5.4 GAMBLER'S RUIN:
A RANDOM WALK ON THE SIMPLEX†

Thomas M. Cover

Departments of Electrical Engineering
and Statistics
Stanford University
Stanford, CA 94305

It is known that if two gamblers with capitals $p$ and $1-p$, respectively, engage in a fair game (we can model it by Brownian motion on the unit interval starting at $p$) until one of the gamblers goes broke, then the gambler with initial capital $p$ will win the game with probability $p$. Now suppose that there are $m$ gamblers with capitals corresponding to a point $p$ in the simplex $p_i \geq 0$, $\sum p_i = 1$. A random walk in the simplex occurs, and the gamblers go broke one by one. Once a gambler goes broke, he stays broke. What is the induced probability distribution on the order in which the gamblers go broke?

† Hajek has exhibited a solution to this problem for $m = 3$ gamblers. See Chapter VI.