Lecture Notes 9

Spatial Resolution

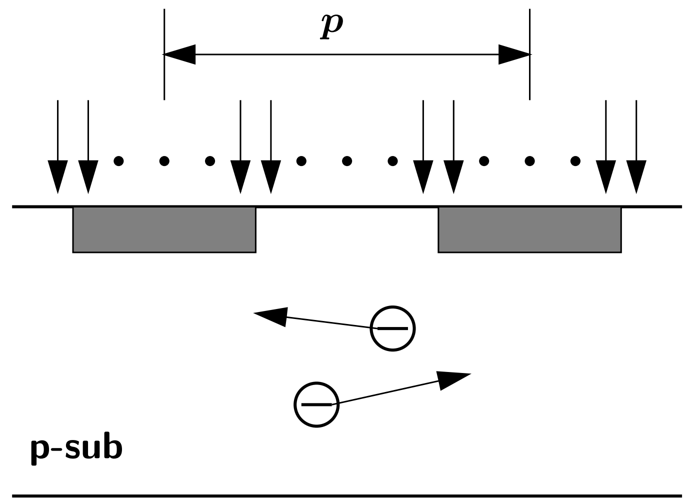
- Sensor Modulation Transfer Function (MTF)
- MTF Calculation
- Aliasing

Preliminaries

- The image sensor is a spatial (as well as temporal) sampling device (of the incident photon flux image) — the sampling theorem sets the limits for the reproducibility in space (and time) of the input spatial (and temporal) frequencies
- So spatial (or temporal) frequency components higher than the respective Nyquist rate cannot be reproduced and cause aliasing
- The image sensor, however, is not a point sampling device in space (or time), and cannot be approximated as such
  - Photocurrent is integrated over the photodetector area (and in time) before sampling
  - Photogenerated carriers in quasi-neutral regions of a pixel may diffuse and be collected by its neighboring pixels

These effects (in addition to the optics) result in *low pass filtering* and *crosstalk* before spatial (and temporal) sampling

- We focus here on spatial sampling



  - Assuming a square pixel with width (pitch) p, the spatial Nyquist rate in each dimension is $f_{\text{Nyquist}} = \frac{1}{2p}$ and is typically reported in line pairs per millimeter (lp/mm)
  - Signals (photon flux images) with spatial frequencies higher than f_{Nyquist} cannot be faithfully reproduced, and cause aliasing
  - The low pass filtering caused by integration and diffusion degrades the reproduction of frequencies below f_{Nyquist} — degradation measured by the Modulation Transfer Function (MTF)

Modulation Transfer Function (MTF)

- The *contrast* in an image can be characterized by the *modulation*

$$M = \frac{S_{max} - S_{min}}{S_{max} + S_{min}},$$

  where S_{max} and S_{min} are the maximum and minimum pixel values over the image

  Note that $0 \leq M \leq 1$

- Let the input to an image sensor be a 1-D sinusoidal monochromatic photon flux

$$F(x, f) = F_o(1 + \cos(2\pi f x)), \text{ for } 0 \leq f \leq f_{\text{Nyquist}}$$

  The sensor *modulation transfer function* is defined as

$$\text{MTF}(f) = \frac{M_{out}(f)}{M_{in}(f)}$$

  From the definition of the input signal, $M_{in} = 1$

- MTF is in general difficult to model and analyze for a real sensor and is determined experimentally

- By making several simplifying assumptions (as we shall see), the sensor can be modeled as a 1-D linear space-invariant system with impulse response $h(x)$ that is real, nonnegative, and even

  In this case the transfer function $H(f) = \mathcal{F}[h(x)]$ is real and even, and the signal at x

$$\begin{aligned} S(x) &= F(x,f) * h(x) \\ &= F_o(1+\cos(2\pi f x)) * h(x) \\ &= F_o\left(H(0) + H(f)\cos(2\pi f x)\right) \end{aligned}$$

  Therefore

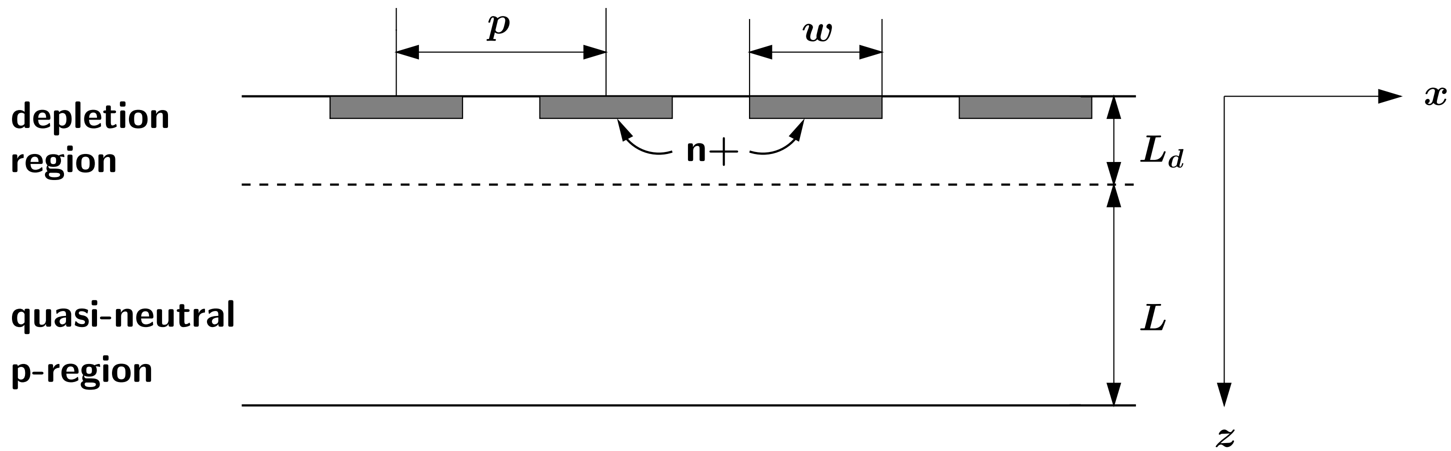
$$\begin{aligned} S_{max} &= F_o(H(0) + |H(f)|) \\ S_{min} &= F_o(H(0) - |H(f)|), \end{aligned}$$

  and the sensor MTF is given by

$$\text{MTF}(f) = \frac{|H(f)|}{H(0)}$$

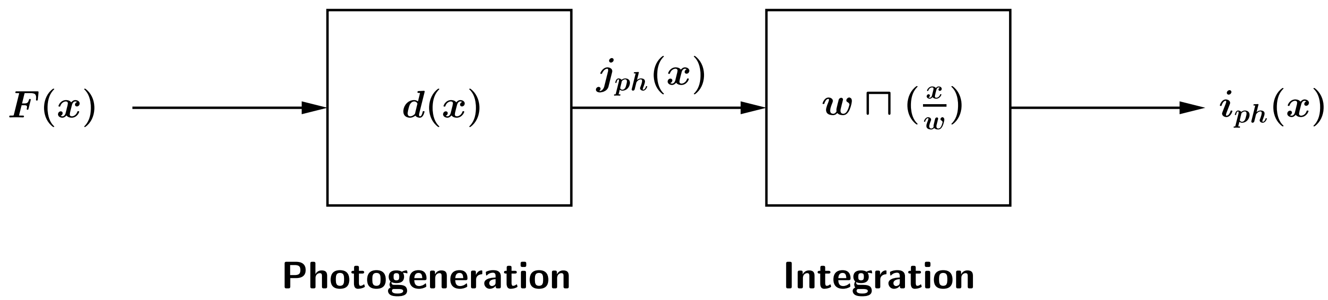
Simplified MTF Derivation

- We consider a 1-D doubly infinite image sensor



- To model the sensor response as a linear space-invariant system
  - We assume n+/p-sub photodiode with very shallow junction depth, and therefore we can neglect generation in the isolated n+ regions and only consider generation in the depletion and p-type quasi-neutral regions
  - We assume a uniform depletion region (from $-\infty$ to ∞)

- The monochromatic input photon flux $F(x)$ to the pixel current $i_{ph}(x)$ can be represented by the linear space invariant system ($i_{ph}(x)$ is sampled at regular intervals p to get the pixel photocurrents)



where

$$\sqcap\left(\frac{x}{w}\right) = \begin{cases} 1 & |x| < \frac{w}{2} \\ 0 & \text{otherwise,} \end{cases}$$

$d(x)$ is the (spatial) impluse response corresponding to the conversion from photon flux to photocurrent density (we will derive it soon), and we assume a square photodetector

- The impulse response of the system is thus given by

$$h(x) = d(x) * w \sqcap \left(\frac{x}{w}\right)$$

and its Fourier transform (transfer function) is given by

$$H(f) = D(f) w^2 \operatorname{sinc}(wf),$$

where

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

- Note that $D(0) = q\mathrm{QE}(\lambda)$, so $D(f)/q$ can be viewed as a generalized quantum efficiency (function of spatial frequency as well as wavelength)

Finding $D(f)$

- To find $D(f)$, note that the photocurrent density

$$j_{ph}(x) = j_{ph}^{sc}(x) + j_{ph}^{n}(x),$$

where $j_{ph}^{sc}(x)$ is the photocurrent density due to generation in the depletion region and $j_{ph}^{n}(x)$ is the photocurrent density due to generation in the quasi-neutral p-region

- Assuming an incident 1-D photon flux $F(x)$ (at $z = 0$) with Fourier Transform $\mathsf{F}(f)$, then

$$D(f) = \frac{\mathcal{F}[j_{ph}]}{\mathsf{F}(f)}$$

so we need to find $\mathcal{F}[j_{ph}]$

- First note that the generation rate at (x, z) is

$$g(x, z) = \alpha e^{-\alpha z} F(x)$$

- Assuming that all photogenerated charge in the depletion region is collected, we obtain

$$j_{ph}^{sc}(x) = q \int_0^{L_d} g(x,z)dz = qF(x)(1 - e^{-\alpha L_d})$$

and $\mathcal{F}[j_{ph}^{sc}] = q(1 - e^{-\alpha L_d})\mathsf{F}(f)$

- Finding $\mathcal{F}[j_{ph}^{n}]$ is more involved, first recall that

$$j_{ph}^{n}(x) = qD_n \frac{\partial n_p(x,z)}{\partial z}\Big|_{z=L_d},$$

where $n_p(x,z)$ is the photogenerated electron concentration at (x,z), so

$$\mathcal{F}[j_{ph}^{n}] = qD_n \frac{\partial N_p(f,z)}{\partial z}\Big|_{z=L_d}$$

where $N_p(f,z)$ is the Fourier Transform w.r.t. x of $n_p(x,z)$

- To find $n_p(x,z)$, we need to solve the 2-D continuity equation (in steady state)

$$0 = D_n\left(\frac{\partial^2 n_p}{\partial x^2} + \frac{\partial^2 n_p}{\partial z^2}\right) - \frac{n_p}{\tau_n} + g(x,z),$$

where the n_p/τ_n term is the recombination (which cannot be ignored here), with the assumed boundary conditions

$$\begin{aligned} n_p(x, L_d) &= 0 \quad \text{(edge of depletion region)} \\ n_p(x, L_d + L) &= 0 \qquad\qquad \text{(ohmic contact)} \end{aligned}$$

- Since we want to find $N_p(f,z)$, we take the Fourier Transform of the continuity equation w.r.t. x and we obtain

$$0 = D_n\left((j2\pi f)^2 \cdot N_p(f,z) + \frac{\partial^2 N_p}{\partial z^2}\right) - \frac{N_p}{\tau_n} + G(f,z),$$

where

$$G(f,z) = \mathcal{F}[g(x,z)] = \alpha e^{-\alpha z}\mathsf{F}(f)$$

Now we define

$$L_f^2 = \frac{L_n^2}{1 + (2\pi f L_n)^2},$$

where $L_n = \sqrt{D_n \tau_n}$ is the diffusion length of electrons in p-sub (around 600μm for our 0.5μ technology)

Substituting in the previous equation we obtain

$$-\frac{\partial^2 N_p}{\partial z^2} + \frac{N_p}{L_f^2} = \frac{G(f,z)}{D_n}$$

The solution w.r.t. z has the form

$$N_p(f,z) = c_1 e^{-\frac{z}{L_f}} + c_2 e^{\frac{z}{L_f}} + c_3 G(f,z)$$

Substituting in the PDE we obtain

$$c_3 = \frac{L_f^2}{D_n(1 - (\alpha L_f)^2)}$$

Using the boundary conditions, we obtain

$$
\begin{aligned}
c_2 &= -c_3 \frac{G(f, L_d + L) - G(f, L_d) e^{-\frac{L}{L_f}}}{e^{\frac{L_d+L}{L_f}} - e^{\frac{L_d-L}{L_f}}}, \text{ and} \\
c_1 &= -c_2 e^{\frac{2L_d}{L_f}} - c_3 G(f, L_d) e^{\frac{L_d}{L_f}}
\end{aligned}
$$

- Thus

$$
\begin{aligned}
\mathcal{F}[j_{ph}^{n}(x)] &= qD_n \frac{\partial N_p(f, z)}{\partial z}\Big|_{z=L_d} \\
&= qD_n \left(-\frac{c_1}{L_f} e^{-\frac{L_d}{L_f}} + \frac{c_2}{L_f} e^{\frac{L_d}{L_f}} - \alpha c_3 G(f, L_d) \right) \\
&= qD_n c_3 \frac{\alpha e^{-\alpha L_d}}{L_f} \left((1 - \alpha L_f) - \frac{e^{-\alpha L} - e^{-\frac{L}{L_f}}}{\sinh\left(\frac{L}{L_f}\right)} \right) \mathsf{F}(f) \\
&= qL_f \alpha e^{-\alpha L_d} \left(\frac{1}{1 + \alpha L_f} - \frac{e^{-\alpha L} - e^{-\frac{L}{L_f}}}{(1 - (\alpha L_f)^2) \sinh\left(\frac{L}{L_f}\right)} \right) \mathsf{F}(f)
\end{aligned}
$$

- Combining the results we obtain

$$\begin{aligned}\mathcal{F}[j_{ph}] &= \mathcal{F}[j_{ph}^{sc}(x) + j_{ph}^{n}(x)] \\ &= q\left(1 - e^{-\alpha L_d} + L_f \alpha e^{-\alpha L_d}\left(\frac{1}{1+\alpha L_f} - \frac{e^{-\alpha L} - e^{-\frac{L}{L_f}}}{(1-(\alpha L_f)^2)\sinh\left(\frac{L}{L_f}\right)}\right)\right)\mathsf{F}(f)\end{aligned}$$

Thus

$$D(f) = \frac{q\left(1+\alpha L_f - e^{-\alpha L_d}\right)}{1+\alpha L_f} - \frac{q L_f \alpha e^{-\alpha L_d}\left(e^{-\alpha L} - e^{-\frac{L}{L_f}}\right)}{(1-(\alpha L_f)^2)\sinh\left(\frac{L}{L_f}\right)},$$

and the system transfer function is given by

$$H(f) = q\left(\frac{1+\alpha L_f - e^{-\alpha L_d}}{1+\alpha L_f} - \frac{L_f \alpha e^{-\alpha L_d}(e^{-\alpha L} - e^{-\frac{L}{L_f}})}{(1-(\alpha L_f)^2)\sinh\left(\frac{L}{L_f}\right)}\right) \cdot w^2 \operatorname{sinc}(wf)$$

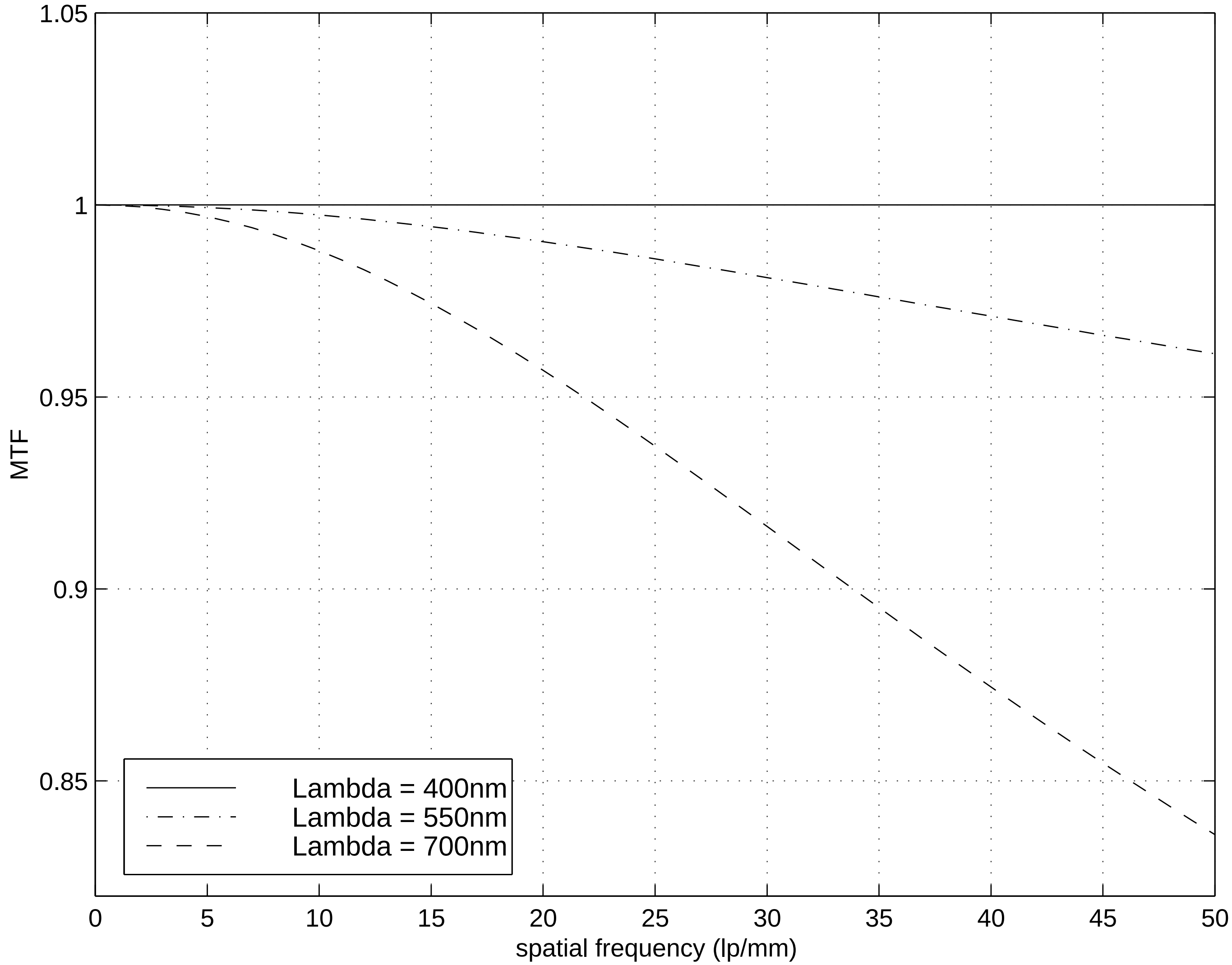
- Finally, the modulation transfer function for $|f| \leq \frac{1}{2p}$ is

$$\mathsf{MTF}(f) = \frac{|H(f)|}{H(0)} = \frac{D(f)}{D(0)} \cdot \operatorname{sinc}(wf)$$

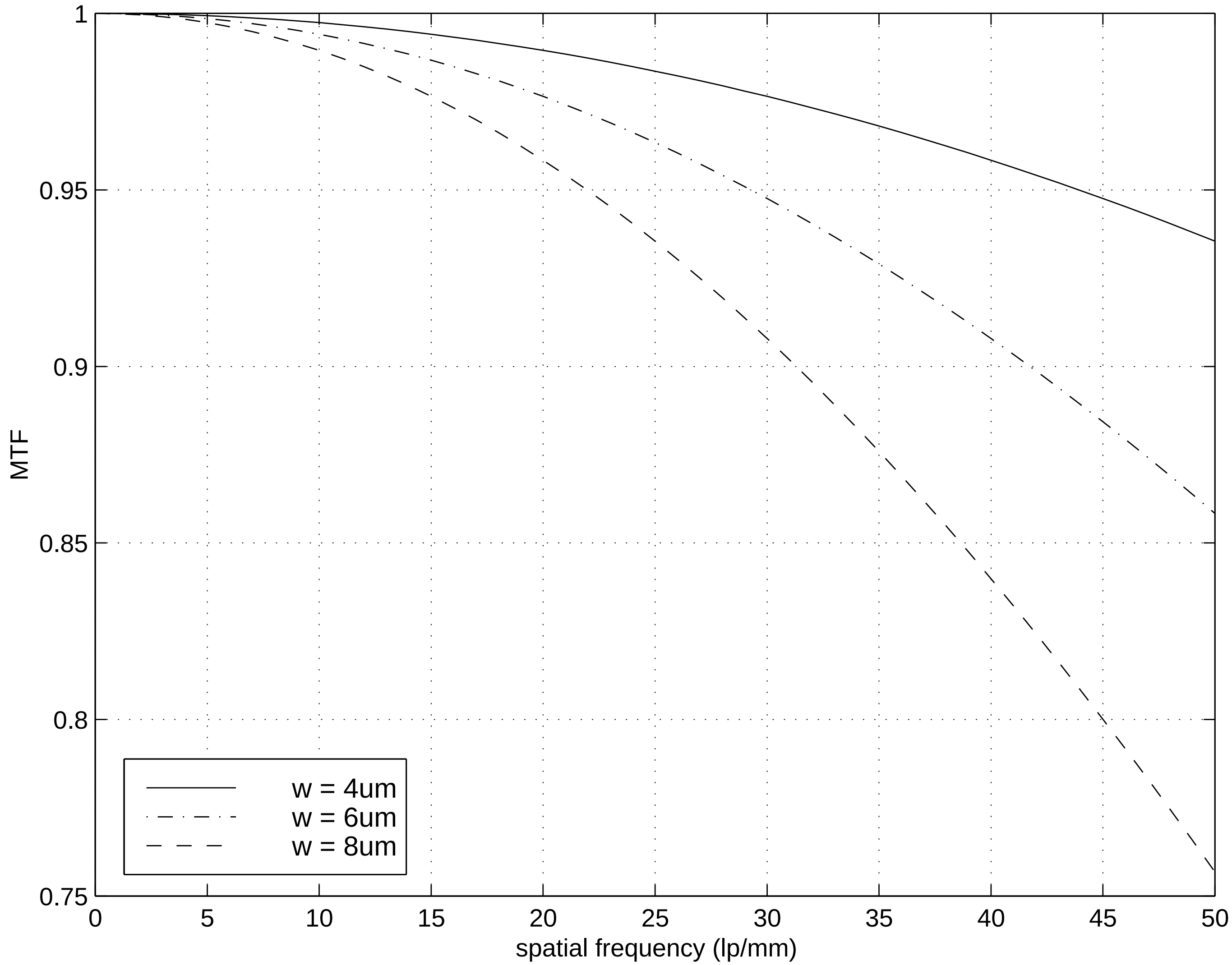
  - $\frac{D(f)}{D(0)}$ is called the *diffusion* MTF
  - $\operatorname{sinc}(wf)$ is called the *geometric* MTF

Example — Diffusion MTF

For the following examples: $p = 10\mu$m, $L_d = 1.8\mu$m, and $L = 10\mu$m

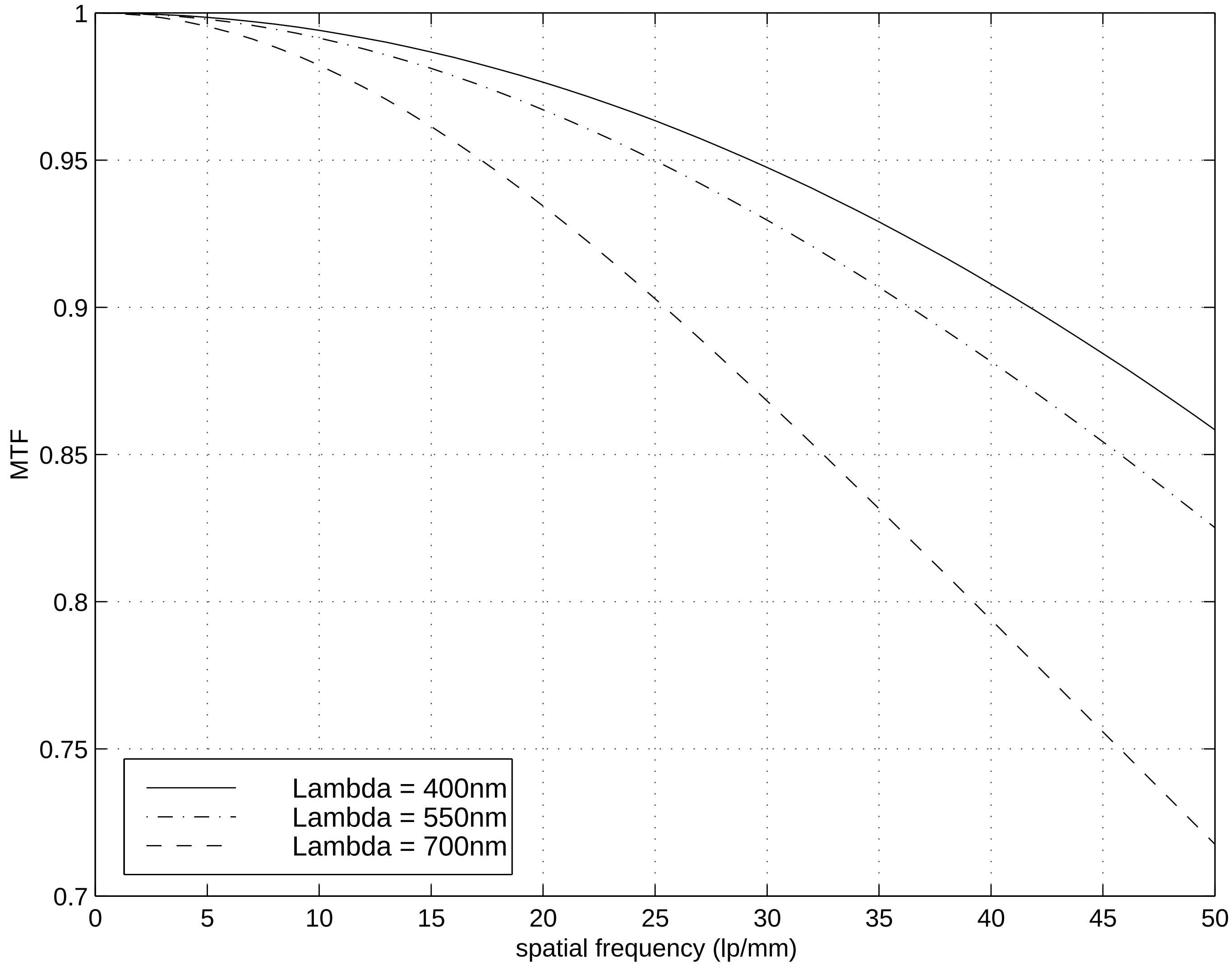


Example — Geometric MTF



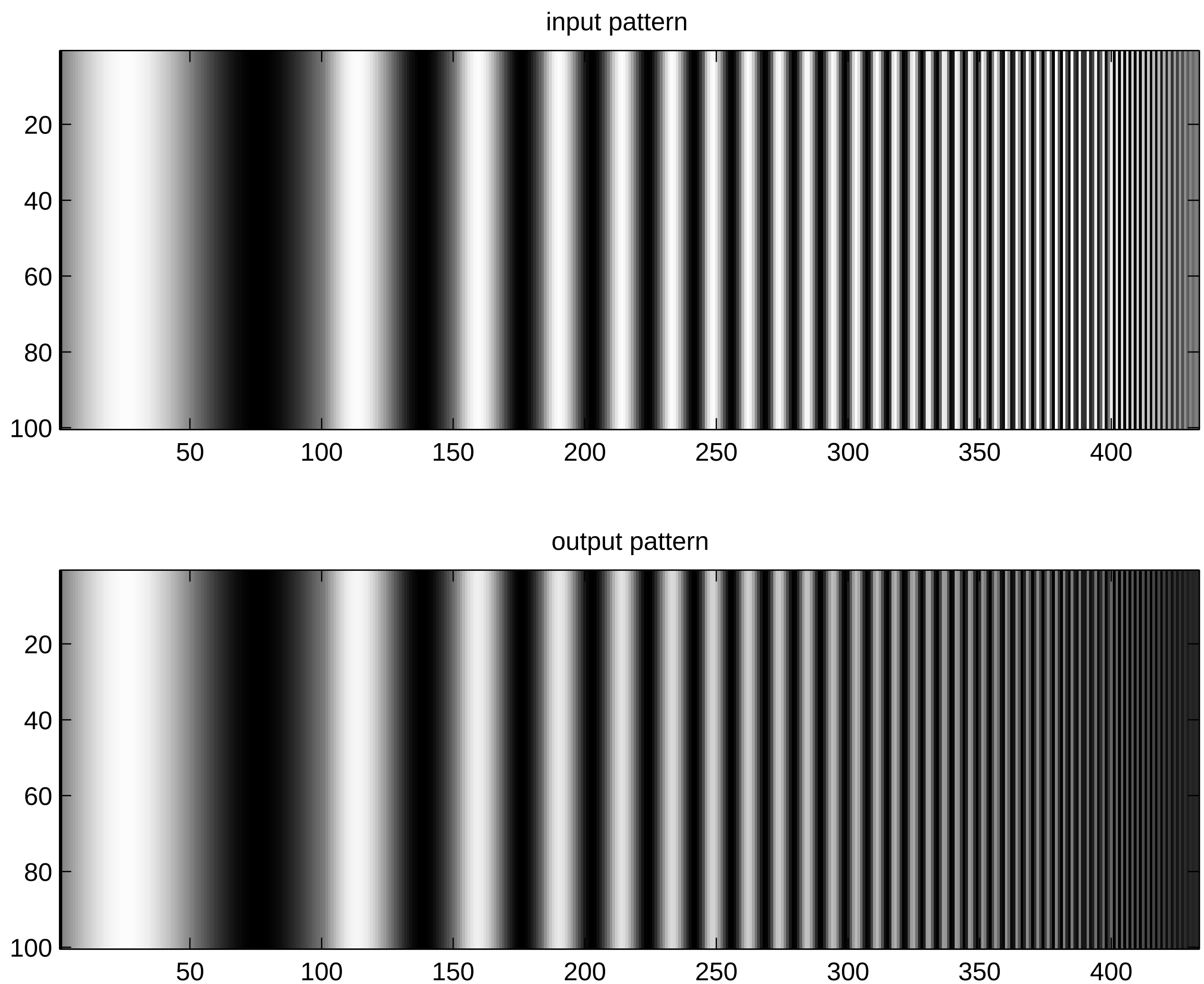
Example — Sensor MTF

Here we take $w = 6\mu$m



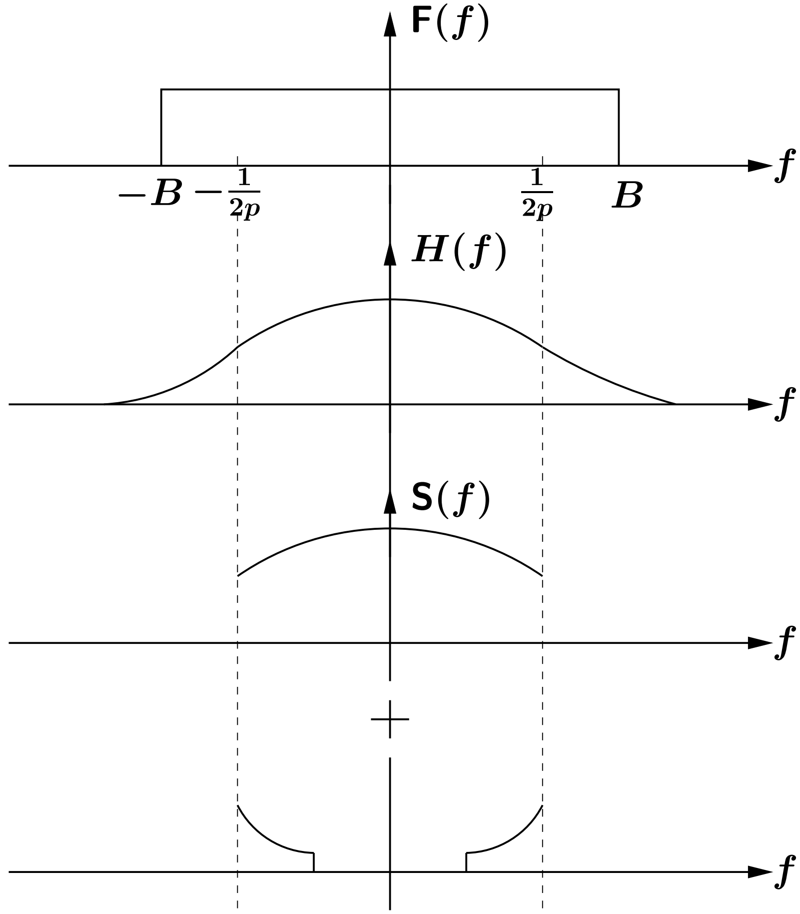
Example — Degradation due to MTF

Here $\lambda = 700$nm



Aliasing

- Aliasing occurs when the bandwidth of the input signal exceeds the Nyquist rate – the high frequency components are "folded" into the band



$S(f)$ refers to the Fourier Transform of the sampled photocurrent for $|f| \leq \frac{1}{2p}$

Aliasing Effect

Here we take $\lambda = 700$nm