Distributed Lossy Computing

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Based on work with Han-I Su, P. Cuff, Young-Han Kim

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ITA 2010
Introduction

- Performance of distributed information processing systems often limited by communication
  - Digital VLSI
  - Multi-processors
  - Data centers
  - Peer-peer networks
  - Sensor networks
  - Networked mobile agents

Purpose of communication is to make decision, compute function, coordinate action based on distributed data.

How much communication is needed to perform such a task?
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Problem Formulations

Problem formulated and studied in several fields:

- **Computer science**: Communication complexity; gossip
- **Information theory**: Coding for computing; \( \mu \)-sum problem
- **Control**: Distributed consensus
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- Formulations differ in:
  - **Data model** (discrete, continuous; deterministic, random)
  - **Type of coding–computing** (scalar, block)
  - **Estimation criterion** (error-free; lossless; lossy)
  - **Metric for communication cost** (bits; rate; rounds)
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This Talk: Lossy Distributed Computing

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- Advantages of IT formulation:
  - Many real-world sources have large (continuous) alphabets
  - Limits that hold “in general”
  - Asymptotic approach simplifies analysis
Distributed Consensus

Distributed coordination, information aggregation in distributed systems

- Flocking and schooling behaviors in nature [Vicsek et al., 95]
- Spread of rumors, epidemics [Demers et al., 87]
- Coordination of autonomous vehicles [Jadbabaie et al., 03]
- Load balancing in parallel computers
- Finding file sizes in peer-to-peer networks [Bawa et al., 03]
- Information aggregation in sensor networks [Kempe et al., 03]
Distributed Averaging [Olfati-Saber et al., 03]

- Undirected graph \((\mathcal{M}, \mathcal{E})\) with \(m\) nodes
- Node \(j\) observes real-valued scalar \(x_j\)
- Each node wishes to estimate the average \(s = (1/m) \sum_{j=1}^{m} x_j\) to some prescribed MSE
Averaging Protocols

- Nodes communicate and perform local computing in *rounds*
  - For example, a node-pair is selected in each round
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- In many applications, communication is asynchronous, subject to link and node failures and topology changes
  - **Distributed protocols**: Do not depend on node identities
  - **Gossip protocols**: Random node subset selections
    [Hedetniemi et al., 88]
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Do the estimates converge to average? How many rounds are needed to achieve prescribed MSE?
Example: Gossip Protocol

- In each round, a node-pair \((j, k)\) is randomly selected.
- Nodes exchange values and average them:
  \[
  s_j(0) = x_j, \quad j = 1, 2, \ldots, m
  \]
  \[
  s_j(t + 1) = \frac{1}{2} s_j(t) + \frac{1}{2} s_k(t)
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Example: Gossip Protocol

- MSE = 0.00875
- Estimates converge to the average
- Bounds on the number of rounds as function of MSE [Boyd et al., 05]
Example: Gossip Protocol

Issues:
- Communicating/computing with infinite precision not realistic
- Number of rounds not good measure of communication cost
Quantized Distributed Averaging

**Control:** Scalar quantization


Signal Processing: Model quantization noise as additive noise

- M. Yildiz, A. Scaglione, ”Coding with side information for rate constrained consensus,” *IEEE Trans. on Sig. Proc.*, 2008

Information Theory: Treat quantization noise indirectly as link capacity constraint

Quantized Distributed Averaging

- **Control**: Scalar quantization

- **Signal Processing**: Model quantization noise as additive noise
Quantized Distributed Averaging

- **Control**: Scalar quantization

- **Signal Processing**: Model quantization noise as additive noise

- **Information Theory**: Treat quantization noise indirectly as link capacity constraint
Distributed Lossy Averaging [Su, EG 09]

- Graph with $m$ nodes. Node $j$ observes source $X_j$
- Assume $(X_1, \ldots, X_m)$ jointly Gaussian
- Each node wishes to estimate $S^n = (1/m) \sum_{j=1}^{m} X^n_j$
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**Averaging protocol:**
- Number of communication rounds $T$
- Sequence of node pairs selected (deterministic or random)
- Block code (two-way) used by each selected pair in each round
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- **Averaging protocol:**
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  - Sequence of node pairs selected (deterministic or random)
  - Block code (two-way) used by each selected pair in each round
- Let $R$ be sum rate over the $T$ rounds
- Let $S^n_j(T)$ be estimate of node $j$ at end of round $T$
- **Per-letter MSE distortion:**
  \[
  D_j^{(n)} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[(S_i - S_{ji}(T))^2]
  \]
Distributed Lossy Averaging

For fixed $T$ and node-pair selections:

- Rate distortion pair $(R, D)$ achievable if there exists a sequence of codes with rate $R$ such that

$$\limsup_{n \to \infty} \frac{1}{m} \sum_{j=1}^{m} D_j^{(n)} \leq D$$

- $R(D) = \inf\{R : (R, D) \text{ is achievable}\}$
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- The network rate–distortion function:
  $$R^*(D) = \inf \{ R(D) : \text{all node-pair selection sequences and } T \}$$
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- The network rate–distortion function:
  \[
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  \]
- $R^*(D)$ is not known in general
$R^*(D)$ for 2-Node Network

- 2-nodes; $(X_1, X_2)$ 2-WGN$(1, \rho)$; arbitrary number of rounds
- Nodes wish to estimate average $S^n = (X_1^n + X_2^n)/2$ with MSE distortion $D$

\[
R^*(D) = \log \left( \frac{1 - \rho^2}{4D} \right)
\]

- Achieved using two independent Wyner-Ziv coding rounds

Theorem (Su, EG 09)
2-Source Distributed Lossy Averaging

- Three nodes; \((X_1, X_2) \sim 2\text{-WGN}(1, \rho)\)
- Communication: 1 → 3, 2 → 3
- Node 3 wishes to estimate \(S^n = (X_1^n + X_2^n)/2\) with distortion \(D\)
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- Problem recently solved (\(\mu\)-sum) by Tse, Ramchandran 04; Oohama 05; Wagner, Tavildar, Viswanath 08
3-Node Cascade Network [Cuff, Su, EG 09]

- Three nodes; \(X_1, X_2\) independent WGN(\(P_1\)), WGN(\(P_2\)) sources
- Two rounds: \(1 \rightarrow 2\) followed by \(2 \rightarrow 3\)
- Node 3 wishes to estimate \(S^n = (X_1^n + X_2^n)/2\) with distortion \(D\)

\[
\begin{align*}
X_1^n & \quad M_1(X_1^n) \\
X_2^n & \quad M_2(X_2^n, M_1) \\
3 & \quad \hat{S}^n
\end{align*}
\]
3-Node Cascade Network [Cuff, Su, EG 09]

- Three nodes; $X_1, X_2$ independent WGN($P_1$), WGN($P_2$) sources
- Two rounds: $1 \rightarrow 2$ followed by $2 \rightarrow 3$
- Node 3 wishes to estimate $S^n_3 = (X^n_1 + X^n_2)/2$ with distortion $D$

Achievability schemes:

- **Compute–Compress:**
  \[
  \hat{S}^n_3 \left( M_1(X^n_1) \right)
  \]

Neither strategy is optimal; problem remains open.
Three nodes; $X_1, X_2$ independent WGN($P_1$), WGN($P_2$) sources
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Achievability schemes:

- **Compute–Compress:**
  
  $M_1(X_1^n) \quad M_2(X_2^n, M_1) \quad \hat{S}^n$

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Cutset Lower Bound on $R^*(D)$

- Independent WGN(1) sources
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Super-node

$$P = \frac{m-1}{m^2}$$
Cutset Lower Bound on $R^*(D)$

- Independent WGN(1) sources

Super-node $P = \frac{m-1}{m^2}$

Theorem (Su,EG 09)

\[ R^*(D) \geq \frac{m}{2} \log \left( \frac{m - 1}{m^2 D} \right) \quad \text{for} \quad D < \frac{(m - 1)}{m^2} \]
Upper Bound on $R^*(D)$

- Use centralized protocol
Upper Bound on $R^*(D)$

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- Round $t = 1, \ldots, (m - 1)$:

\[ r_j(t) = \frac{1}{2} \log \left( \frac{1}{d} \right) \]

\[ d = \frac{mD}{2} \]
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- Round $t = m, \ldots, (2m - 2)$:
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Upper Bound on $R^*(D)$

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- Upper bound:
  \[
  R^*(D) \leq (m - 1) \log \left( \frac{2}{mD} \right)
  \]
  Within 2x of cutset bound
Distributed Weighted-Sum Protocols

- $T$ rounds of node-pair, two-way communication/computing
- Estimate of node $j$ at $t = 0$, $S^n_j(0) = X^n_j$
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- Assume $(j, k)$ selected in round $t > 0$:
  - Node $j$: Sends description $\hat{S}_j^n(t)$ to node $j$
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  - Similarly, node $k$ sends description $\hat{S}_k^n(t)$ to node $j$
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  - Nodes update their estimates

\[
S^n_v(t + 1) = \frac{1}{2} S^n_v(t) + \frac{1}{2(1 - d)} \hat{S}^n_{j+k-v}(t) \text{ for } v = j, k
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- Define $R^*_{WS}(D) \geq R^*(D)$
Distributed Weighted-Sum Protocols

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- Define $R_{WS}^*(D) \geq R^*(D)$

Theorem (Lower Bound on $R_{WS}^*(D)$)

\[
R_{WS}^*(D) \geq \left( \frac{m}{2} \log \frac{1}{\sqrt{D} + 1/m} \right) \left( \log \frac{1}{4mD} \right)
\]
Gossip-Based Weighted-Sum Protocol

- Node-pair selected independently at random in each round
Gossip-Based Weighted-Sum Protocol

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- Expected weighted-sum network rate distortion function

\[ E(R_{WS}(D)) = \inf \{ E(R) : (R, \Delta) \text{ is achievable}, E(\Delta) \leq D \} \]
Gossip-Based Weighted-Sum Protocol

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- **Expected weighted-sum network rate distortion function**

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- \( R^*_{WS}(D) \leq E(R_{WS}(D)) \)

**Theorem (Upper bound on \( E(R_{WS}(D)) \))**

\[
E(R_{WS}(D)) \leq (m - 1) \left( \ln \frac{2}{D} \right) \left( \log \frac{(m - 1) \ln(2/D)}{m^2 D} \right)
\]
Summary of Bounds

- Complete graph
### Summary of Bounds

- **Complete graph**

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The penalty of using a distributed protocol is a factor of $\log m$ in the sum rate.
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Penalty of using distributed protocol is factor of $\log m$ in sum rate
Conclusion

- Distributed lossy averaging as example of lossy distributed computing
  - Upper and lower bounds on network rate distortion function
  - Formulation allowed us to quantify the penalty of distributedness
  - Network rate distortion function not known in general

To learn more:
Conclusion

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- To learn more:
  - Su, EG, “Distributed Lossy Averaging,”
    http://arxiv.org/abs/0901.4134
  - Cuff, Su, EG, “Cascade Multiterminal Source Coding,”
    http://arxiv.org/abs/0905.1883
  - Kim, EG, “Lecture notes on network information theory,”
Thank You
Effect of Using Correlation

- We ignored the build up in correlation
- Can achieve better rate using Wyner-Ziv coding
- Very difficult to analyze
Effect of Using Correlation

- We ignored the build up in correlation
- Can achieve better rate using Wyner-Ziv coding
- Very difficult to analyze
- Using simulations ($m = 50$):

![Graph showing effect of using correlation with and without correlation, and an upper bound.]
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