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Withits 2010

Brief Bio

- Born in 1941, Budapest Hungary
- PhD from Eötvös Loránd University in 1965
- Department of Numerical Mathematics, Central Research Institute for Physics, Budapest, 1965–1973
- Mathematical Institute of the Hungarian Academy of Sciences, 1973–present
- Visited Institute of Information Transmission, Moscow, USSR, 1969
- Visited MIT, 1980
- Research contributions and interests:
  - Information Theory
  - Measure concentration
  - Applications in Probability Theory

Selected Contributions to Information Theory

- Broadcast channels:
  J. Körner, K. Marton, "Comparison of two noisy channels," *Colloquia Mathematica Societatis*, János Bolyai, 16, *Topics in Information Theory*, North Holland, pp. 411-424, 1977
  J. Körner, K. Marton, "Images of a set via two different channels and their role in multiuser communication," *IEEE Transactions on Information Theory*, IT-23, pp. 751-761, Nov. 1977
  J. Körner, K. Marton, "General broadcast channels with degraded message sets," *IEEE Trans. on Information Theory*, IT-23, pp. 60-64, Jan. 1977
  K. Marton, "A coding theorem for the discrete memoryless broadcast channel," *IEEE Trans. on Information Theory*, IT-25, pp. 306-311, May 1979
- Strong converse:
  K. Marton, "A simple proof of the Blowing-Up Lemma," *IEEE Trans. on Information Theory*, IT-32, pp. 445-446, 1986
- Coding for computing via structured codes:
  J. Körner, K. Marton, "How to encode the mod-2 sum of two binary sources?," *IEEE Trans. on Information Theory*, Vol. 25, pp. 219-221, March 1979
- Rate distortion theory:
  K. Marton, "Asymptotic behavior of the rate distortion function of discrete stationary processes," *Problemy Peredachi Informatsii*, VII, 2, pp. 3-14, 1971
  K. Marton, "On the rate distortion function of stationary sources," *Problems of Control and Information Theory*, 4, pp. 289-297, 1975
- Error exponents:
  K. Marton, "Error exponent for source coding with a fidelity criterion," *IEEE Trans. on Information Theory*, Vol. 29, pp. 197-199, March 1974
  I. Csiszár, J. Körner, K. Marton, "A new look at the error exponent of coding for discrete memoryless channels," *IEEE Symposium on Information Theory*, Oct. 1977
- Isomorphism:
  K. Marton, The problem of isomorphy for general discrete memoryless sources, *Z. Wahrscheinlichkeitstheorie verw. Geb.*, 53. pp. 51-58, 1983
- Entropy and capacity of graphs:
  J. Körner, K. Marton, "Random access communication and graph entropy," *IEEE Trans. on Inform. Theory*, Vol. 34, No. 2, 312-314, 1988
  K. Marton, "On the Shannon capacity of probabilistic graphs," *J. of Combinatorial Theory*, 57, pp. 183-195, 1993

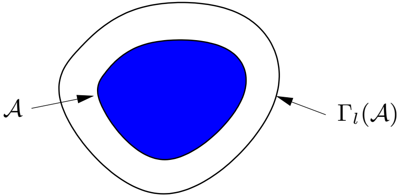
Blowing-Up Lemma

K. Marton, "A simple proof of the Blowing-Up Lemma," *IEEE Trans. on Information Theory*, IT-32, pp. 445-446, 1986

- Lemma first proved by Ahlswede, Gacs, Körner (1976)
- Used to prove strong converse, e.g., for degraded DM-BC
- Complicated, combinatorial proof

Blowing-Up Lemma

- Let $x^n, y^n \in \mathcal{X}^n$ and $d(x^n, y^n)$ be Hamming distance between them
- Let $\mathcal{A} \subseteq \mathcal{X}^n$. For $l \leq n$, let $\Gamma_l(\mathcal{A}) = \{x^n : \min_{y^n \in \mathcal{A}} d(x^n, y^n) \leq l\}$



Blowing up Lemma

Let $X^n \sim \mathrm{P}_{X^n} = \prod_{i=1}^n \mathrm{P}_{X_i}$ and $\epsilon_n \to 0$ as $n \to \infty$. There exist $\delta_n, \eta_n \to 0$ as $n \to \infty$ such that if $\mathrm{P}_{X^n}(\mathcal{A}) \geq 2^{-n\epsilon_n}$, then $\mathrm{P}_{X^n}(\Gamma_{n\delta_n}(\mathcal{A})) \geq 1 - \eta_n$

Marton's Simple Proof

- The proof uses the following information theoretic coupling inequality

Lemma 1

Let $X^n \sim \prod_{i=1}^n \mathrm{P}_{X_i}$ and $\hat{X}^n \sim \mathrm{P}_{\hat{X}^n}$. Then, there exists a joint probability measure $\mathrm{P}_{X^n,\hat{X}^n}$ with these given marginals such that

$$\frac{1}{n}\mathrm{E}(d(X^n,\hat{X}^n)) = \frac{1}{n}\sum_{i=1}^n \mathrm{P}\{X_i \neq \hat{X}_i\} \leq \left(\frac{1}{n}D\Big(\mathrm{P}_{\hat{X}^n}\,\Big\|\,\prod_{i=1}^n \mathrm{P}_{X_i}\Big)\right)^{1/2}$$

- Now, define

$$\mathrm{P}_{\hat{X}^n}(x^n) = \mathrm{P}_{X^n|\mathcal{A}}(x^n) = \begin{cases} \frac{\mathrm{P}_{X^n}(x^n)}{\mathrm{P}_{X^n}(\mathcal{A})} & \text{if } x^n \in \mathcal{A}, \\ 0 & \text{if } x^n \notin \mathcal{A} \end{cases}$$

Then,

$$D\Big(\mathrm{P}_{\hat{X}^n}\,\Big\|\,\prod_{i=1}^n \mathrm{P}_{X_i}\Big) = -\log \mathrm{P}_{X^n}(\mathcal{A}) \leq n\epsilon_n$$

- By Lemma 1, there exists $\mathrm{P}_{X^n,\hat{X}^n}$ with given marginals such that

$$\mathrm{E}(d(X^n,\hat{X}^n)) \leq n\sqrt{\epsilon_n}$$

- By the Markov inequality,

$$\mathrm{P}_{X^n,\hat{X}^n}\{d(X^n,\hat{X}^n) \leq n\delta_n\} \geq 1 - \frac{\sqrt{\epsilon_n}}{\delta_n} = 1 - \eta_n,$$

where we choose $\delta_n \to 0$ such that $\eta_n \to 0$ as $n \to \infty$

- We therefore have

$$\begin{aligned}
\mathrm{P}_{X^n}(\Gamma_{n\delta_n}(\mathcal{A})) &= \mathrm{P}_{X^n,\hat{X}^n}(\Gamma_{n\delta_n}(\mathcal{A}) \times \mathcal{A}) + \mathrm{P}_{X^n,\hat{X}^n}(\Gamma_{n\delta_n}(\mathcal{A}) \times \mathcal{A}^c) \\
&= \mathrm{P}_{X^n,\hat{X}^n}(\Gamma_{n\delta_n}(\mathcal{A}) \times \mathcal{A}) \\
&\stackrel{*}{=} \mathrm{P}_{X^n,\hat{X}^n}\{d(X^n,\hat{X}^n) \leq n\delta_n\} \\
&\geq 1 - \eta_n
\end{aligned}$$

* follows since $\mathrm{P}_{X^n,\hat{X}^n}(x^n,\hat{x}^n) = 0$ if $\hat{x}^n \notin \mathcal{A}$

Impact on My Work

J. Körner, K. Marton, "Comparison of two noisy channels," *Colloquia Mathematica Societatis*, János Bolyai, 16, *Topics in Info. Th.*, North Holland, pp. 411-424, 1977

- Defined less noisy and more capable BC
- I established the capacity region of the more capable class in 1977

K. Marton, "A coding theorem for the discrete memoryless broadcast channel," *IEEE Trans. on Information Theory*, IT-25, pp. 306-311, May 1979

- Best known inner bound on capacity region of BC
- van der Meulen and I provided an alternative proof in 1980
- Motivated the EGC region for multiple description coding in 1981

J. Körner, K. Marton, "General broadcast channels with degraded message sets," *IEEE Trans. on Information Theory*, IT-23, pp. 60-64, Jan. 1977

- Nair and I in 2008 showed that the extension of their superposition region to > 2 receivers is not optimal in general

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