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Withits 2010
Brief Bio

- Born in 1941, Budapest Hungary
- PhD from Eötvös Loránd University in 1965
- Department of Numerical Mathematics, Central Research Institute for Physics, Budapest, 1965–1973
- Mathematical Institute of the Hungarian Academy of Sciences, 1973–present
- Visited Institute of Information Transmission, Moscow, USSR, 1969
- Visited MIT, 1980

Research contributions and interests:
  - Information Theory
  - Measure concentration
  - Applications in Probability Theory
Selected Contributions to Information Theory

**Broadcast channels:**

**Strong converse:**

**Coding for computing via structured codes:**

**Rate distortion theory:**

**Error exponents:**

**Isomorphism:**

**Entropy and capacity of graphs:**
Blowing-Up Lemma


- Lemma first proved by Ahlswede, Gacs, Körner (1976)
- Used to prove strong converse, e.g., for degraded DM-BC
- Complicated, combinatorial proof
Blowing-Up Lemma

- Let $x^n, y^n \in \mathcal{X}^n$ and $d(x^n, y^n)$ be Hamming distance between them.
- Let $\mathcal{A} \subseteq \mathcal{X}^n$. For $l \leq n$, let $\Gamma_l(\mathcal{A}) = \{x^n : \min_{y^n \in \mathcal{A}} d(x^n, y^n) \leq l\}$.

Blowing up Lemma

Let $X^n \sim P_{X^n} = \prod_{i=1}^n P_{X_i}$ and $\epsilon_n \to 0$ as $n \to \infty$. There exist $\delta_n, \eta_n \to 0$ as $n \to \infty$ such that if $P_{X^n}(\mathcal{A}) \geq 2^{-n\epsilon_n}$, then $P_{X^n}(\Gamma_{n\delta_n}(\mathcal{A})) \geq 1 - \eta_n$.
Marton’s Simple Proof

- The proof uses the following information theoretic coupling inequality

**Lemma 1**

Let $X^n \sim \prod_{i=1}^{n} P_{X_i}$ and $\hat{X}^n \sim P_{\hat{X}_n}$. Then, there exists a joint probability measure $P_{X^n, \hat{X}^n}$ with these given marginals such that

$$\frac{1}{n} \mathbb{E}(d(X^n, \hat{X}^n)) = \frac{1}{n} \sum_{i=1}^{n} P\{X_i \neq \hat{X}_i\} \leq \left( \frac{1}{n} D\left( P_{\hat{X}_n} \mid \mid \prod_{i=1}^{n} P_{X_i} \right) \right)^{1/2}$$

- Now, define

$$P_{\hat{X}_n}(x^n) = P_{X^n|A}(x^n) = \begin{cases} \frac{P_{X^n}(x^n)}{P_{X^n}(A)} & \text{if } x^n \in A, \\ 0 & \text{if } x^n \notin A \end{cases}$$

Then,

$$D\left( P_{\hat{X}_n} \mid \mid \prod_{i=1}^{n} P_{X_i} \right) = - \log P_{X^n}(A) \leq n \epsilon_n$$
By Lemma 1, there exists $P_{X^n, \hat{X}^n}$ with given marginals such that
\[
\mathbb{E}(d(X^n, \hat{X}^n)) \leq n\sqrt{\epsilon_n}
\]

By the Markov inequality,
\[
P_{X^n, \hat{X}^n}\{d(X^n, \hat{X}^n) \leq n\delta_n\} \geq 1 - \frac{\sqrt{\epsilon_n}}{\delta_n} = 1 - \eta_n,
\]
where we choose $\delta_n \to 0$ such that $\eta_n \to 0$ as $n \to \infty$

We therefore have
\[
P_{X^n}(\Gamma_n\delta_n(\mathcal{A})) = P_{X^n, \hat{X}^n}(\Gamma_n\delta_n(\mathcal{A}) \times \mathcal{A}) + P_{X^n, \hat{X}^n}(\Gamma_n\delta_n(\mathcal{A}) \times \mathcal{A}^c)
\]
\[
= P_{X^n, \hat{X}^n}(\Gamma_n\delta_n(\mathcal{A}) \times \mathcal{A})
\]
\[
\geq 1 - \eta_n
\]

* follows since $P_{X^n, \hat{X}^n}(x^n, \hat{x}^n) = 0$ if $\hat{x}^n \notin \mathcal{A}$
Impact on My Work


- Defined less noisy and more capable BC
- I established the capacity region of the more capable class in 1977


- Best known inner bound on capacity region of BC
- van der Meulen and I provided an alternative proof in 1980
- Motivated the EGC region for multiple description coding in 1981


- Nair and I in 2008 showed that the extension of their superposition region to \( > 2 \) receivers is not optimal in general
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