On the Capacity of AWGN Relay Channels with Linear Relaying Functions

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We consider the relay channel models depicted in Figure 1. We assume that at time i, the transmitted relay signal X_{1i} is a linear function of its past received signals $Y_{11}, Y_{12}, \ldots, Y_{1i-1}$. Thus for k transmissions, the relay sends $\mathbf{X}_1 = D\mathbf{Y}_1$, where D is a $k \times k$ strictly lower triangular matrix.

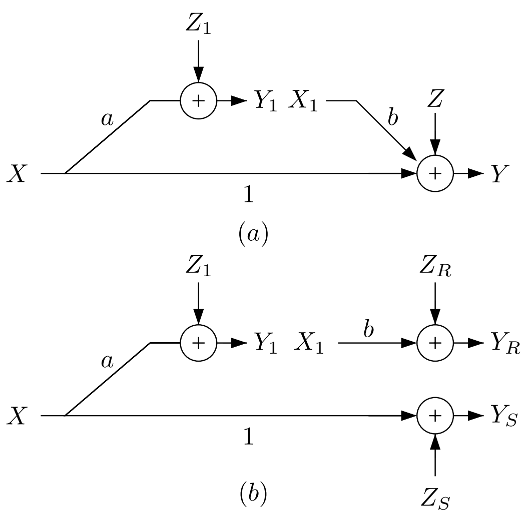

Figure 1: (a) General AWGN relay channel model. (b) Frequency-Division AWGN relay channel. The relay transmission at time i, x_{1i}, is a function of the past received symbols $y_{11}, \ldots, y_{1(i-1)}$. The noise components in each model are i.i.d. $\mathcal{N}(0, N)$ random variables. An average power constraint P on the sender and γP, for $\gamma \geq 0$, on the relay transmitter are assumed.

First we consider the general relay channel model (a). It can be shown that the channel capacity with linear relaying is given by $C^{(l)}(P, \gamma P) = \lim_{k\to\infty} \frac{1}{k} C_k^{(l)}(P, \gamma P)$, where $C_k^{(l)}(P, \gamma P) = \frac{1}{2} \max_{K_x, D} \log |I + GK_xG^T|$, for $G = \sqrt{N}(I + b^2DD^T)^{-1/2}(I + abD)$, subject to $K_x \succeq 0$, $\mathrm{tr}(K_x) \leq kP$, $\mathrm{tr}(a^2K_xD^TD + ND^TD) \leq k\gamma P$, and $d_{ij} = 0$, for $j \geq i$. Thus finding $C^{(l)}(P, \gamma P)$ involves solving a sequence of nonconvex optimization problems, which appears to be intractable. Linear relaying can outperfom more complex coding schemes, however. For example, consider the suboptimal linear relaying scheme with $k = 2$ and

$$K_x = 2P \begin{bmatrix} \alpha & \sqrt{\alpha(1-\alpha)} \\ \sqrt{\alpha(1-\alpha)} & (1-\alpha) \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ \sqrt{\frac{2\gamma P}{2a^2\alpha P + N}} & 0 \end{bmatrix}.$$

Assuming $a = b\sqrt{\gamma} = 1$, and $P/N = 0.1$, this scheme achieves a rate of 0.08829 bits/transmission, while the block-Markov and side information schemes in [1] only achieve 0.06875 and 0.07378 bits/transmission, repectively.

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Now, we consider the frequency-division relay channel model (b). For this model, linear capacity can again be expressed as the limit of the sequence of nonconvex optimization problems $C_{\mathrm{FD},k}^{(l)}(P, \gamma P) = \frac{1}{2} \max_{K_x, D} \log \left| I + GK_xG^T \right|$, where

$$G = \frac{1}{\sqrt{N}} \begin{bmatrix} I \\ ab(I + b^2DD^T)^{-1/2}D \end{bmatrix},$$

subject to $Kx \succeq 0$, $\mathrm{tr}(K_x) \leq kP$, $\mathrm{tr}(a^2K_xD^TD + ND^TD) \leq k\gamma P$ and $d_{ij} = 0$ for $j > i$. Surprisingly, these nonconvex optimization problems can be solved analytically, yielding the following key result of the paper.

Theorem 1 *The capacity of frequency division AWGN relay channel with linear relaying function is given by*

$$C_{\mathrm{FD}}^{(l)}(P, \gamma P) = \max_{0<\alpha\leq 1} \alpha \mathcal{C}\left(\frac{P}{\alpha N}\left(1 + \frac{a^2b^2\gamma P}{(a^2 + b^2\gamma)P + \alpha N}\right)\right).$$

Note that $C_{\mathrm{FD}}^{(l)}(P, \gamma P) = \max_{0<\alpha\leq 1} \alpha C_{\mathrm{FD},1}^{(l)}\left(\frac{P}{\alpha}, \frac{\gamma P}{\alpha}\right)$. Thus for large vaues of P, amplify-and-forward scheme is optimal, whereas for small values of P, proper time-sharing is also needed. It can be shown that the capacity with linear relaying is very close to the achievable rate given by the more complex side information scheme and can be higher than the achievable rate given by the block-Markov scheme (see Figure 2). Linear capacity approaches the upper bound as $b\sqrt{\gamma} \to \infty$.

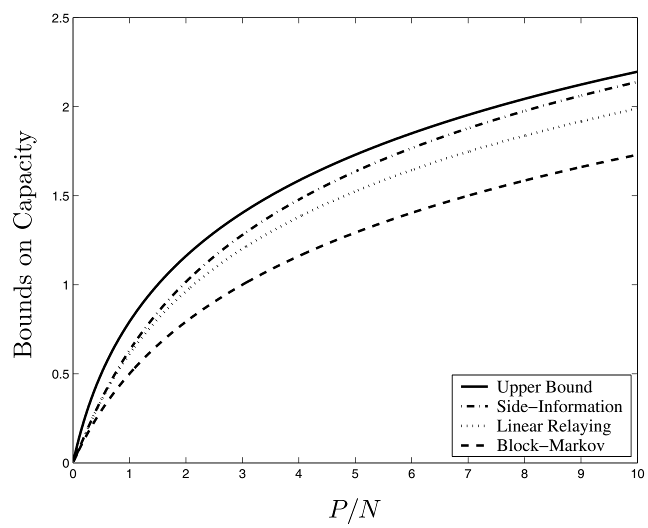

Figure 2: Bounds on capacity of frequency-division AWGN relay channel with $a = b\sqrt{\gamma} = 1$.

References

[1] T. M. Cover, A. El Gamal, "Capacity Theorems for the Relay Channel," *IEEE Transcations on Information Theory*, Vol. 25, No. 5, pp. 572-584, September 1979.

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