Lecture Notes 10
Image Sensor Optics

- Imaging optics
  - Space-invariant model
  - Space-varying model
- Pixel optics
  - Transmission
  - Vignetting
- Microlens
Image sensor optics consist of (a) imaging optics and (b) pixel optics.
A generalized model for rotationally symmetric imaging optics

Imaging Optics: Camera Equation

\[ I_{\text{ideal}}(x, y; \lambda) \equiv \pi \frac{T(\lambda)R(x, y; \lambda)}{1 + 4(1 + m)^2 (f/\#)^2} L_{\text{scene}} \left( \frac{x}{m}, \frac{y}{m}; \lambda \right) \]

- \( I_{\text{ideal}} \): ideal, unaberrated, geometric image irradiance distribution \( (W/m^2) \)
- \( L_{\text{scene}} \): Lambertian object radiance distribution \( (W/sr \ m^2) \)

- \( m = -|z_i/z_o| \): magnification
- \( T \): transmittance
- \( R \): relative illumination factor (e.g., \( \cos^4 \) fall-off)
- \( f/\# = f/D \): f-number

Imaging optics map 2D object radiance distribution \( (W/sr \ m^2) \) into 2D image irradiance distribution \( (W/m^2) \)

Diffraction, which is caused at aperture edges, is the fundamental reason why a plane wave can not be focussed into a perfect geometric point.
Diffraction: Space Domain Description

Image formation process with incoherent illumination can be described in the space domain by a convolution operation

\[ I_{\text{image}}(x, y; \lambda) = \text{PSF}(x, y; \lambda) \ast I_{\text{ideal}}(x, y; \lambda) \]

\( I_{\text{image}} \): blurred, aberrated, distorted image irradiance distribution
\( I_{\text{ideal}} \): ideal, unaberrated, geometric image irradiance distribution

PSF: point spread function, i.e., response of optical system in image plane to a point excitation in object plane
\( \ast \): convolution operator
Diffraction: Frequency Domain Description

Alternatively, image formation in shift-invariant systems can be viewed as a linear filtering process in the frequency domain

\[
F \{I_{image}(x, y; \lambda)\} = F \{\text{PSF}(x, y; \lambda)\} \cdot F \{I_{ideal}(x, y; \lambda)\} = \text{OTF}(f_x, f_y; \lambda) \cdot F \{I_{ideal}(x, y; \lambda)\}
\]

\[
I_{image}(x, y; \lambda) = F^{-1}\{\text{OTF}(f_x, f_y; \lambda) \cdot F \{I_{ideal}(x, y; \lambda)\}\}
\]

The optical transfer function (OTF) is normalized to preserve radiometry. Its magnitude is the modulation transfer function (MTF)

\[
\text{MTF}(f_x, f_y; \lambda) = |\text{OTF}(f_x, f_y; \lambda)| = \left. \frac{F \{\text{PSF}(x, y; \lambda)\}}{F \{\text{PSF}(x, y; \lambda)\}} \right|_{f_x=0, f_y=0}
\]
Example: Diffraction-limited Lens

\[ \text{MTF}(f_r) = \frac{2}{\pi} (\varphi - \cos \varphi \sin \varphi) \text{ with } \varphi = \cos^{-1}(f_r \lambda f/\#) \text{ and } f_{r, \text{cutoff}} = \frac{1}{\lambda f/\#} \]
System MTF

If the system is linear and space-invariant, the system MTF (optics + sensor) in the frequency domain can then be easily computed

$$MTF_{system}(f_x, f_y; \lambda) = MTF_{optics}(f_x, f_y; \lambda) \cdot MTF_{geometric}(f_x, f_y) \cdot MTF_{diffusion}(f_x, f_y; \lambda)$$

To obtain the image spectrum we apply the MTF as a linear filter

$$I_{image}(f_x, f_y; \lambda) = MTF_{system}(f_x, f_y; \lambda) \cdot I_{object}(f_x, f_y; \lambda)$$
Modeling Real Imaging Lenses

Vignetting  Distortion  Illumination Effects

Wavefront Aberrations  Point Spread Function (PSF)  Modulation Transfer Function (MTF)

Image plane is segmented into isoplanatic regions: $\Omega_1, \Omega_2, \ldots, \Omega_n$
Let $\Omega_k$, $k = 1, \ldots, n$ be aplanatic image segments, then

$$I_{image} (x, y; \lambda) = \sum_k \text{PSF}_{\Omega_k} (x, y; \lambda) \ast I_{\text{ideal},\Omega_k} (x, y; \lambda)$$

or

$$I_{image} (x, y; \lambda) = \mathcal{F}^{-1} \left\{ \sum_k \text{OTF}_{\Omega_k} (f_x, f_y; \lambda) \cdot \mathcal{F} \left\{ I_{\text{ideal},\Omega_k} (x, y; \lambda) \right\} \right\}$$

Example: Double Gauss f/2.0 Lens

Double Gauss f/2.0 Lens: MTF

Pixel Optics

Incident Photons

Pixel Transmittance

\[ F^+(z') \quad F^-(z') \]
Layer 0

\[ L_0 \]
Layer 1

\[ L_1 \]
Layer j-1

\[ L_{j-1,j} \]
Layer j

\[ L_j \]
Layer m

\[ L_m \]
Layer m+1

\[ L_{m,m+1} \]

\[ F^+(z'') \quad F^-(z) \]

**Transmission**

F. Abeles, Ann. de Phys. 5, pp. 596-640 (1950)
Example: Transmittance 0.18\(\mu\)m CMOS

Pixel transmittance is \(\lambda\)-dependent (even for dispersion-free materials)

Example: Transmittance 0.18μm CMOS

Pixel transmittance (λ-averaged) is approximately independent of angle

Pixel Vignetting

Pixel Vignetting: Effect of Pixel Height

Reduction in optical efficiency as a function of the number of metal layers in a 0.18\(\mu\)m standard CMOS process (f/1.8 imaging lens)

Pixel Vignetting: Effect of Technology Scaling

Reduction in optical efficiency for a standard APS pixel with a 30% fill-factor using 2 metal layers as a function of the feature size of CMOS technology

Optical Efficiency

- **Definition:**
  - Optical efficiency (OE) is the ratio of the photons incident of the substrate and the photons incident on the pixel surface.

- **Sources of photon loss:**
  - Back-reflections in dielectric stack (air-SiO$_2$, SiO$_2$-Si)
  - Photons absorbed in dielectric stack (SiON)
    - \( \Rightarrow \) Pixel transmittance \( T(\lambda,\theta) \)
  - Photons scattered away from pixel (pixel cross-talk)
  - Photons rejected by metal
    - \( \Rightarrow \) Pixel vignetting \( V(x,y,\theta) \)

- **Description:**
  - \( OE(x,y,\lambda,\theta) = T(\lambda,\theta) V(x,y,\theta) \)
Microlens

- Focus light onto photo-sensitive region – increases effective fill factor from 25-40% to 60-80% (and sensitivity by $\geq 2X$)
- Less effective if photosensitive area is irregularly shaped

Microlens Fabrication

- Lens material requirements:
  - Highly transparent in the visible light region
  - Index of refraction $> 1.59$
  - Can be applied below $500^\circ C$
  - No degradation or aging
  - Semiconductor processing compatible
  - Can be patterned with feature size commensurate with the pixel size
- Lens materials are typically i-line or DUV resists
- Base materials are acrylic-based resists, polyimide resists, epoxy resists, polyorganosiloxane, polyorganosilicate
Microlens and the Main Lens

- Microlens is optimized for a specific main lens system
- Rays incident on the microlens form a cone with $\text{NA} = \sin \varphi$
- NA varies as a function of the size and position of the exit pupil
- Principle ray at the periphery of the sensor has an angle $\delta$, chief ray angle (CRA), between the ray and the optical axis ($\delta$ depends on the position of the pixel on the sensor)

Microlens and F-Number

- High F-number: rays are parallel (NA ≈ 0)
- Low F-number: rays arrive at an angle (NA large) – microlens effectiveness low

Micro lens: How to concentrate photons

Shallow Pixel

Deep Pixel
Micro lens: How to concentrate photons

Shallow Pixel

Deep Pixel
Micro lens: How to concentrate photons

\[ \text{Shallow Pixel} \]

\[ NA = \sqrt{\frac{1}{1+4(f/#)^2}} \]

\[ \text{Deep Pixel} \]

\[ NA' = \sqrt{\frac{1}{1+4(f/#')^2}} \]
Micro lens: How to concentrate photons

**Shallow Pixel**

\[ f / \# = \frac{f_{\mu \text{Lens}}}{D_{\mu \text{Lens}}} \]

**Deep Pixel**

\[ f / \#' = \frac{f'_{\mu \text{Lens}}}{D_{\mu \text{Lens}}} \]
Micro lens: How to concentrate photons

- Conservation of etendue
  \[ G = 2NA_{\text{imaging}}w_{\text{pixel}} = 2NA_{\mu\text{lens}}w_{\text{diode}} \]

- Etendue limits light collection efficiency (NA)
  \[ \Rightarrow \] Bigger NA allows bigger etendue

- Concentration depends on ratio of NA of microlens and imaging lens
  \[ \Rightarrow \] For a 2x space concentration \((w_{\text{diode}} = w_{\text{pixel}}/2)\)
  \[ \Rightarrow 2NA_{\mu\text{lens}} > NA_{\text{imaging}} \]

- Diffraction limits spot size (f-number)
  \[ \Rightarrow \] Smaller f-number allows smaller spot size
Micro lens: Concentration

Pixel Position

- For large CRA (pixel away from the center of the lens, or exit pupil close to the sensor), ray may not focus on the photo-sensitive region
- Effect: non-uniform sensitivity profile across image sensor

Micro lens: Redirection (w/o offset)

Micro lens: Redirection (with offset)

Pixel width: 3\(\mu\)m  
Pixel Height: 8\(\mu\)m

\[
\text{Optical Efficiency} = f_{\text{ml}} \tan(\theta)
\]

Optical Efficiency: Summary

- Without microlens:
  \[ \text{OE}(x,y,\lambda,\theta) = T(\lambda,\theta) \ V(x,y,\theta) \]

- With microlens:
  \[ \text{OE}(x,y,\lambda,\theta) = T(\lambda,\theta) \ V(x,y,\theta) \ ML(x,y,\theta) \]
  *where \( ML(x,y,\theta) \) represents a correction factor to account for the concentration and/or redirection performed by the microlens
Improving OE

- Optimizing dielectric stack thickness
- Make sure dielectrics are not light absorbing
- Utilize different dielectric materials to achieve total internal reflection
- Add an airgap between pixels (total internal reflection)