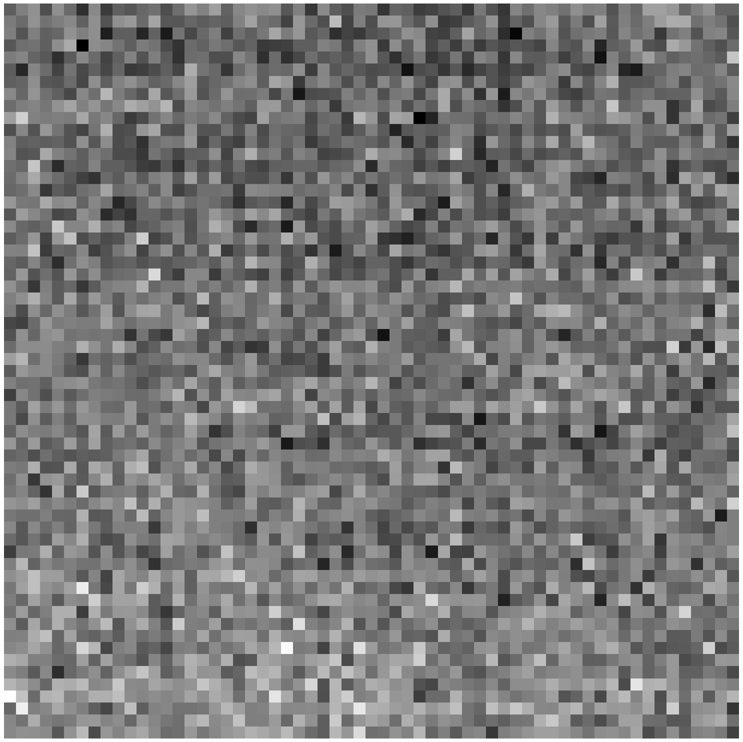
- Definition
- Sources of FPN
- Analysis of FPN in PPS and APS
- Total Noise Model
- Correlated Double Sampling

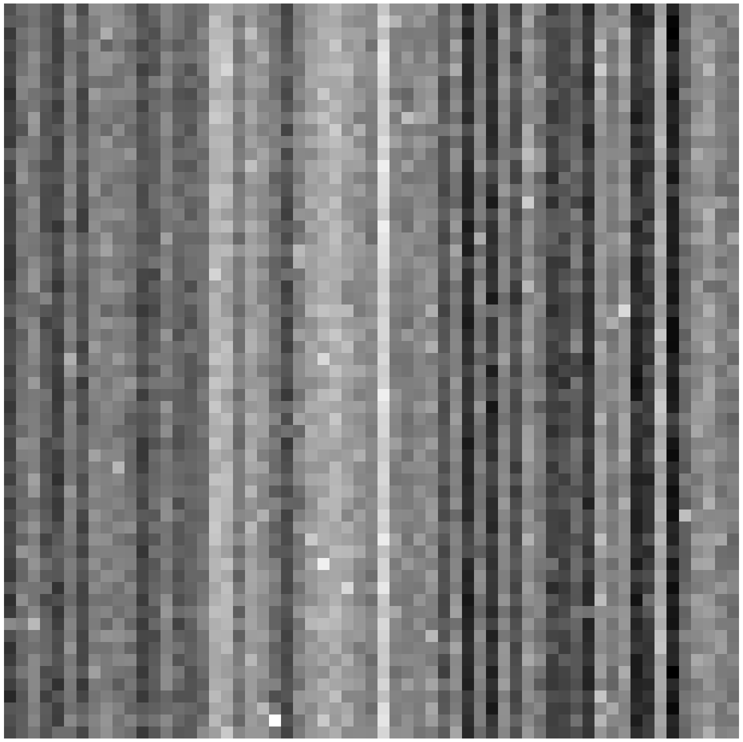
# Fixed Pattern Noise (FPN)

- FPN (also called *nonuniformity*) is the spatial variation in pixel output values under uniform illumination due to device and interconnect parameter variations (mismatches) across the sensor
- It is *fixed* for a given sensor, but varies from sensor to sensor, so if $v_o$ is the nominal pixel output value (at unifrom illumination), and the output pixel values (excluding temporal noise) from the sensor are $v_{ij}$ for $1 \leq i \leq n$ and $1 \leq j \leq m$, then the fixed pattern noise is the set of values $\Delta v_{oij} = v_{oij} - v_o$
- FPN consists of *offset* and *gain* components – increases with illumination, but causes more degradation in image quality at low illumination
- FPN for CCD image sensors appears random
- CMOS (PPS and APS) sensors have higher FPN than CCDs and suffer from *column* FPN, which appears as "stripes" in the image and can result in significant image quality degradation

# FPN Images



For CCD sensor

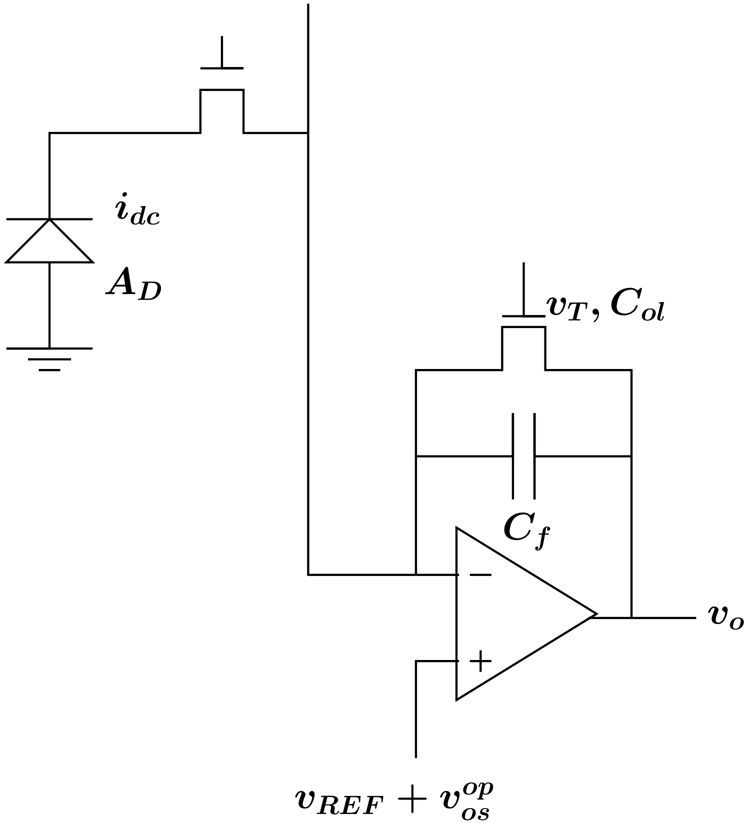


For CMOS sensor

# Sources of FPN

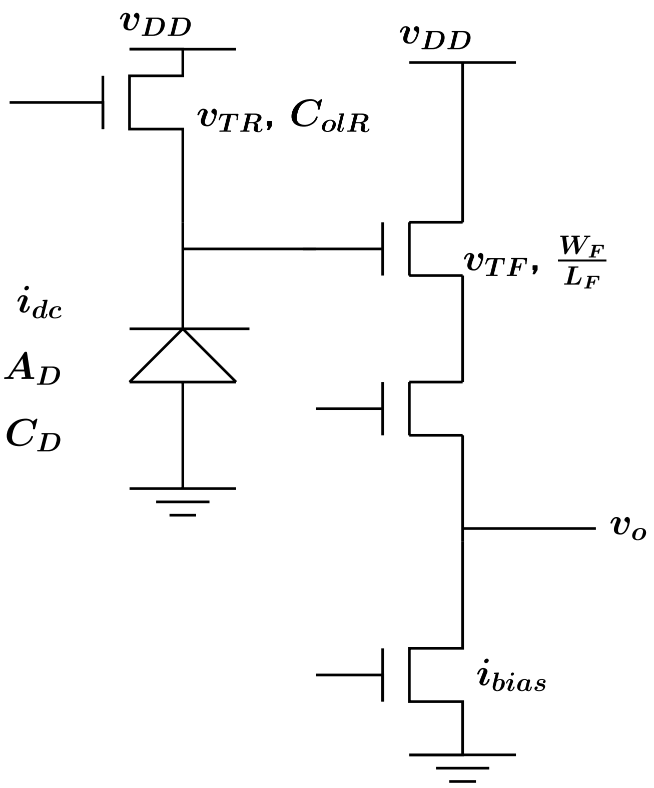
- CCD image sensors only suffer from *pixel* FPN due to spatial variation in photodetector device parameters and dark current – neither the CCDs nor the output amplifier (which is shared by all pixels) cause FPN (additional nonuniformity can result if more than one output amplifier is used, however)
- In CMOS image sensors pixel transistors cause additional *pixel* FPN and column amplifiers cause *column* FPN. As a result FPN is in general higher than in CCDs

- Main sources of FPN in PPS:



  - Pixel FPN is mainly due to the variation in the photodetector parameters (e.g., area $A_D$) and dark current
  - Column FPN is due to the variation in the column amplifier parameters, e.g., offset voltage $v_{os}^{op}$, feedback capacitor value, reset transistor threshold voltage and overlap capacitance value $C_{ol}$

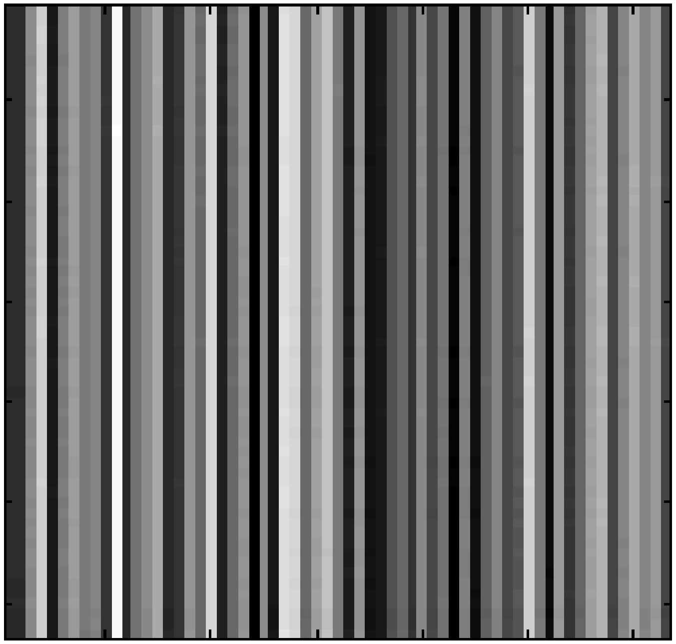
- In APS



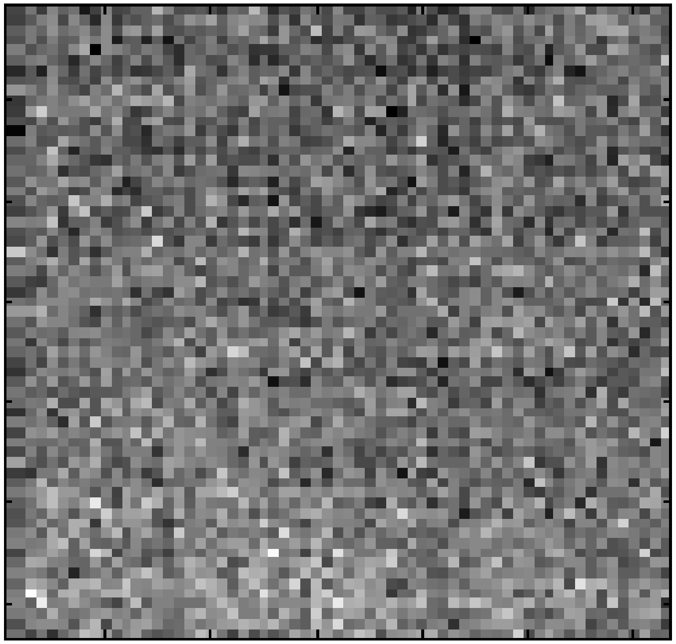
  - In addition to variation in the photodetector parameters and dark current, pixel FPN is caused also by variations in transistor parameters
  - Column FPN is mainly due to variation $i_{bias}$

# PPS and APS FPN

- APS suffers from higher pixel FPN than PPS but PPS generally suffers from higher column FPN



PPS



APS

# Quantifying FPN

- FPN is quantified by the standard deviation of the spatial variation in pixel outputs under uniform illumination (not including temporal noise). It is typically reported as a $\%$ of voltage swing (or well capacity)
    - FPN standard deviation values of $< 0.1\%$ to $> 4\%$ of well capacity have been reported
- Experimentally, FPN is measured as follows:
    - Set a constant uniform illumination level (including no illumination)
    - Take many images
    - For each pixel compute the average output value (to average out temporal noise)
    - Estimate the standard deviation of the average pixel values
    - Repeat the procedure for several uniform illumination levels

# Analysis of FPN

- Suppose we are given the standard deviation of each parameter that casues FPN, we now show how to compute its contribution to the total FPN
- Assume the parameter values to be random variables $Z_1$,$Z_2, \ldots$, $Z_k$ expressed as

$$Z_i = z_i + \Delta Z_i,$$

where $z_i$ is the mean of $Z_i$ (i.e., nominal value of the device parameter) and $\Delta Z_i$ is the variation of $Z_i$ from its mean, and has zero mean and standard deviation $\sigma_{Z_i}$

- Assuming sufficiently small device parameter variations, we can approximate the pixel output voltage (for a given illumination) as a function of the device parameters using the Taylor series expansion, as

$$V_o(Z_1, Z_2, \ldots, Z_k) \approx v_o(z_1, z_2, \ldots z_k) + \sum_{i=1}^{k} \frac{\partial v_o}{\partial z_i}\Big|_{z_1, z_2, \ldots z_k} \cdot \Delta Z_i$$

where $v_o(z_1, z_2, \ldots z_k)$ is the nominal output voltage and $\partial v_o / \partial z_i$ is the *sensitivity* of $v_o$ w.r.t. the $i$th parameter (evaluated at the nominal parameter values)

- So the variation in $V_o$ can be represented by the random variable

$$\Delta V_o = \sum_{i=1}^{k} \frac{\partial v_o}{\partial z_i}\Big|_{z_1, z_2, \ldots z_k} \cdot \Delta Z_i$$

- To quantify FPN, we find the standard deviation of the output voltage, $\sigma_{V_o}$, i.e., the standard deviation of the r.v. $\Delta V_o$

- Assuming that the $\Delta Z_i$s are uncorrelated (may not be a good assumption in general), we can write

$$\sigma_{V_o} = \sqrt{\sum_{i=1}^{k} \left( \frac{\partial v_o}{\partial z_i}\Big|_{z_1, z_2, \ldots z_k} \right)^2 \cdot \sigma_{Z_i}^2}$$

# Column and Pixel FPN

- For a CMOS (PPS or APS) image sensor, let the column device parameters be $Z_1$, $Z_2, \ldots$, $Z_l$ and the rest be the pixel device parameters, we can define the column variation as

$$Y = \sum_{i=1}^{l} \frac{\partial v_o}{\partial z_i}\bigg|_{z_1, z_2, \ldots z_k} \cdot \Delta Z_i$$

  and the pixel variation as

$$X = \sum_{i=l+1}^{k} \frac{\partial v_o}{\partial z_i}\bigg|_{z_1, z_2, \ldots z_k} \cdot \Delta Z_i$$

- We quantify column FPN by $\sigma_Y$ and pixel FPN by $\sigma_X$ (vary with illumination)
- Since (by assumption) $X$ and $Y$ are uncorrelated

$$\sigma_{V_o}^2 = \sigma_Y^2 + \sigma_X^2$$

# Offset and Gain FPN

- The pixel output voltage $v_o$ and FPN $\sigma_{V_o}$ vary with illumination
- The nominal output voltage from a pixel can be expressed in terms of the photocurrent density as

  $$v_o = h j_{ph} + v_{os}$$

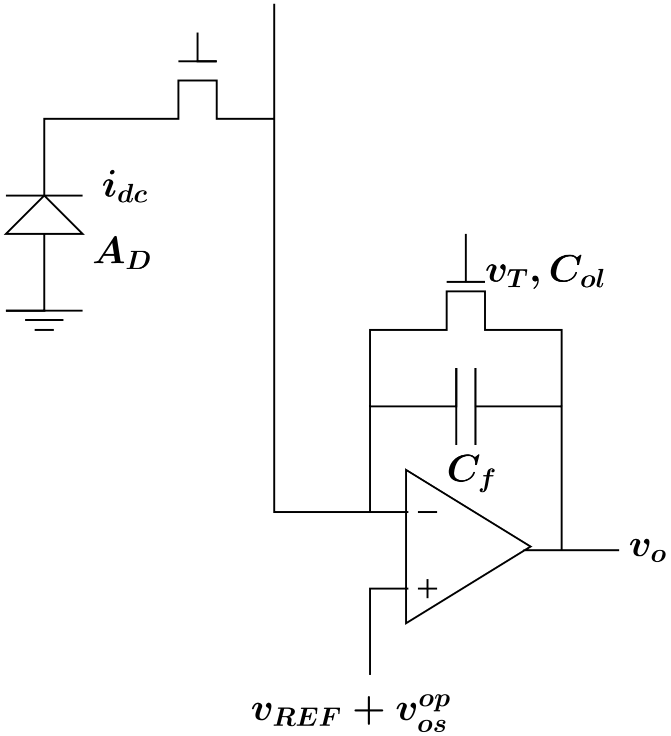
  where $h$ is the pixel *gain* in V·cm$^2$/A (not to be confused with sensor conversion gain $g$) and $v_{os}$ is the pixel *offset* (which includes the dark signal as well as the offset voltages due to the amplifiers used, e.g., $v_{os}^{op}$ for PPS)
- Assuming all photodetectors have the same QE, and thus under uniform illumination, they have the same photocurrent density, we can now write the pixel output voltage variation as

  $$\begin{aligned}\Delta V_o &= \left(\sum_{i=1}^{k} \frac{\partial h}{\partial z_i}\Big|_{z_1,z_2,\ldots z_k} \Delta Z_i\right) j_{ph} + \left(\sum_{i=1}^{k} \frac{\partial v_{os}}{\partial z_i}\Big|_{z_1,z_2,\ldots z_k} \Delta Z_i\right) \\ &= \Delta H\, j_{ph} + \Delta V_{os}\end{aligned}$$

- We quantify offset FPN by $\sigma_{V_{os}}$ and gain FPN by $\sigma_H \cdot j_{ph}$
- Offest FPN is reported as $\%$ of well capacity
- Gain FPN is referred to as Pixel Response Nonuniformity (PRNU) and is reported as $\%$ of gain factor variation, i.e., $100\sigma_H/h$
- Note that $\Delta H$ and $\Delta V_{os}$ are not necessarily uncorrelated, since some device parameters can affect both offset and gain

# Analysis of FPN in PPS

- The figure shows the device parameters considered



$A_D$ is the photodiode area, $i_{dc}$ is its dark current, $v_{os}^{op}$ is the opamp offset voltage, $C_{ol}$ is the overlap capacitance, and $v_T$ is the threshold voltage

- The output voltage in steady state is given by

$$v_o = (Q + C_{ol} v_T) \cdot \frac{1}{C_f} + v_{REF} + v_{os}^{op},$$

where $C_{ol} v_T$ is the "feedthrough" charge (when the reset transistor is turned off), and the charge $Q$ accumulated on the photodiode capacitance

$$Q = (j_{ph} A_D + i_{dc}) t_{int}$$

- The following table lists the absolute values of the parameter senitivities $\frac{\partial v_o}{\partial z_i}$ and effect on FPN

| Parameter | Sensitivity | Effect on FPN |
|---|---|---|
| $A_D$ | $\frac{t_{int}}{C_f} \cdot j_{ph}$ | pixel/gain |
| $i_{dc}$ | $\frac{t_{int}}{C_f}$ | pixel/offset |
| $v_{os}^{op}$ | $1$ | column/offset |
| $C_f$ | $\frac{i_{dc} t_{int} + C_{ol} v_T}{C_f^2}$ | column/offset |
| | $+\frac{A_D t_{int}}{C_f^2} \cdot j_{ph}$ | column/gain |
| $v_T$ | $\frac{C_{ol}}{C_f}$ | column/offset |
| $C_{ol}$ | $\frac{v_T}{C_f}$ | column/offset |

- Offset FPN

$$\sigma_{V_{os}} = \sqrt{\left(\frac{t_{int}}{C_f}\sigma_{i_{dc}}\right)^2 + \sigma^2_{v^{op}_{os}} + \left(\left(\frac{i_{dc}t_{int} + C_{ol}v_T}{C_f^2}\right)\sigma_{C_f}\right)^2 + \left(\frac{v_T}{C_f}\sigma_{C_{ol}}\right)^2 + \left(\frac{C_{ol}}{C_f}\sigma_{v_T}\right)^2}$$

- Gain FPN

$$\sigma_H \cdot j_{ph} = j_{ph}\sqrt{\left(\frac{t_{int}}{C_f}\sigma_{A_D}\right)^2 + \left(\frac{A_D t_{int}}{C_f^2}\sigma_{C_f}\right)^2}$$

- Pixel FPN

$$\sigma_X = \sqrt{\left(\frac{j_{ph}t_{int}}{C_f}\sigma_{A_D}\right)^2 + \left(\frac{t_{int}}{C_f}\sigma_{i_{dc}}\right)^2}$$

- Column FPN

$$\sigma_Y = \sqrt{\sigma^2_{v^{op}_{os}} + \left(\left(\frac{i_{dc}t_{int} + C_{ol}v_T + A_D j_{ph} t_{int}}{C_f^2}\right)\sigma_{C_f}\right)^2 + \left(\frac{v_T}{C_f}\sigma_{C_{ol}}\right)^2 + \left(\frac{C_{ol}}{C_f}\sigma_{v_T}\right)^2}$$

- Note that the FPN variance $\sigma^2_{V_o} = \sigma^2_X + \sigma^2_Y$ can be written as the sum of three terms, a term that is independent of the signal, a term that increases linearly with the signal, and a term that increases quadratically with the signal

# Example

- Assume the following device parameter means, standard deviations, and that $t_{int} = 30$ms

| Parameter | Mean | $\sigma$ | Sensitivity |
|---|---|---|---|
| $A_D$ | $50\mu$m$^2$ | $0.4\%\overline{A}_D$ | $15 \times 10^3 j_{ph}$V/$\mu$m$^2$ |
| $i_{dc}$ | 5fA | $2\%\bar{i}_{dc}$ | 1.5mV/fA |
| $v_{os}^{op}$ | 0V | 2mV | 1 |
| $C_f$ | 20fF | $0.2\%\overline{C}_f$ | $11.6 \times 10^{11}$V/F |
| | | | $37500 j_{ph}$V/fF |
| $v_{TR}$ | 0.8V | $0.2\%\overline{v}_{TR}$ | 0.02 |
| $C_{ol}$ | 0.4fF | $0.4\%\overline{C}_{ol}$ | 0.04V/fF |

- Offset FPN

| Parameter | Contribution to $\sigma_{V_{os}}$ |
|---|---|
| $i_{dc}$ | $0.15$ mV |
| $v_{os}^{op}$ | $2$ mV |
| $C_f$ | $0.0464$ mV |
| $v_{TR}$ | $0.032$ mV |
| $C_{ol}$ | $0.064$ mV |

  and

$$\sigma_{V_{os}} \approx 2\text{mV},$$

  which is basically equal to the opamp offset $\sigma_{v_{os}^{op}}$ value

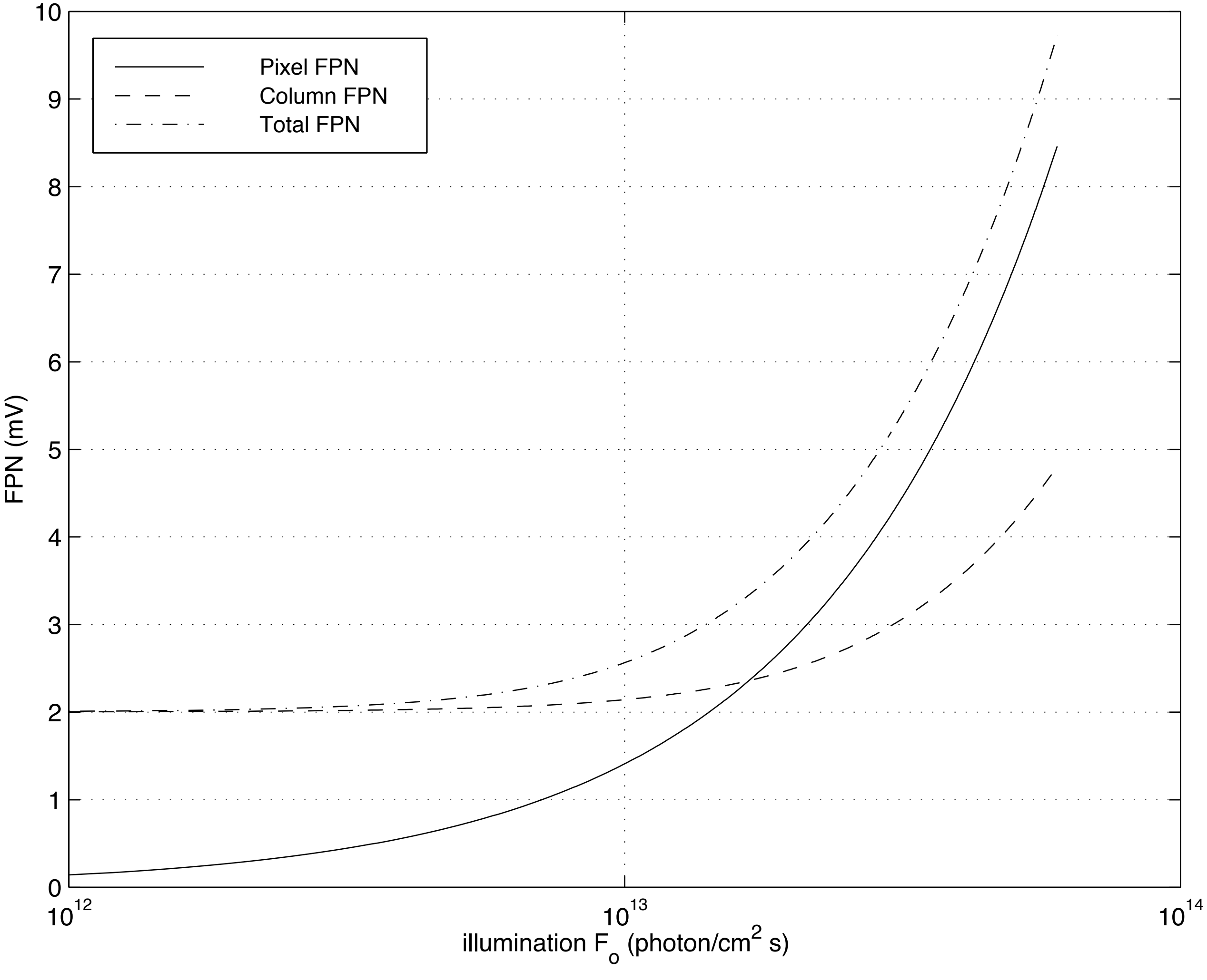
- Gain FPN at $j_{ph} = 2.64 \times 10^{-6}$A/cm$^2$ (high illumination)

| Parameter | Contribution to $\sigma_H \cdot j_{ph}$ |
|---|---|
| $A_D$ | $7.92$ mV |
| $C_f$ | $3.96$ mV |

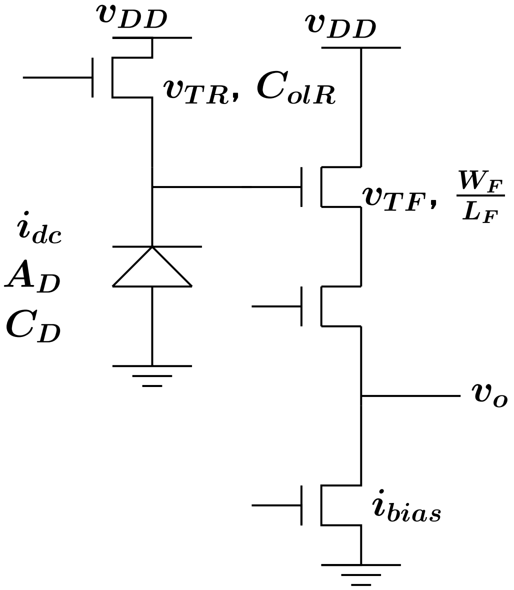
  and

$$\sigma_H \cdot j_{ph} = 8.85\text{mV}$$

- The following figure plots total FPN $\sigma_{V_o}$, pixel FPN $\sigma_X$, and column FPN $\sigma_Y$, assuming monochromatic illumination $F_0$ photons/cm$^2$.s at quantum efficiency QE $= 0.3$



# Analysis of FPN in APS



- In steady state and assuming soft reset, the output voltage is given by

$$v_o = v_{DD} - v_{TR} - \frac{Q}{C_D} - \left( v_{TF} + \sqrt{\frac{2L_F}{k_n W_F} i_{bias}} \right),$$

where the charge accumulted on the photodiode is given by

$$Q = (A_D j_{ph} + i_{dc}) t_{int} + C_{olR} v_{DD}$$

The $C_{olR} v_{DD}$ term is the “feedthrough” charge (when the reset transistor is turned off)

# Example

- Consider the following parameter means and standard deviations

| parameter | mean | $\sigma$ | effect on FPN |
|---|---|---|---|
| $i_{dc}$ | 5fA | $2\%\bar{i}_{dc}$ | pixel/offset |
| $A_D$ | $50\mu$m$^2$ | $0.4\%\overline{A}_D$ | pixel/gain |
| $C_D$ | 20fF | $0.4\%\overline{C}_D$ | pixel/offset,gain |
| $v_{TR}$ | 1.1V | $0.2\%\overline{v}_{TR}$ | pixel/offset |
| $C_{olR}$ | 0.4fF | $0.4\%\overline{C}_{olR}$ | pixel/pffset |
| $v_{TF}$ | 0.9V | $0.2\%\overline{v}_{TF}$ | pixel/offset |
| $\frac{W_F}{L_F}$ | $\frac{4}{2}$ | $0.2\%\frac{\overline{W}_F}{\overline{L}_F}$ | pixel/offset |
| $i_{bias}$ | $1.88\mu$A | $1\%\bar{i}_{bias}$ | column/offset |

- You will compute the FPN component values in the homework

# Image Sensor Total Noise Model

- Combining temporal noise and FPN, we can express the total input referred noise charge as

$$Q_{\mathrm{n}} = Q_{\mathrm{shot}} + Q_{\mathrm{reset}} + Q_{\mathrm{readout}} + Q_{\mathrm{fpn}},$$

  where

  - $Q_{\mathrm{shot}}$ is the r.v. representing the noise charge due to photodetector photo and dark current shot noise and is Gaussian with zero mean and variance $\frac{1}{q}(i_{ph} + i_{dc})t_{int}$ electrons$^2$
  - $Q_{\mathrm{reset}}$ is the r.v. representing the reset noise and is basically independent of the signal
  - $Q_{\mathrm{readout}}$ is the r.v. representing the readout circuit noise (possibly including quantization) and is basically independent of the signal

  - $Q_{\text{fpn}}$ is the r.v. representing FPN (in electrons), and can be represented either as a sum of pixel and column components

    $$Q_{\text{fpn}} = \frac{1}{g}(X + Y)$$

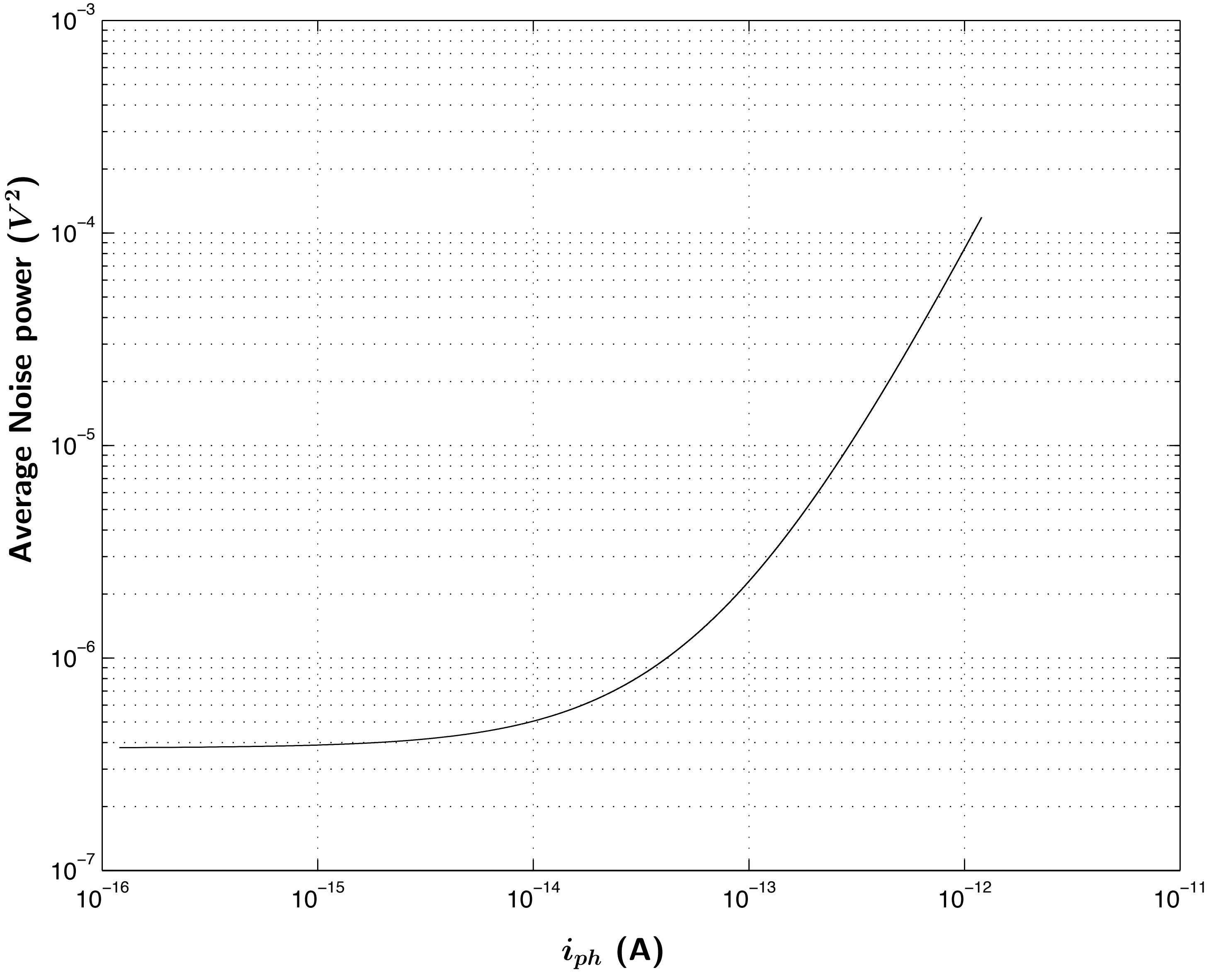
    where $g$ is the sensor conversion gain in V/electron, or offset and gain components

    $$Q_{\text{fpn}} = \frac{1}{g}(\Delta H j_{ph} + \Delta V_{os})$$

    Thus it has one component that is independent of signal and one that grows with the signal
  - The noise components are assumed independent

- Thus the total average noise power is the sum of three components:
  - One that does not depend on the signal (due to reset and readout noise and offset FPN)
  - One that increases linearly with the signal ($i_{ph}$ or $j_{ph}$) (due to shot noise and gain FPN)
  - One that increases quadratically with the signal (due to gain FPN)

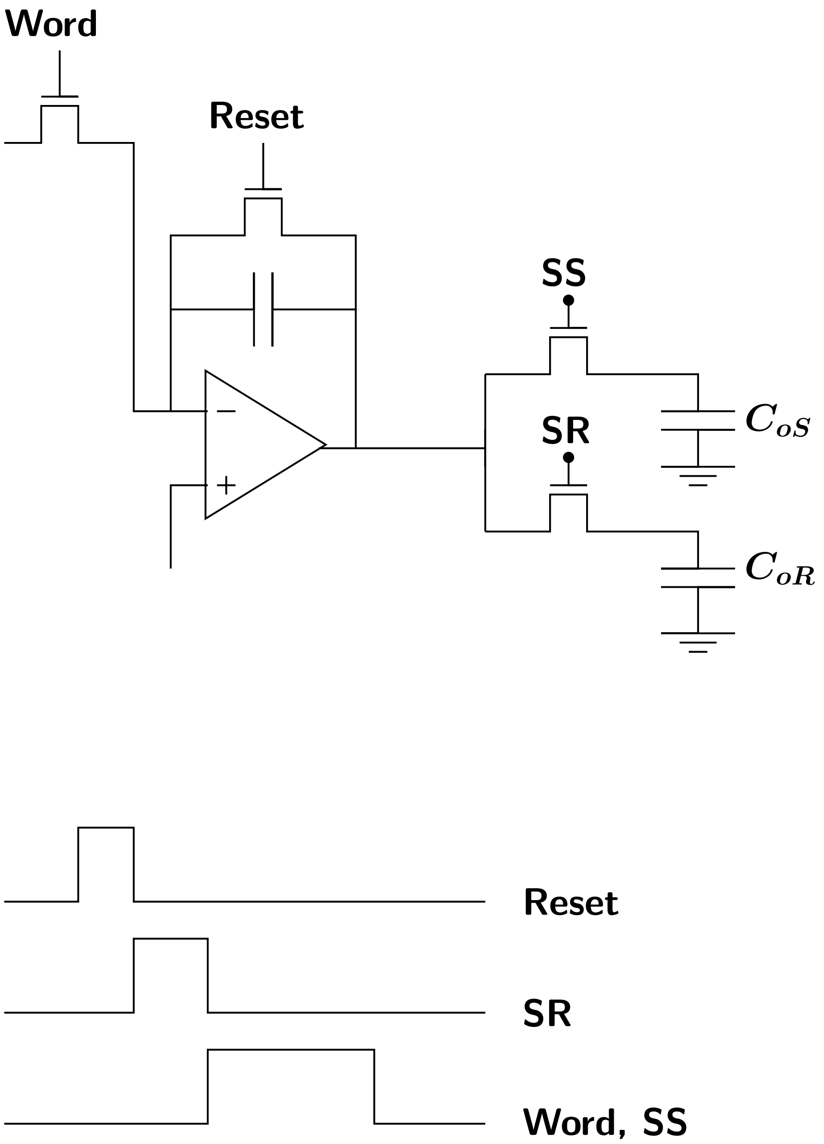
# Noise as Function of Photocurrent



# Correlated Double Sampling (CDS)

- CDS is a multiple sampling technique commonly used in image sensors to reduce FPN, and reset and 1/f noise
- You sample the output twice; once right after reset and a second time with the signal present. The output signal is the difference between the two samples
  - ○ CDS only reduces offset FPN (does not reduce gain FPN)
  - ○ CDS does not cancel offset FPN due to dark current variation
  - ○ In CCDs, PPS, photogate and pinned diode APS, CDS cancels reset noise. In photodiode APS it increases it

# CDS in PPS



- Cancells
  - FPN due to $v_{os}^{op}$, $v_T$, and $C_{ol}$
  - Temporal noise due to reset (terms $\overline{V_{o2}^2}$ and $\overline{V_{o3}^2}$ in our analysis)
  - Readout noise due to op-amp 1/f noise
- Does not cancel
  - Offset FPN due to $i_{dc}$ variation. This is called Dark Signal Non-uniformity (DSNU)
  - Gain FPN (or PRNU)
  - Other temporal noise components
- Adds
  - Opamp noise due to reset read ($\overline{V_{o4}^2}$ term)
  - $\frac{KT}{C}$ noise due to SS and SR transistors

- To summarize, the total noise charge for the two samples are given by:

$$\begin{aligned} Q_{n1} &= Q_{\text{reset}} + Q_{\text{read1}} + Q_{\text{fpn1}} \\ Q_{n2} &= Q_{\text{shot}} + Q_{\text{reset}} + Q_{\text{readout2}} + Q_{\text{fpn2}} \end{aligned}$$

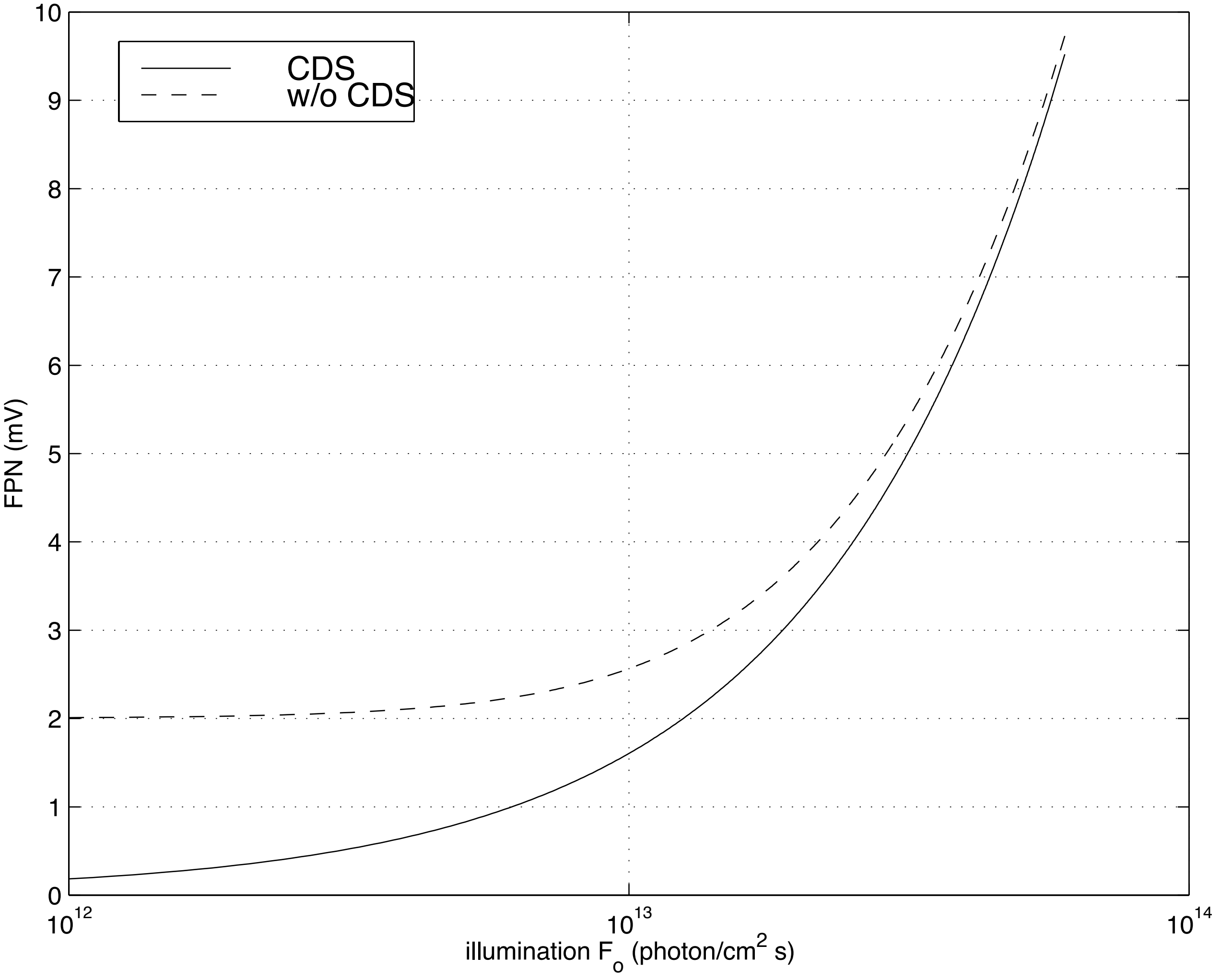
  Note that $Q_{\text{fpn1}}$ is simply an offset FPN whereas $Q_{\text{fpn2}}$ is the sum of offset and gain FPN (PRNU). However, $Q_{\text{fpn1}}$ does not include the offset FPN due to dark current variation (DSNU), whereas the offset part of $Q_{\text{fpn1}}$ includes it

  The difference between the two samples is thus:

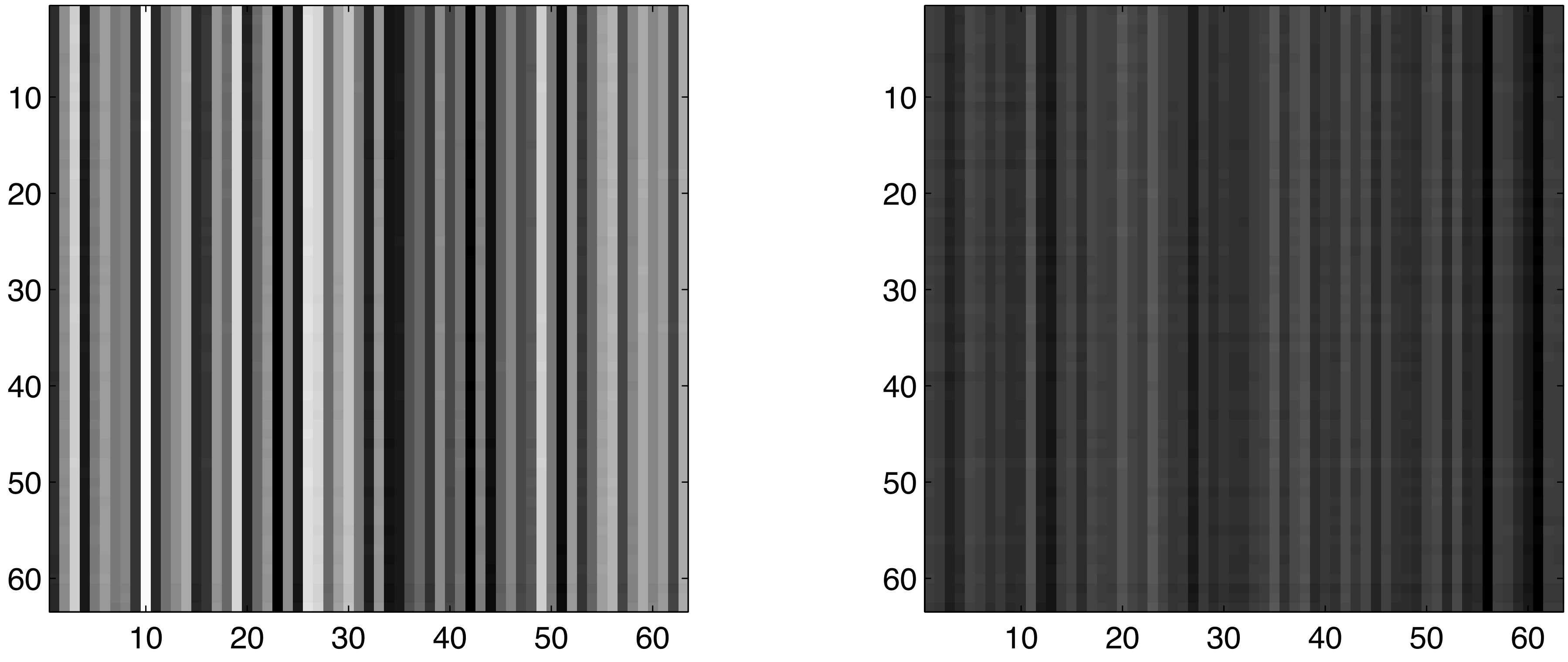
$$Q_{n2} - Q_{n1} = Q_{\text{shot}} + (Q_{\text{readout2}} - Q_{\text{readout1}}) + Q_{\text{prnu}} + Q_{\text{dsnu}}$$

# PPS FPN With and Without CDS

- The following figure plots PPS FPN with and without CDS (assuming that $v_{os}^{op}$, $v_T$, and $C_{ol}$ are eliminated)



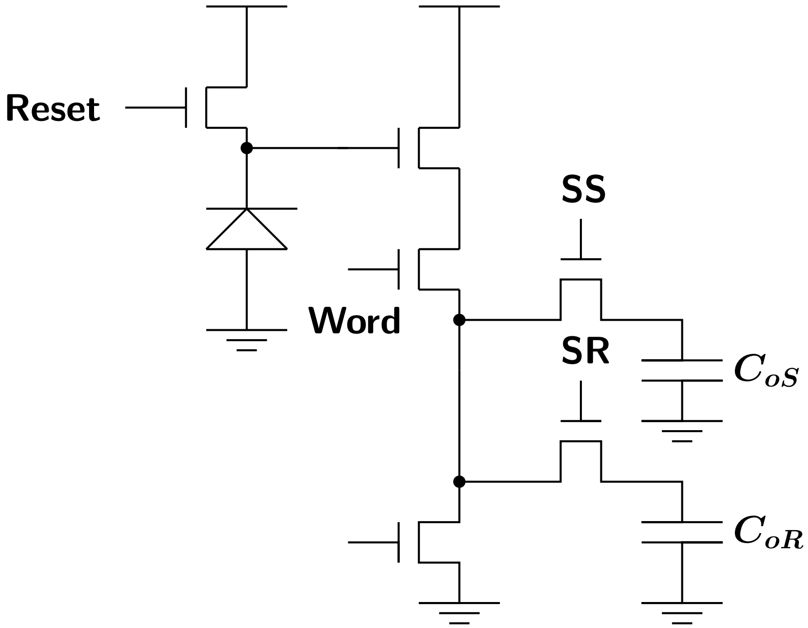
# PPS Offset FPN With and Without CDS

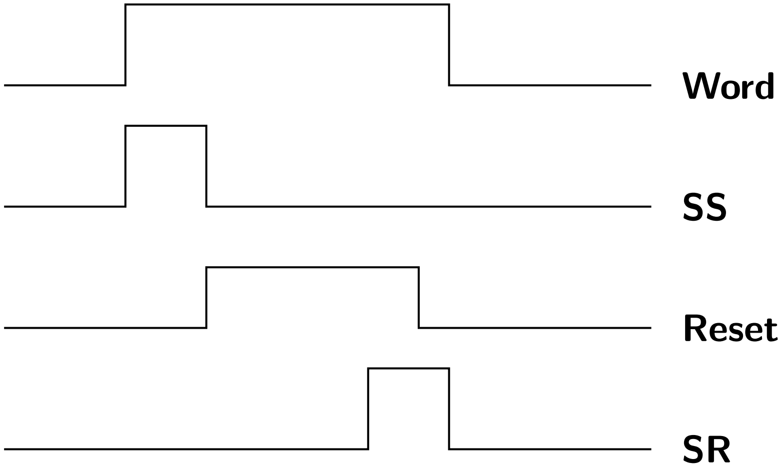


without CDS

with CDS

# CDS in 3T APS





- Cancells
  - All offset FPN terms involving $v_{TR}$, $v_{TF}$, $C_{olR}$, $\frac{W_F}{L_F}$, and $i_{bias}$
- Does not cancel
  - DSNU
  - Reset noise
  - PRNU
  - Readout noise
- Adds
  - Reset noise $\frac{kT}{2C_D}$ (reset noise component during reset readout independent of that during signal readout)
  - Readout noise during reset readout
  - $\frac{kT}{C}$ due to SS and SR transistors

- To summarize, the total noise charge for the two samples are given by:

$$\begin{aligned} Q_{n1} &= Q_{\text{shot}} + Q_{\text{reset1}} + Q_{\text{readout1}} + Q_{\text{fpn1}} \\ Q_{n2} &= Q_{\text{reset2}} + Q_{\text{readout2}} + Q_{\text{fpn2}} \end{aligned}$$

  The difference is:

$$Q_{n1} - Q_{n2} = Q_{\text{shot}} + (Q_{\text{reset1}} - Q_{\text{reset2}}) + (Q_{\text{read1}} - Q_{\text{read2}}) + Q_{\text{prnu}} + Q_{\text{dsnu}}$$

- An important advantage of photogate and pinned diode APS is that reset noise is eliminated using CDS instead of doubled