Lecture Notes 1

Silicon Photodetectors

- Light Intensity and Photon Flux
- Photogeneration in Silicon
- Photodiode
    - Basic operation
    - Photocurrent derivation
    - Quantum efficiency
    - Dark current
- Direct Integration
- Photogate
- Appendices
    - Appendix I: Derivation of Continuity Equation
    - Appendix II: Depletion Width for PN Junction
    - Appendix III: MOS Capacitor
    - Appendix IV: Useful Data

Preliminaries

- Photodetector is the *front end* of the image sensor. It converts light incident on it into *photocurrent* that is (hopefully) proportional to its intensity
- Conversion is done in two steps:
  - Incident photons generate e-h pairs in the detector (e.g., silicon)
  - Some of the generated carriers are converted into photocurrent
- Photocurrents are typically very small (10s to 100s of fA)
  - Direct measurement is difficult
  - Usually integrated into charge on a capacitor and then converted to voltage before readout

Photonflux → Current density → Charge → Voltage

Quantum Efficiency

Integration space/time

Conversion Gain

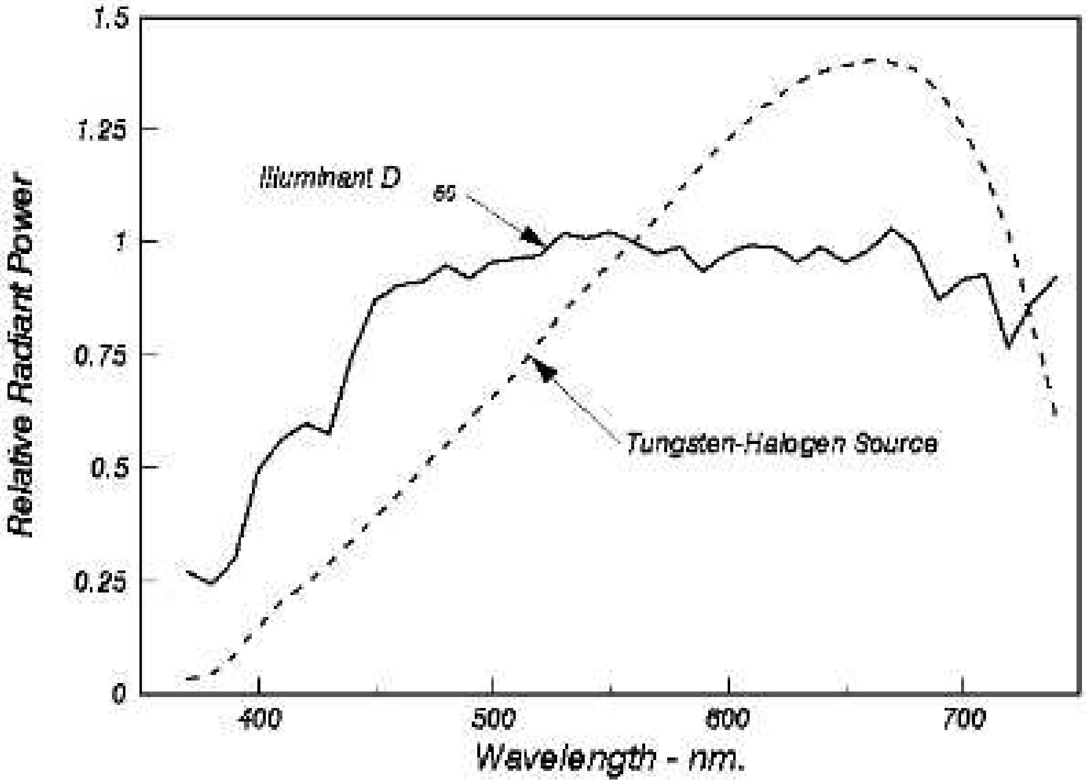
Photonflux	Current density	Charge	Voltage
$\mathrm{ph/cm^2 \cdot sec}$	$\mathrm{A/cm^2}$	Col	V

Visible Light

- We are mainly concerned with visible light image sensors
- Recall that the energy of a photon is given by $E_{ph} = hc/\lambda$, where $h = 4.135 \times 10^{-15}$eV.sec is Planck's constant, $c = 3 \times 10^{8}$m/s is the speed of light, and λ is the wavelength
- Visible light wavelengths (λ) range from 400 nm to 700 nm

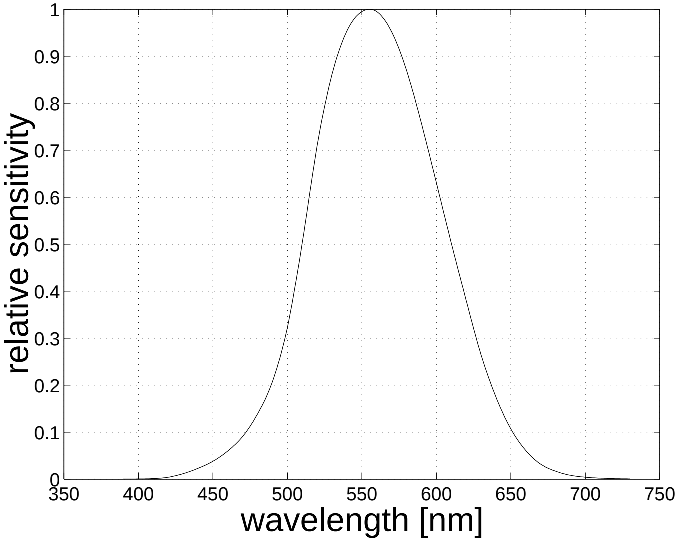
Violet: 400 nm ($E_{ph} = 3.1$ eV)
Blue: 450 nm ($E_{ph} = 2.76$ eV)
Cyan: 500 nm ($E_{ph} = 2.48$ eV)
Green: 550 nm ($E_{ph} = 2.27$ eV)
Yellow: 600 nm ($E_{ph} = 2.08$ eV)
Red: 700 nm ($E_{ph} = 1.77$ eV)
Infrared: > 800 nm ($E_{ph} < 1.55$ eV)

- The amount of light incident on an image sensor surface depends on
    - The light source
    - The surface reflectance of the object being imaged
    - The imaging optics used
- Different visible light sources, e.g., daylight (D65), incandescent, halogen, fluorescent have different power spectra



Radiometry and Photometry

- Two ways to measure the intensity of light incident on a surface:
  - Radiometry measures it as *irradiance* E W/m^2
  - Photometry measures it as *illuminance* E_ν in lux or lumens/m^2, which is defined as $\frac{1}{683}$W/m^2 at $\lambda = 555$nm
- Illuminance takes into account the sensitivity of the human eye to different wavelengths; $\lambda = 555$nm is the wavelength for which the human eye is most sensitive and the value for which the *photopic vision curve* is normalized



- Translating from irradiance to illuminance: Denote the vision photopic curve as $Y(\lambda)$ and the irradinace density $E(\lambda)$ W/m^2.nm, then the illuminance is given by

$$E_\nu = 683 \int_{400}^{700} Y(\lambda) E(\lambda)\, d\lambda \text{ lux}$$

Photon Flux

- Photon flux F_0 is the number of photons per cm^2.sec incident on a surface
- Using the photon energy $E_{ph}(\lambda)$, we can readily translate irradiance density $E(\lambda)$ into photon flux

$$F_0 = \int_{400}^{700} \frac{10^{-4}E(\lambda)}{E_{ph}(\lambda)}\, d\lambda \text{ photons/cm}^2\text{.sec}$$

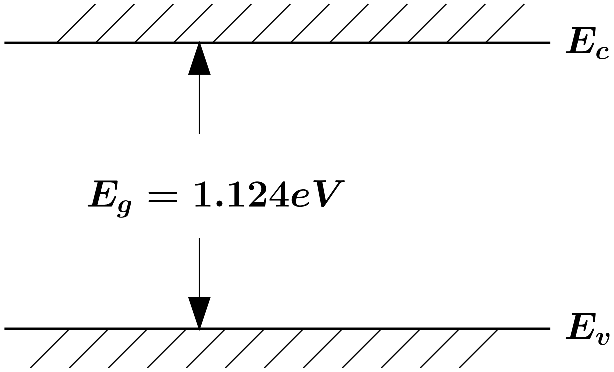
- Translating from illuminance to photon flux:
  - At $\lambda = 555$nm, $E_{ph} = 35.8 \times 10^{-20}$Joule; thus 1 lux corresponds to $F_0 = 10^{16}/683 \times 35.8 = 4.09 \times 10^{11}$photons/cm^2·sec, or, 133 photons strike a 1μm × 1μm surface per 1/30 sec
  - A typical light source (e.g., D65) has a wide range of wavelengths and 1 lux roughly corresponds to $F_0 \approx 10^{12}$photons/cm^2.sec, or, 333 photons strike a 1μm× 1μm surface per 1/30 sec

- Photon flux values encountered vary over a very wide range:

$$\begin{aligned} \text{clear sky} &\approx 10^4 \text{ Lux, or } F_0 = 10^{17} \\ \text{room light} &\approx 10 \text{ Lux, or } F_0 = 10^{13} \\ \text{full moon} &\approx 0.1 \text{ Lux, or } F_0 = 10^{11} \\ \text{moonless night} &\approx 10^{-4} \text{Lux, or } F_0 = 10^{8} \end{aligned}$$

Photocharge Generation in Semiconductors

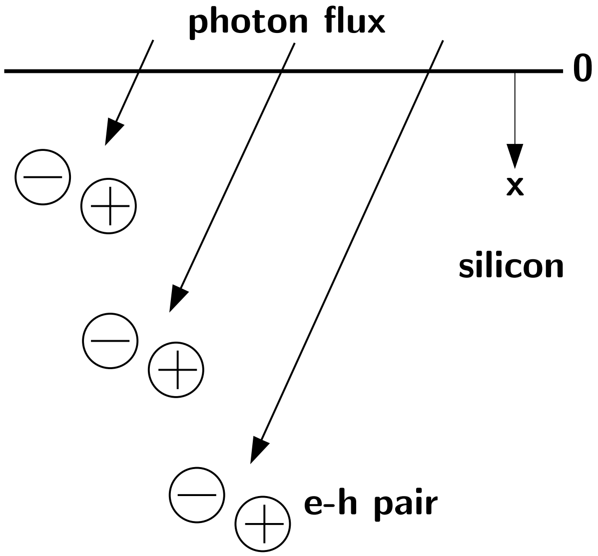
- Incident photon energy must be $>$ band gap energy (E_g) to generate an electron-hole pair
  - Electrons go to the conduction band (E_C)
  - Holes go to the valence band (E_V)
- Energy band diagram of silicon:



- Coincidentally (and luckily) photons in the visible range have enough energy to generate e-h pairs
  - No photon can generate more than one e-h pair
- Energy gap of other semiconductors: Ge (0.66 eV), GaAs (1.42 eV)

Photocharge Generation Rate in Silicon

- Assume a monochromatic photon flux F_0 photons/cm^2.sec at wavelength λ incident at the surface (i.e., $x = 0$) of silicon



- The photon absorption in a material is governed by its *absorption coefficient* $\alpha(\lambda)$ cm^{-1}
- Let $F(x)$ be the photon flux at depth x, then the number of photons absorbed per second between x and $x + \Delta x$ is given by

$$F(x) - F(x + \Delta x) \approx \alpha F(x) \Delta x,$$

We can write this equation in the limit as

$$\frac{dF(x)}{dx} = -\alpha F(x)$$

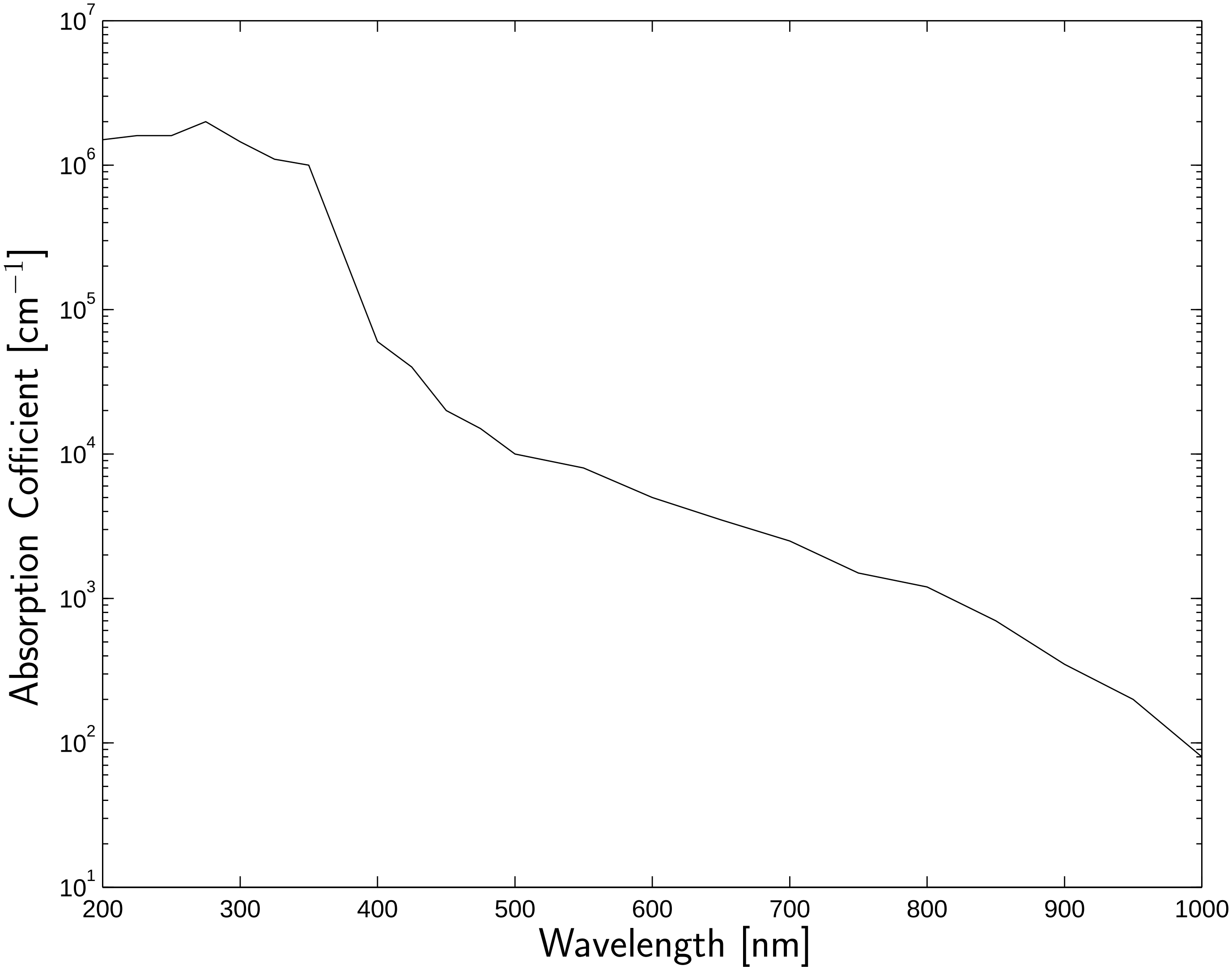
Solving we obtain

$$F(x) = F_0 e^{-\alpha x} \text{ photons/cm}^2\text{.sec}$$

Thus the rate of e-h pairs generated at x is

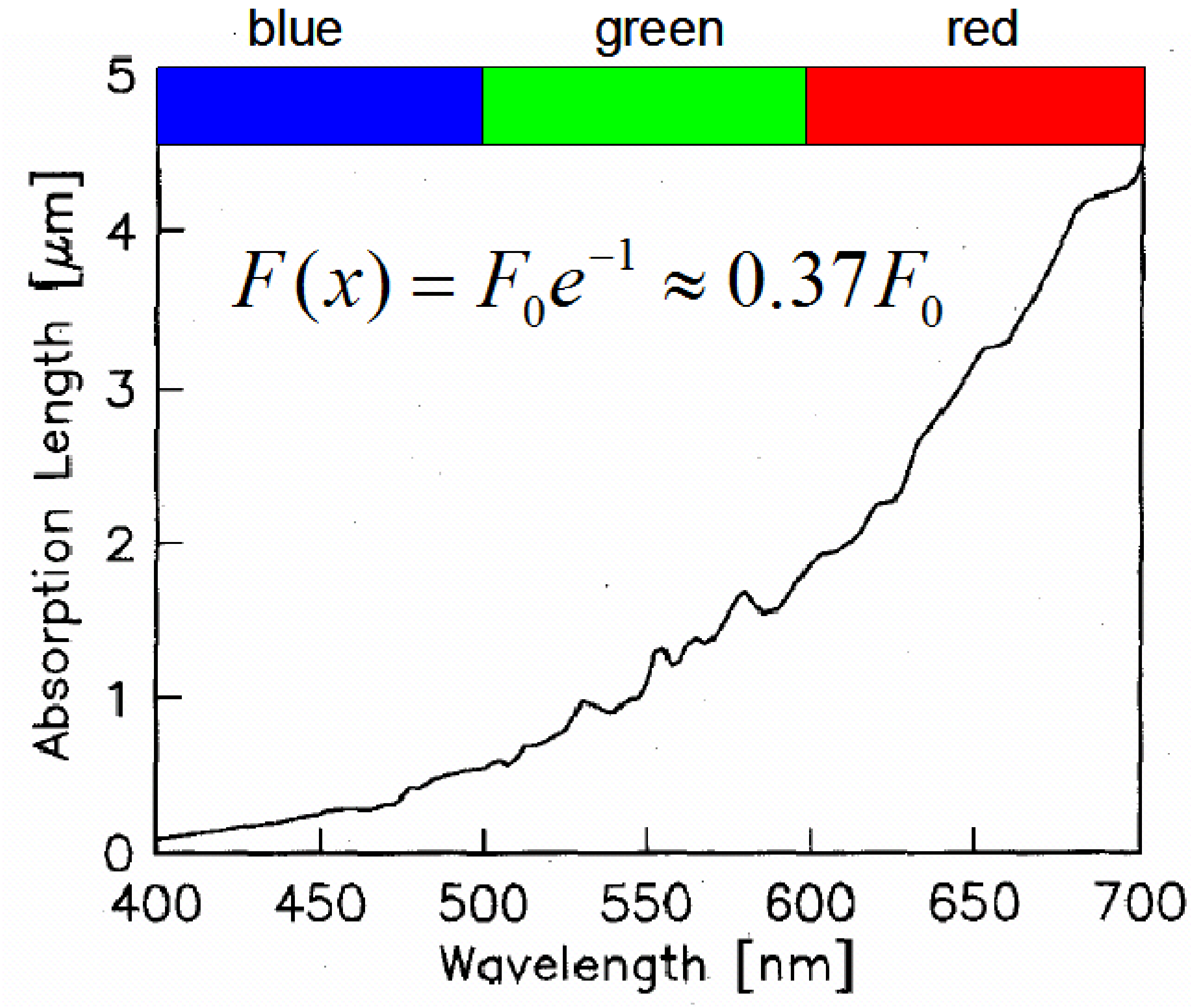
$$G(x) = \frac{d}{dx}(F_0 - F(x)) = \alpha F_0 e^{-\alpha x} \text{ e-h pair/cm}^3\text{.sec}$$

Absorption Coefficient of Silicon



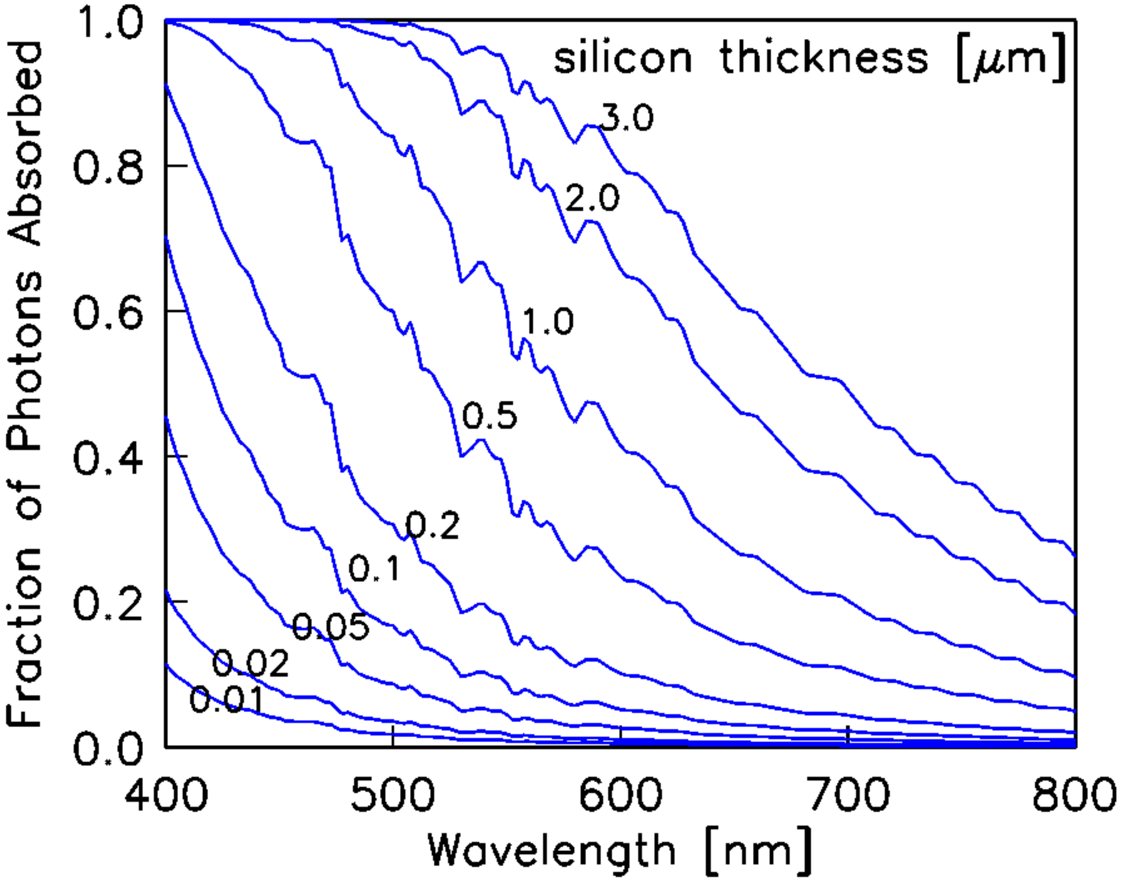
E. Palik, "Handbook of Optical Constant of Solids," Academic, New York, 1985

Absorption Length of Visible Light in Silicon



$$F(x) = F_0 e^{-1} \approx 0.37 F_0$$

Light Absorption in a Silicon Slab



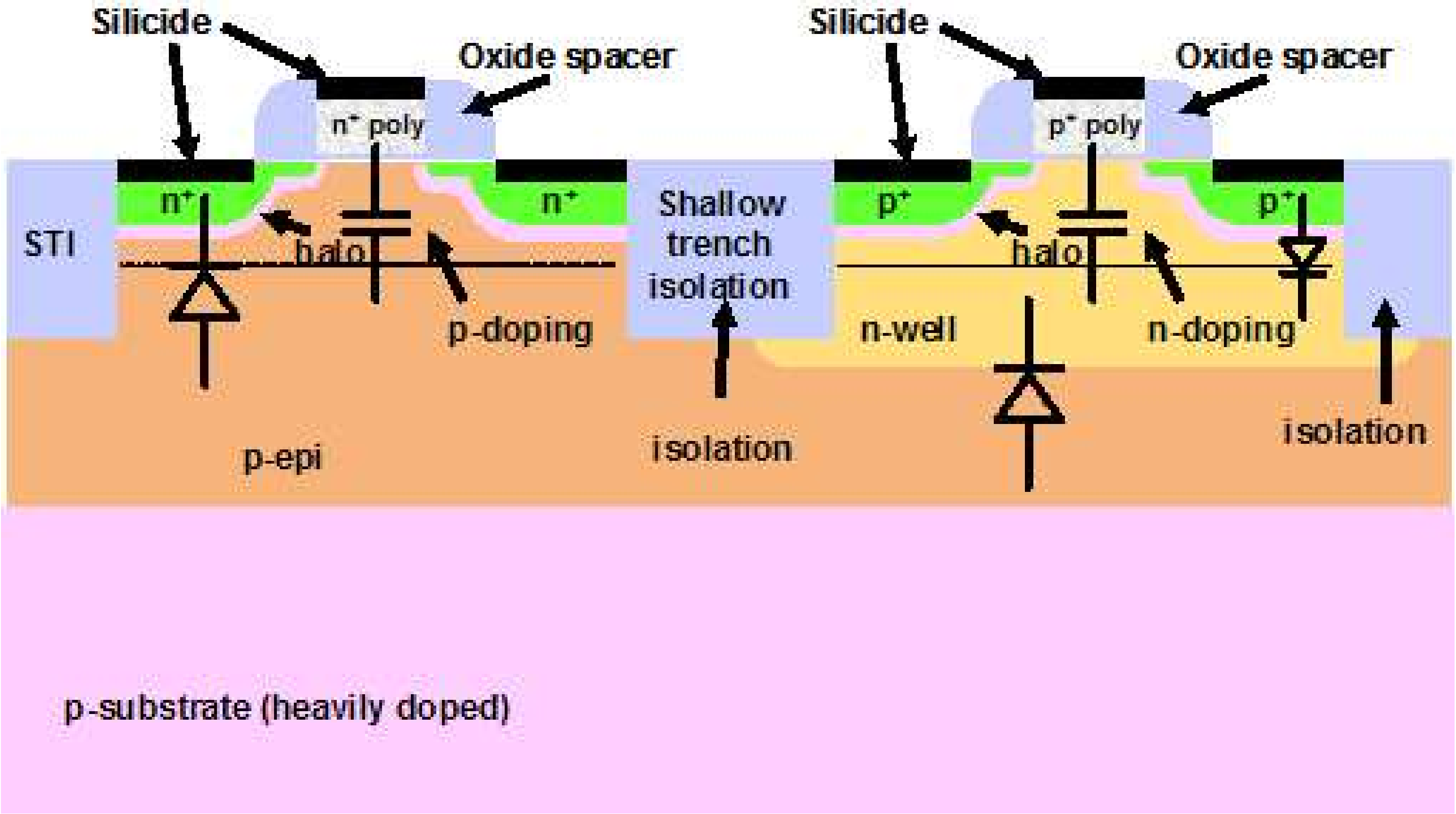
Comments

- $F(x)$ and $G(x)$ are average values assuming a large ensemble of photons (approaching continuum values)
  - The photon absorption process is actually discrete and random
- Note that:
  - 99% of blue light is absorbed within 0.6 μm
  - 99% of red light is absorbed within 16.6 μm
- These depths (surprisingly) are quite consistent with the junction and well depths of a CMOS process
- But, this is not the whole story . . .
  - Photocharge needs to be collected and converted into electrical signal

Photodetectors in Silicon

- A photodetector is used to convert the absorbed photon flux into photocurrent
- There are three types of photodetectors used, *photodiode*, which is a reverse biased pn junction, *photogate*, and *pinned diode*
- In a standard CMOS process there are three types of photodiodes available
  - nwell/psub
  - n+/psub
  - p+/nwell

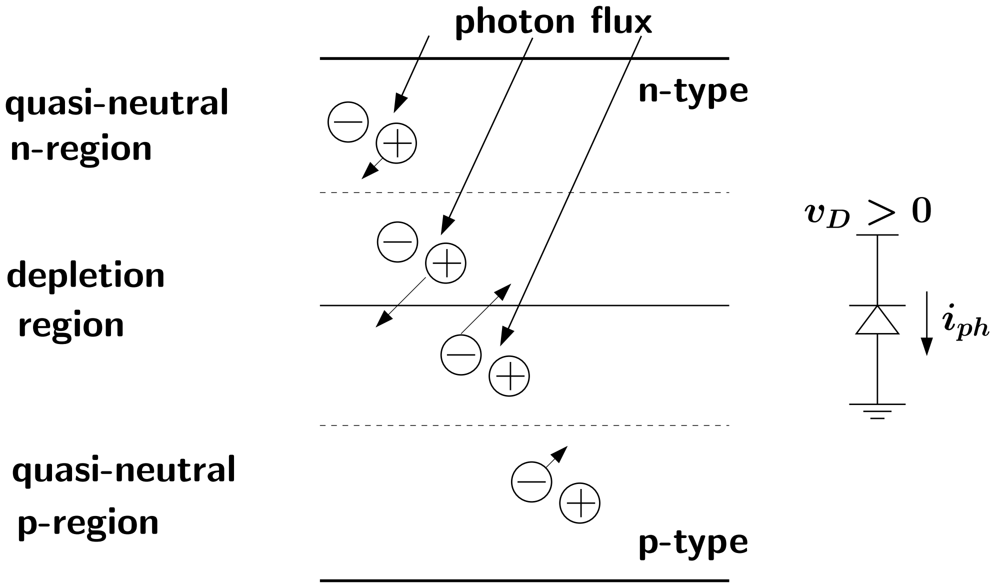
  and two types of photogates
  - nMOS transistor gate to drain
  - pMOS transistor gate to drain



- In this lecture notes we discuss the photodiode and photogate operation. The pinned diode will be discussed in the following lecture notes

Photodiode Operation

- Assume the *depletion approximation* of a reverse biased pn junction



- The photocurrent, i_{ph}, is the sum of three components:
  - Current due to electrons generated in the depletion (space charge) region, i_{ph}^{sc}
  - Current due to holes generated in the quasi-neutral n-region, i_{ph}^{p}
  - Current due to electrons generated in the quasi-neutral p-region, i_{ph}^{n}

- Most electrons generated in the depletion region are converted into current by strong electric field
- Carriers generated in the quasi-neutral regions need to diffuse to the depletion region to be collected
  - Some charge is lost through recombination
  - The diffusion length determines the fraction of charge that is not recombined

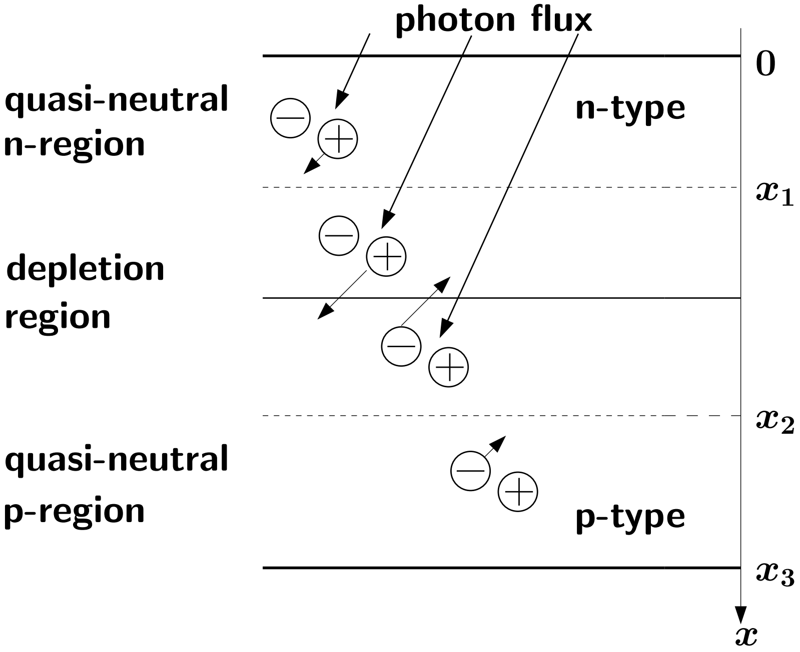
Photocurrent Derivation

- Assumptions
  - Abrupt pn junction
  - Depletion approximation
  - Low level injection, i.e., flux induced carrier densities $<<$ majority carrier densities
  - Short base region approximation, i.e., junction depths $<<$ diffusion lengths. This is is quite reasonable for advanced CMOS processes
- Our results are inaccurate but will help us understand the dependence of i_{ph} on various device parameters

References:

- F. Van de Wiele, “Photodiode Quantum Efficiency,” in P. G. Jespers, F. van de Wiele, M. H. White eds. “Solid State Imaging,” p. 47, Noordhoff (1976).
- J.C. Tandon, D.J. Roulston, S.G. Chamberlain, *Solid State Electronics*, vol. 15, pp. 669 – 685, (1972).
- R.W. Brown, S.G. Chamberlain, *Physica Status Solidi (a)*, vol. 20, pp. 675 – 685 (1973)

- Consider the depletion approximation for a reverse biased pn junction



- Assume a monochromatic photon flux F_0 photon/cm^2·sec incident at the surface ($x = 0$), the e-h generation rate at depth x is given by

$$G(x) = \alpha F_0 e^{-\alpha x} \text{ ph/cm}^3\text{.sec}$$

- Assuming all generated electrons in the space charge region are collected, the current density due to generation in the space charge region is

$$j_{ph}^{sc} = qF_0(e^{-\alpha x_1} - e^{-\alpha x_2}) \text{ A/cm}^2,$$

where $q = 1.6 \times 10^{-19}$Col is the electron charge

- The current density due to generation in n-type quasi-neutral region, which is diffusion current (since there is no field in this region), is given by

$$j_{ph}^{p} = -qD_p \frac{\partial p_n^{'}(x)}{\partial x}\Big|_{x=x_1}$$

where $p_n^{'}$ is the photogenerated minority carrier (hole) density, and D_p is the *diffusion constant* of holes (in cm^2/sec)

- To find the current density due to generation in the n-type quasi-neutral region, we first need to find $p_n^{'}(x)$. This can be done by solving the *continuity equation* (see derivation in Appendix I) with current density substituted for by the j_{ph}^{p} expression above

$$\frac{\partial p_n^{'}}{\partial t} = D_p \frac{\partial^2 p_n^{'}}{\partial x^2} + G(x) - R(x),$$

where $G(x)$ is the hole photogeneration rate and $R(x)$ is their *recombination rate*

By the short base assumption, the recombination rate is negligible and we set $R(x) = 0$

Now, assuming steady state, the continuity equation simplifies to

$$0 = D_p \frac{d^2 p_n^{'}}{dx^2} + G(x),$$

which has solution of the form

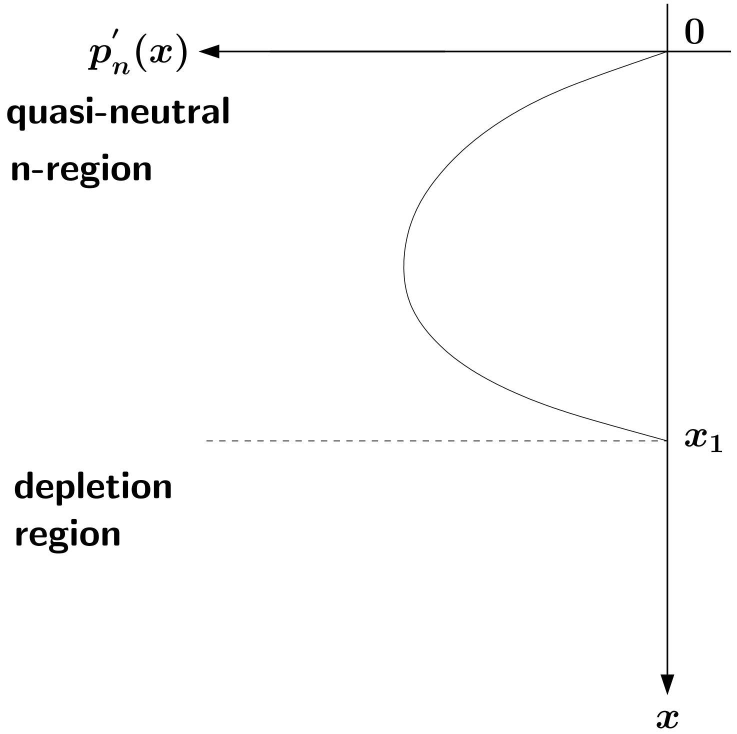
$$p_n^{'}(x) = a + bx - \frac{F_0}{\alpha D_p} e^{-\alpha x}$$

To find a and b, we assume that:

- ∘ at $x = 0$, we have an *ohmic contact*, which gives $p_n^{'}(0) = 0$
- ∘ at $x = x_1$, i.e., at the edge of the depletion region, $p_n^{'}(x_1) = 0$

Substituting, we obtain

$$p_n^{'}(x) = \frac{F_0}{\alpha D_p}(1 - \frac{x}{x_1}(1 - e^{-\alpha x_1}) - e^{-\alpha x})$$



We can now find the diffusion current density

$$\begin{aligned} j_{ph}^{p} &= -qD_p \frac{\partial p_n^{'}(x)}{\partial x}\Big|_{x=x_1} \\ &= \frac{qF_0}{\alpha x_1}(1-(\alpha x_1+1)e^{-\alpha x_1}) \end{aligned}$$

- The current density due to generation in the p-type can be similarly found, and we obtain
$$j_{ph}^{n} = \frac{qF_0}{\alpha(x_3 - x_2)}((\alpha(x_3 - x_2) - 1)e^{-\alpha x_2} + e^{-\alpha x_3})$$
Here we assumed that an ohmic contact at $x = x_3$, which is quite arbitrary (you will derive it with more reasonable assumptions in HW1)

- The total photogenerated current density is thus given by
$$j_{ph} = \frac{qF_0}{\alpha}\left(\frac{(1 - e^{-\alpha x_1})}{x_1} - \frac{(e^{-\alpha x_2} - e^{-\alpha x_3})}{(x_3 - x_2)}\right) \text{ A/cm}^2$$

- To find x_1 and x_2, we can use the simplifying assumptions to derive the depletion region width (see Appendix II), and we obtain
$$x_2 - x_1 = \sqrt{\frac{2\epsilon_s}{q}(v_D + \phi_n + \phi_p)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)},$$
and use the fact that $x_n/x_p = N_a/N_d$, where
$\epsilon_s = 10.45 \times 10^{-13}$F/cm is the permittivity of Si
N_d and N_a are the donor and acceptor densities in cm^{-3}
ϕ_n and ϕ_p are the potentials in the n and p regions

Example

- Consider the nwell/psub diode in the generic 0.5μm CMOS process described in Handout 4 with $v_D = 2$V and $F_0 = 4.09 \times 10^{12}$ photons/cm^2·sec at $\lambda = 555$nm (room light), find the photocurrent density components
- Using the depletion equation, we find that $x_1 = 2 - 0.176 = 1.824\mu$m and $x_2 = 3.76\mu$m

  The photocurrent density components are

$$\begin{aligned} j_{ph}^{sc} &= 120 \text{ nA/cm}^2 \\ j_{ph}^{p} &= 192 \text{ nA/cm}^2 \\ j_{ph}^{n} &= 28 \text{ nA/cm}^2 \end{aligned}$$

  Thus the total photocurrent density $j_{ph} = 340$ nA/cm^2

  So, for a photodiode of area $30\mu^2$, $i_{ph} = 102$fA

Factors Affecting Photocurrent

- i_{ph} is *linear* in F_0, i.e., proportional to illumination
- i_{ph} is nonlinear in α and λ
- i_{ph} increases as x_1 decreases and x_2 increases, i.e., as the depletion width $(x_2 - x_1)$ increases, which can be achieved by a combination of:
  - shallow pn junction,
  - low doping, and/or
  - by increasing reverse bias voltage
- Depletion region width, however, increases slowly with reverse bias voltage (high reverse bias voltage also increases dark current as we shall soon see)

Quantum Efficiency

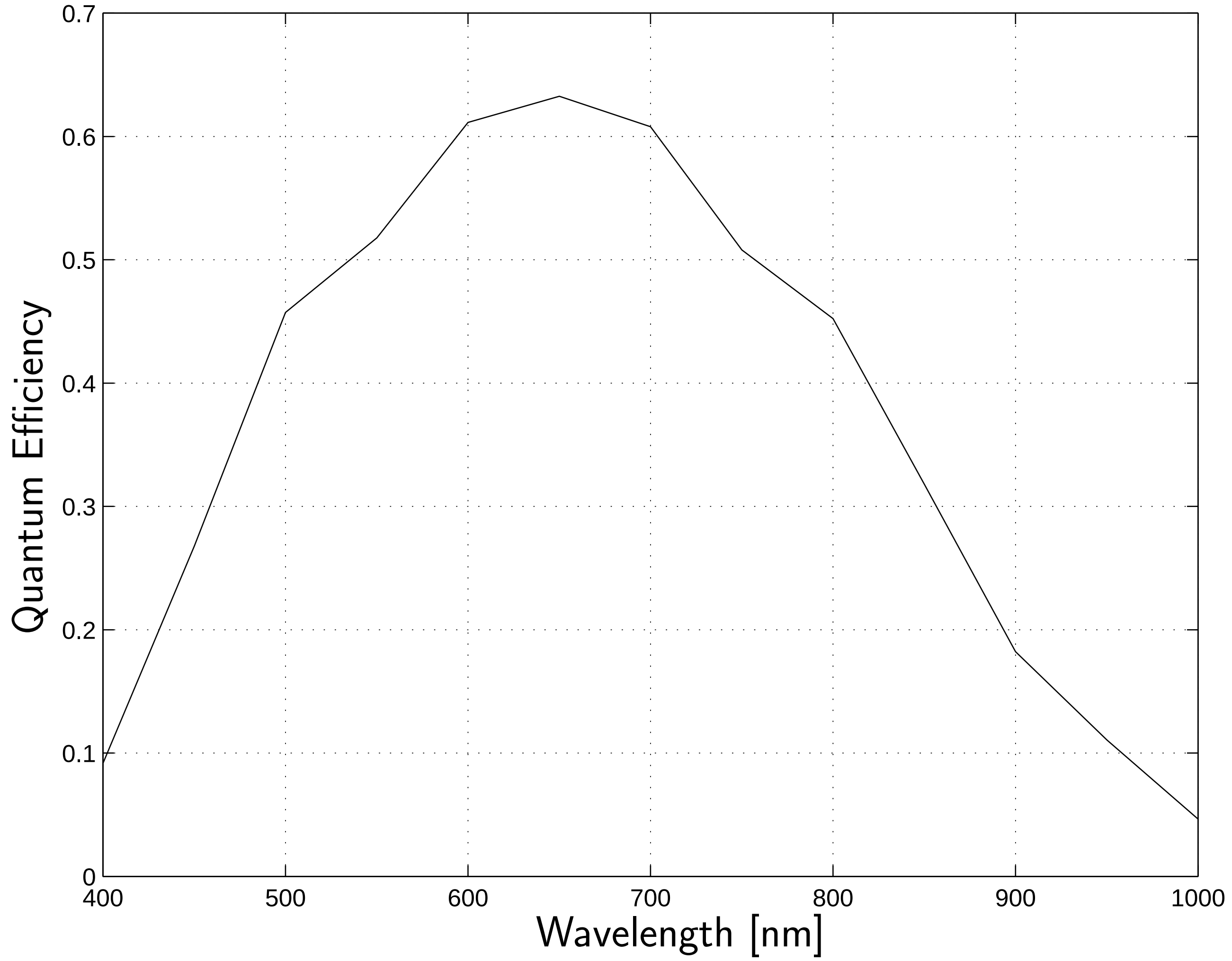
- Quantum efficiency $\mathrm{QE}(\lambda)$ is the fraction of photon flux that contributes to photocurrent as a function of the wavelength λ
- Using our derived photocurrent equation, we obtain

$$\mathrm{QE}(\lambda) = \frac{1}{\alpha}\left(\frac{(1-e^{-\alpha x_1})}{x_1} - \frac{(e^{-\alpha x_2}-e^{-\alpha x_3})}{(x_3-x_2)}\right) \text{ electrons/photons}$$

- This, in addition to being inaccurate due to the approximations we made, ignores:
  - reflection at the surface of the chip
  - reflections and absorptions in layers above the photodetector
  - variation of j_{ph} over the photodetector area (edge effects)

Example

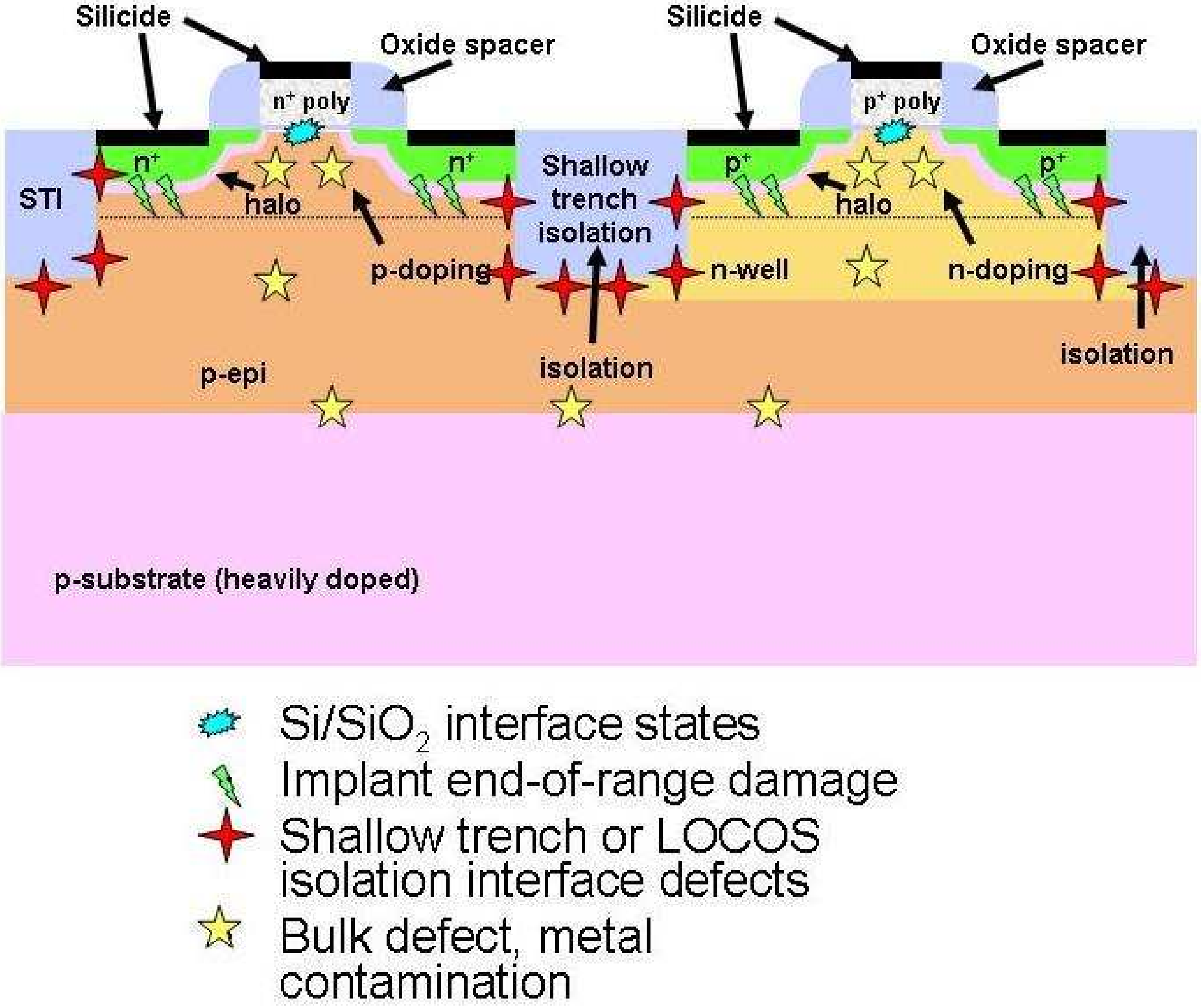
Consider the nwell/psub diode with $v_D = 2$V



Dark Current

- There are sources other than photon flux that lead to current in the photodetector – the sum of these currents is called ”dark current”
- It is called ”dark current” because it is the the photodetector current with no illumination present (in the dark)
- Dark current is *bad*. It limits the image sensor performance:
    - Introduces unavoidable shot noise
    - Can vary substantially over the image sensor array causing Dark Signal Non-uniformity (DSNU)
    - Reduces signal swing

Sources of Dark Current



Dark Current Contributions

- Dark current due to thermal generation (dominated by traps with energy in the middle of the bandgap) – can be calculated
  - Generation current is exponentially dependent on temperature
- Dark current due to interface defects, material (crystal) defect, and metal contamination, such as:
  - Edge of the STI or LOCOS
  - $\mathrm{Si/SiO_2}$ interface
  - Edge of junctions (end-of-range implant damage)

  These sources are difficult to model and can only be experimentally measured
  - Highly fabrication process dependent
  - Mitigated by careful pixel layout and dopant profiling (more on this later)

Calculation of Generation Dark Current

- Thermally generated dark current density due to bulk defects consists of three components:
  - Current due to carrier diffusion from the quasi-neutral regions, j_{dc}^{p} and j_{dc}^{n} $(v_D > 0)$
  - Current due to generation in the space charge region, j_{dc}^{sc}
- We first analyze the first two components in the same way we analyzed j_{ph}^{p} and j_{ph}^{n}, assuming abrupt pn junction and short base approximation let $p_n(x)$ be the *thermally* generated minority carriers in the n-type quasi-neutral region

  Ignoring recombination and assuming steady state, the continuity equation reduces to

$$0 = D_p \frac{d^2 p_n(x)}{dx^2},$$

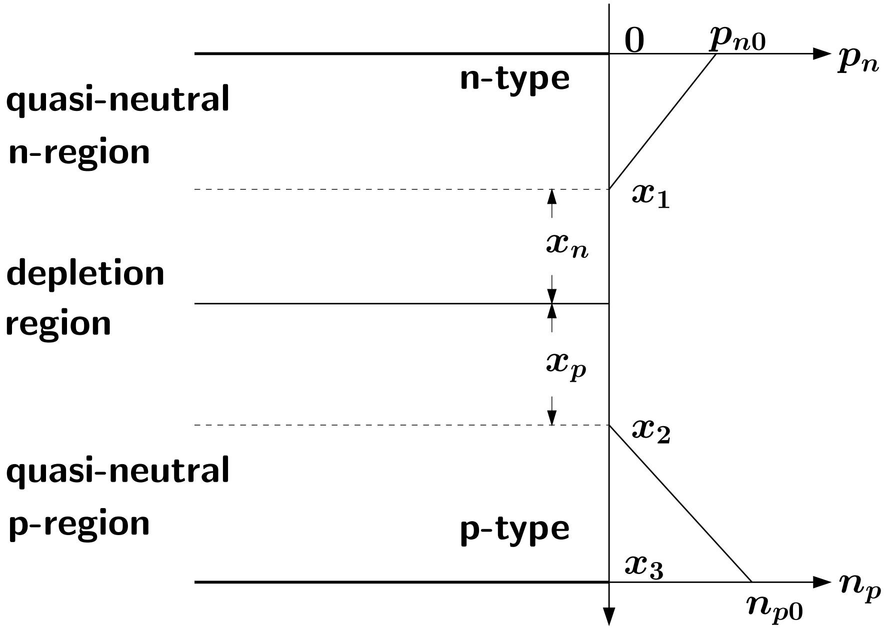
  with the general solution

$$p_n(x) = ax + b$$

Assuming ohmic contact at $x = 0$ we get $p_n(0) = p_{n0}$, the minority carrier concentration at thermal equilibrium, and assuming no free carriers at the edge of the depletion region, we have $p_n(x_1) = 0$

Thus

$$p_n(x) = p_{n0}\left(1 - \frac{x}{x_1}\right)$$



The (diffusion) current density is given by

$$\begin{aligned} j_{dc}^{p} &= -qD_p \frac{dp_n(x)}{dx}\Big|_{x=x_1} \\ &= qD_p \frac{p_{n0}}{x_1} = qD_p \frac{n_i^2}{N_d x_1} \end{aligned}$$

Similarly

$$j_{dc}^{n} = qD_n \frac{n_i^2}{N_a(x_3 - x_2)}$$

- Derivation of the current due to generation in the space charge region, j_{dc}^{sc}, is more complicated (see below). It yields

$$j_{dc}^{sc} \approx \frac{qn_i}{2}\left(\frac{x_n}{\tau_0^{\mathrm{n}}} + \frac{x_p}{\tau_0^{\mathrm{p}}}\right),$$

where τ_0^{n} and τ_0^{p} are the excess carrier lifetimes in the n and p type material, respectively

In practice the depletion region is much wider on one side. In this case we can express the current density as

$$j_{dc}^{sc} \approx \frac{q n_i x_d}{2\tau_o},$$

where $x_d = x_n + x_p$ is the depletion width and $\tau_o = \tau_n = \tau_p$ is the excess carrier lifetime of the wider side

- Example: again consider the nwell/psub diode with $v_D = 2$V, at room temperature

$$\begin{aligned} j_{dc}^{sc} &= 3.977 \text{ nA/cm}^2 \\ j_{dc}^{p} + j_{dc}^{n} &= 1.9611 \text{ nA/cm}^2 \\ j_{dc} &= 5.938 \text{ nA/cm}^2 \end{aligned}$$

So, for a photodiode of area 30μm^2, $i_{dc} \approx 1.78$fA

Factors Affecting i_{dc}

- i_{dc} increases dramatically with temperature T, since it increases with the intrinsic carrier concentration n_i, which is proportional to $T^{1.5}e^{-\frac{E_g}{2kT}}$
- i_{dc}^{sc} is the dominant component of i_{dc}
  - it increases with doping concentration, since τ is proportional to $\frac{1}{N}$
  - it decreases with decrease in depletion width, thus reducing reverse bias voltage reduces i_{dc} (but also reduces i_{ph}!)
  - the calculated i_{dc}^{sc} is only valid for low electric field, at higher electric fields (which occurs in the shallower and more highly doped junctions of advanced processes), i_{dc}^{sc} increases much faster with reverse bias voltage

Generation-Recombination in Depletion Region

- Here, we derive the generation-recombination current in the depletion region of a reverse biased pn-junction
- The analysis is referred to as the Shockley, Read, Hall (SRH) model
- Salient features of the SRH model:
  - Generation and recombination of carriers occur through localized states (recombination centers) with energy within the bandgap
  - Overall population of the recombination center is fairly constant
  - Recombination centers quickly capture the majority carriers, but have to wait for the arrival of a minority carrier

- The generation-recombination rate is given by

$$\begin{aligned} U &= \frac{N_t v_{th}\sigma_n\sigma_p(pn - n_i^2)}{\sigma_p\left[p + n_i\exp\left(\frac{E_i - E_t}{kT}\right)\right] + \sigma_n\left[n + n_i\exp\left(\frac{E_t - E_i}{kT}\right)\right]} \\ &= \frac{(pn - n_i^2)}{\tau_n\left[p + n_i\exp\left(\frac{E_i - E_t}{kT}\right)\right] + \tau_p\left[n + n_i\exp\left(\frac{E_t - E_i}{kT}\right)\right]}, \end{aligned}$$

  where N_t is the generation-recombination center density, $\sigma_{n,p}$ is the capture cross section, and the minority carrier lifetimes are given as

$$\tau_{n,p} = 1/N_t v_{th}\sigma_{n,p}$$

  We assume that $\tau_n = \tau_p \equiv \tau_o$ (see Handout 4)

  Recombination: $U > 0$
  Generation: $U < 0$

- In the depletion region

$$\begin{aligned} n &= n_i\exp\left(\frac{E_{fn} - E_i(x)}{kT}\right) \ll n_i \\ p &= n_i\exp\left(\frac{E_i(x) - E_{fp}}{kT}\right) \ll n_i \end{aligned}$$

- Then

$$U \cong \frac{-n_i^2}{\tau_p n_i \exp\left(\frac{E_t - E_i}{kT}\right) + \tau_n n_i \exp\left(\frac{E_i - E_t}{kT}\right)}$$
$$= \frac{-n_i}{\tau_o \left[\exp\left(\frac{E_t - E_i}{kT}\right) + \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

- Define

  $$U_T = \exp\left(\frac{E_i - E_t}{kT}\right)$$

  Then, the maximum generation rate is obtained when

  $$\frac{\partial U}{\partial U_T} = 0 \quad \Rightarrow \quad U_T = 1 \quad \Rightarrow \quad E_t = E_i,$$

  and is given by

  $$G_{\max} = \frac{n_i}{2\tau_o},$$

- The generation current is

  $$j_{dc}^{sc} = q \int_{x_1}^{x_2} G_{\max}\, dx \approx \frac{q n_i x_d}{2\tau_o}$$

Generation Current at Si/SiO$_2$ Surface

- The semiconductor surface has plenty of localized states having energies within the bandgap
- The kinetics of generation-recombination at the surface is similar to trap states in the bulk except that the trap density N_{st} is an areal density (# / cm^2)
- Again, using the SRH model, the surface generation-recombination rate is given by

$$\begin{aligned} U_s &= \frac{N_{st} v_{th} \sigma_n \sigma_p (p_s n_s - n_i^2)}{\sigma_p \left[p_s + n_i \exp\left(\frac{E_i - E_{st}}{kT}\right)\right] + \sigma_n \left[n_s + n_i \exp\left(\frac{E_{st} - E_i}{kT}\right)\right]} \\ &\cong N_{st} v_{th} \sigma \frac{(p_s n_s - n_i^2)}{\left[p_s + n_s + 2n_i \cosh\left(\frac{E_i - E_{st}}{kT}\right)\right]}, \end{aligned}$$

where again we assumed that $\sigma_n = \sigma_p \equiv \sigma$

Recombination: $U_s > 0$

Generation: $U_s < 0$

- Surface generation depends on carrier density
  - When the surface has plenty of carriers, either due to inversion or accumulation,

$$U_s \cong N_{st} v_{th} \sigma \frac{(p_s n_s - n_i^2)}{\left[p_s + n_s + 2 n_i \cosh\left(\frac{E_i - E_{st}}{kT}\right)\right]}$$

    is small
  - When the surface is depleted, p_s and n_s are small, and

$$U_s \cong -\frac{N_{st} v_{th}}{2} \sigma n_i = -\frac{s n_i}{2},$$

    Here we assume that $E_{st} = E_i$ and $s = N_{st} v_{th} \sigma$ has the unit of velocity (cm·sec^{-1})

    Surface generation current density is thus

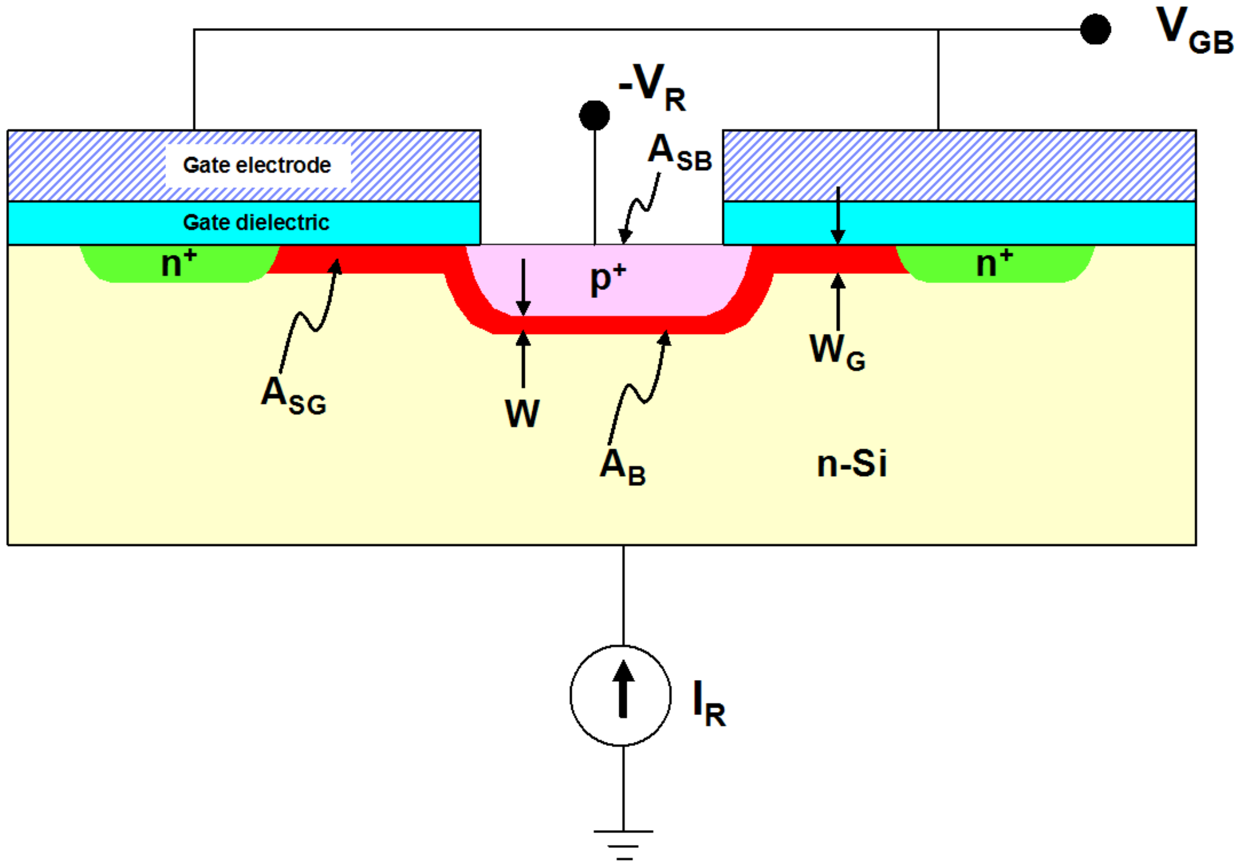
$$j_{dc}^{s} = \frac{q s n_i}{2}$$

Activation Energy of Dark Current Components

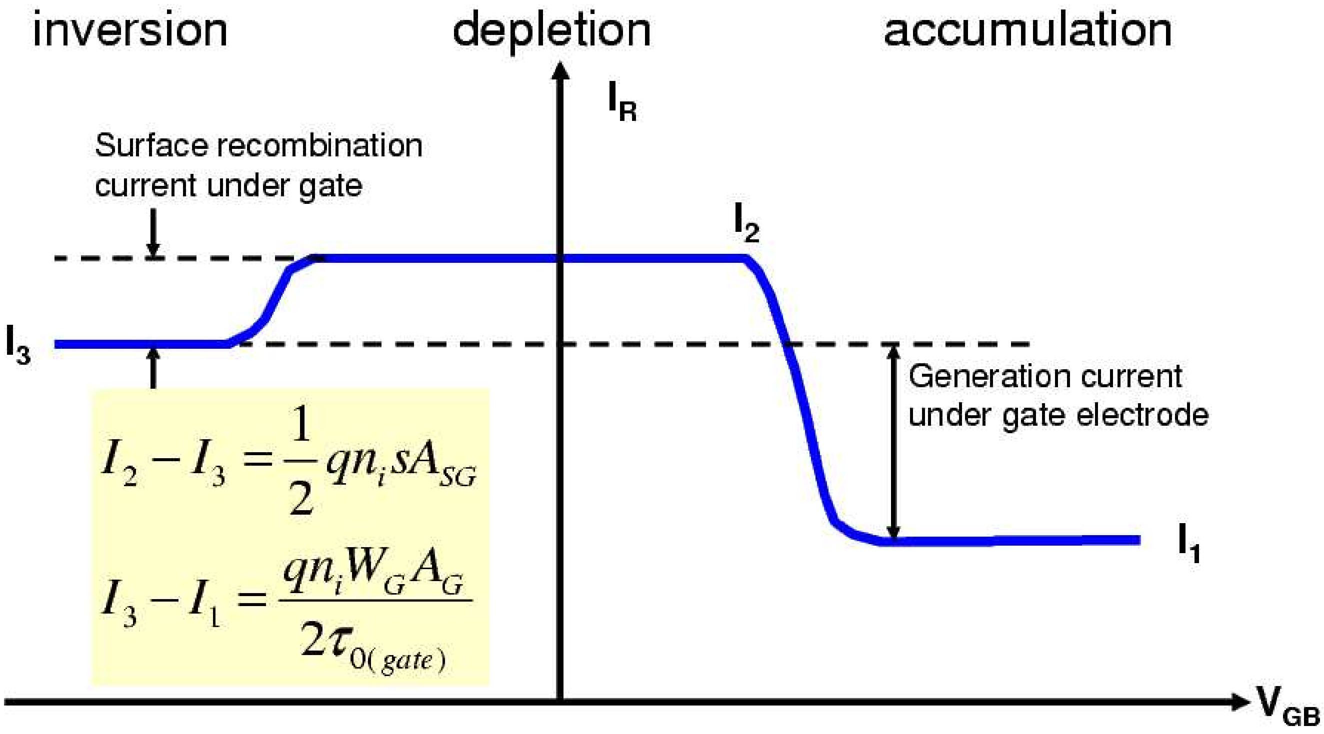
- Notice that the generation currents due to bulk traps and surface traps are proportional to n_i
- And the diffusion current in the quasi-neutral region is proportional to n_i^2
- Therefore, in a plot of dark current vs $\log(1/T)$
    - the activation energy of bulk trap and surface trap dark current is E_g
    - the dark current that is due to diffusion in the quasi-neutral region has activation energy of $E_g/2$

Surface Recombination Velocity

- It is difficult to "derive" the surface recombination velocity
- It is obtained experimentally



- Experimental procedure:
  - Reverse bias the gated diode
  - Sweep gate bias from inversion to accumulation
  - Measure DC current from substrate in inversion (I_3), depletion (I_2), and accumulation (I_1)



$$I_2 - I_3 = \frac{1}{2} q n_i s A_{SG}$$

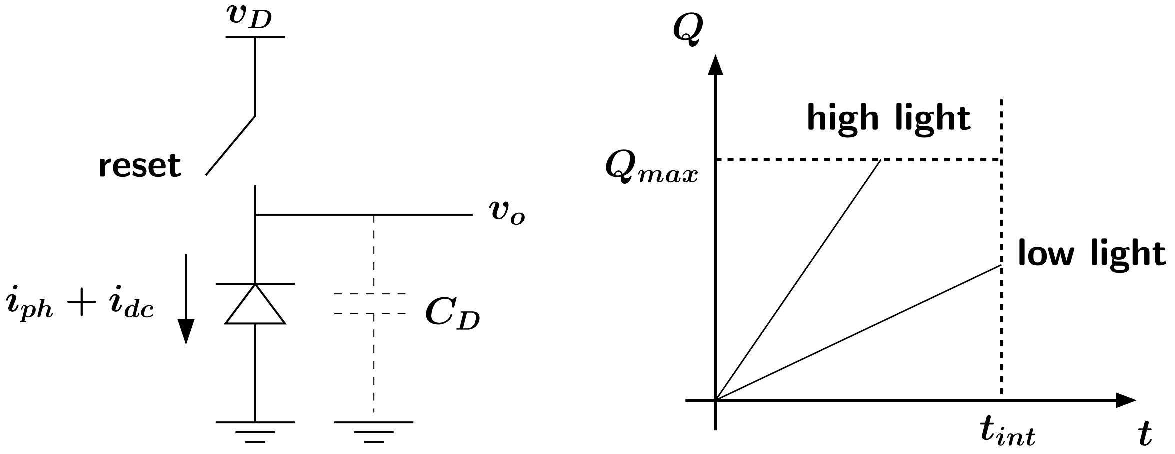
$$I_3 - I_1 = \frac{q n_i W_G A_G}{2\tau_{0(gate)}}$$

- The currents are given by:

$$\begin{aligned}
I_1 &= \frac{qn_iWA_B}{2\tau_{o(\text{diode})}} + \frac{qn_isA_{SB}}{2} + \frac{qn_i^2}{N_B}\sqrt{\frac{D_p}{\tau_p}}A_B \\
I_2 &= I_1 + \frac{qn_iW_GA_G}{2\tau_{o(\text{gate})}} + \frac{qn_isA_{SG}}{2} \\
I_3 &= I_2 - \frac{qn_isA_{SG}}{2} \\
I_2 - I_3 &= \frac{qn_isA_{SG}}{2} \\
I_3 - I_1 &= \frac{qn_iW_GA_G}{2\tau_{o(\text{gate})}}
\end{aligned}$$

Direct Integration

- As discussed earlier, photocurrent is typically too small to measure directly
- The most commonly used mode of photodiode operation in an image sensor is *direct integration*, where the photocurrent (and dark current) are directly integrated over the diode capacitance



  - ○ The photodetector is *reset* to the reverse bias voltage v_D
  - ○ The diode current discharges C_D for t_{int} seconds, which is called *integration time* or *exposure time*
  - ○ At the end of the integration time the accumulated charge $Q(t_{int})$ (in electrons) or voltage $v_o(t_{int})$ is read out

- Assuming that the photo and dark currents do not change with reverse bias voltage, we obtain

  $$Q(t_{int}) = \frac{1}{q}(i_{ph} + i_{dc})t_{int} \text{ electrons}$$

  Assuming that C_D does not vary with reverse bias voltage, we get

  $$v_o(t_{int}) = v_D - \frac{(i_{ph} + i_{dc})t_{int}}{C_D} \text{ V}$$

- The maximum nonsaturating photocurrent is thus given by

  $$i_{ph}^{max} = \frac{qQ_{max}}{t_{int}} - i_{dc}$$

  Q_{max} is called the *well capacity*

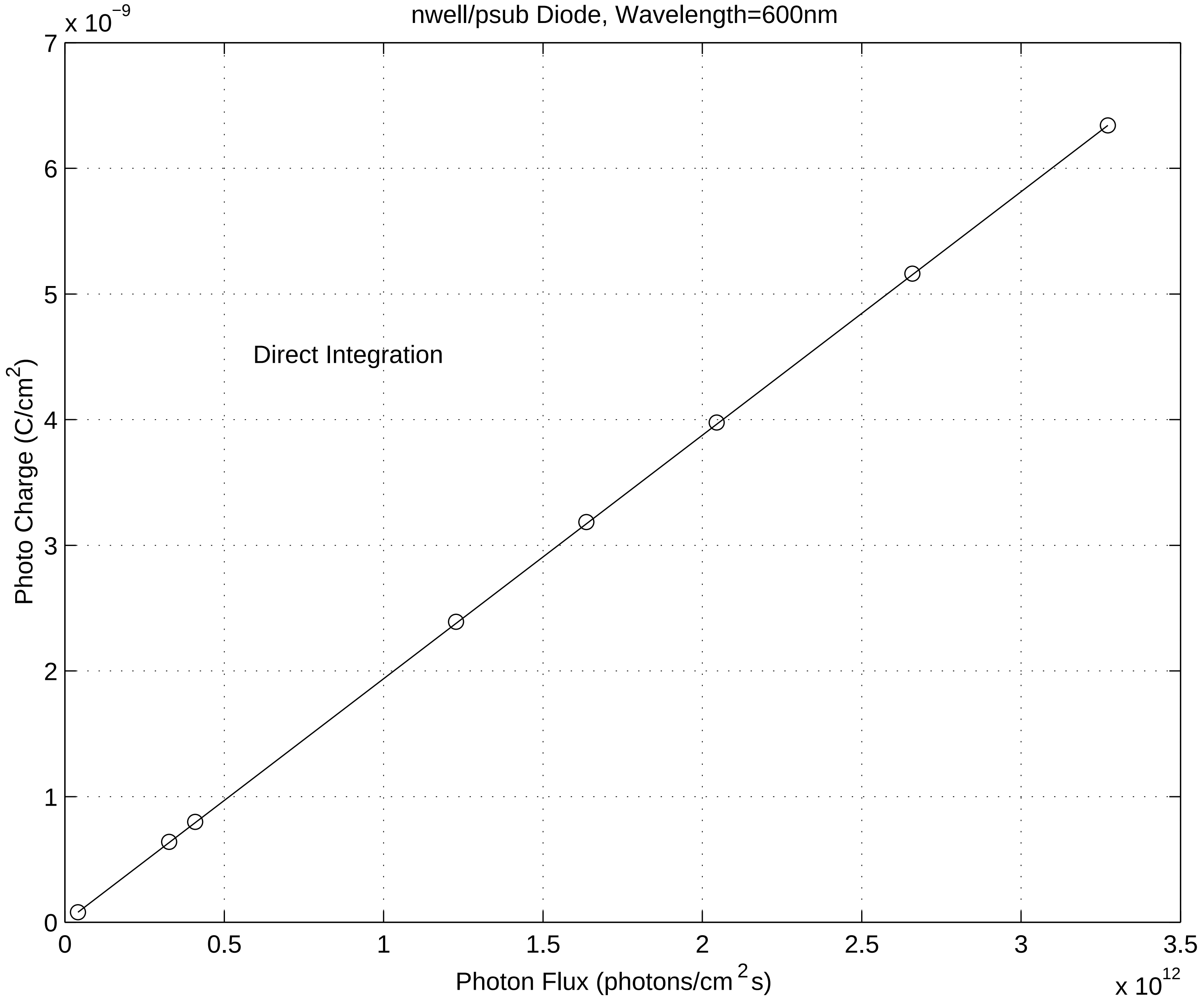
- To avoid *blooming*, i.e., overflowing of charge to neighboring photodetectors in the image sensor, we ensure that the diode is reverse biased , i.e., $v_o(t_{int}) > 0$V
  Thus $qQ_{max} \leq v_D \times C_D$ (very often the voltage swing is lower than v_D resulting in well capacity lower than $v_D \times C_D$)

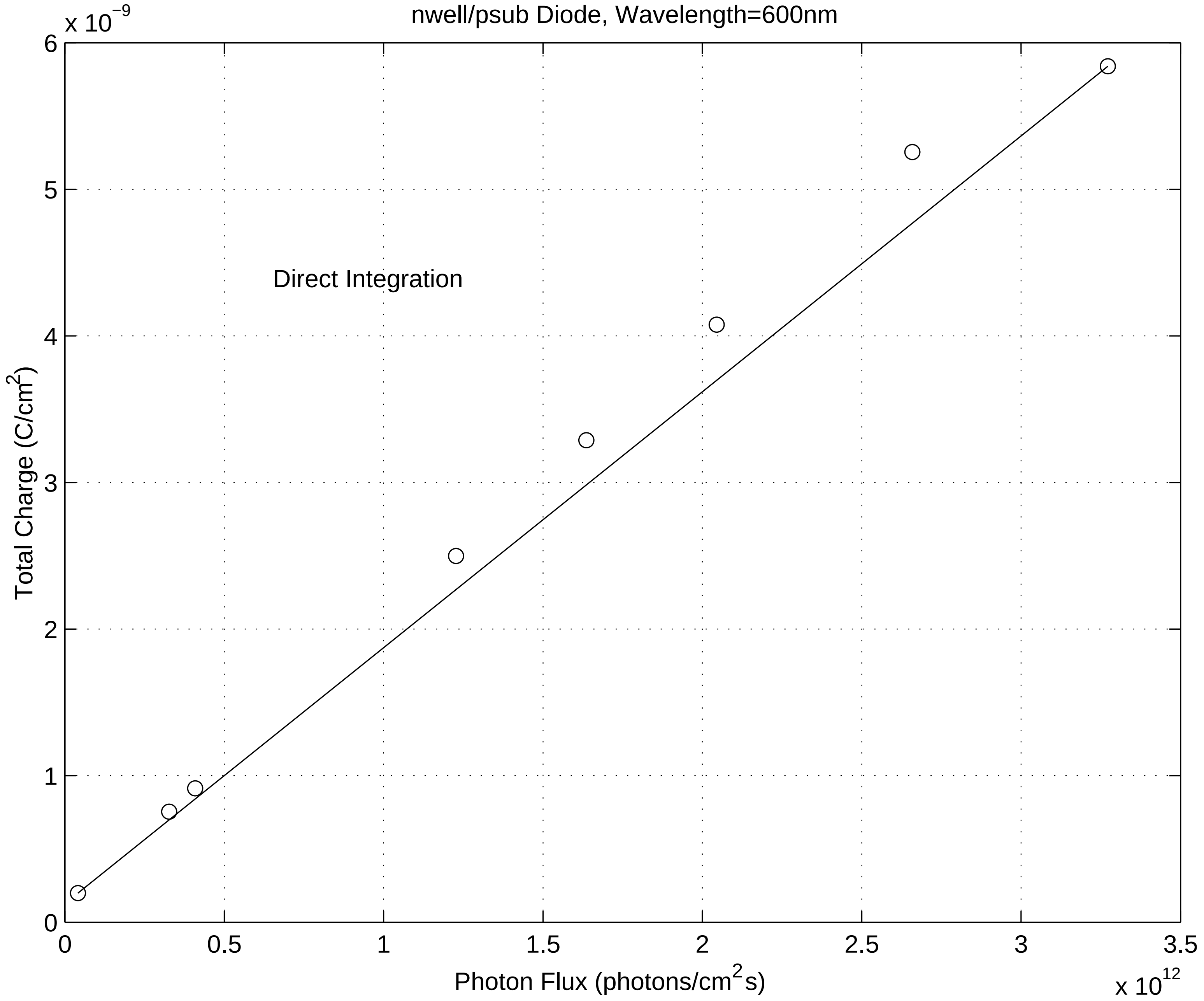
Example

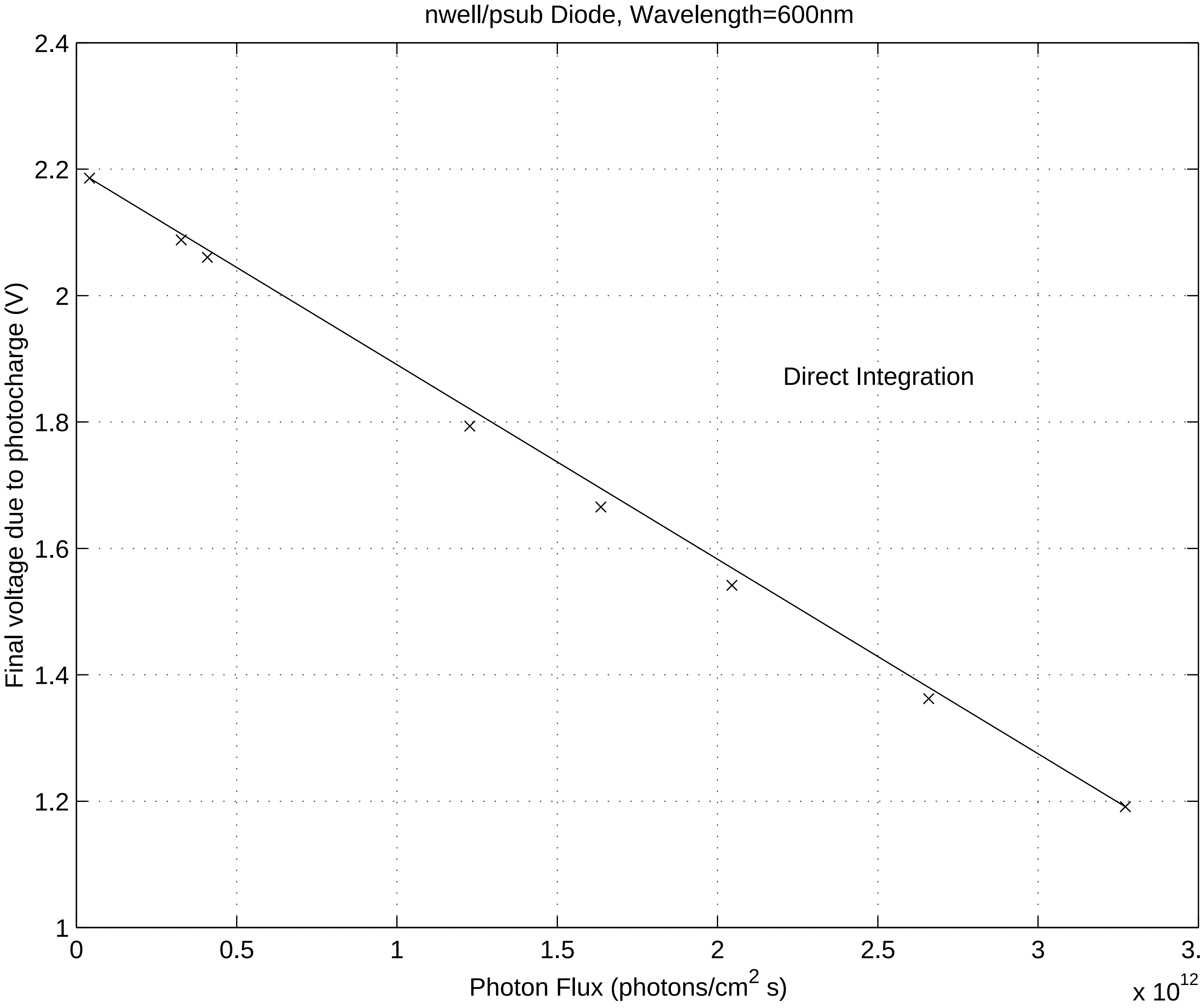
- Consider the nwell/psub diode with $v_D = 2.2$V, and area $A_D = 30\mu$m^2 the photodiode capacitance
$$C_D = \frac{\epsilon_s}{x_n + x_p} A_D = 1.55 f\text{F}$$
Note: this is unrealistically small since it does not include edge capacitance and the capacitances of interconnect and other devices connected to the photodetetcor
- Thus (ignoring these other capacitances) the well capacity
$$Q_{\max} = 3.41 \times 10^{-15} / 1.6 \times 10^{-19} = 21312 \text{ electrons}$$
- Assuming $t_{int} = 20$ms and dark current $i_{dc} = 2$fA we get that the maximum nonsaturating photocurrent
$$i_{ph}^{\max} = 167.55\text{fA},$$
which corresponds to 16.5 lux at $\lambda = 555$nm
- To put this in perspective, a typical DRAM cell holds $\sim$ 160,000 to 200,000 electrons

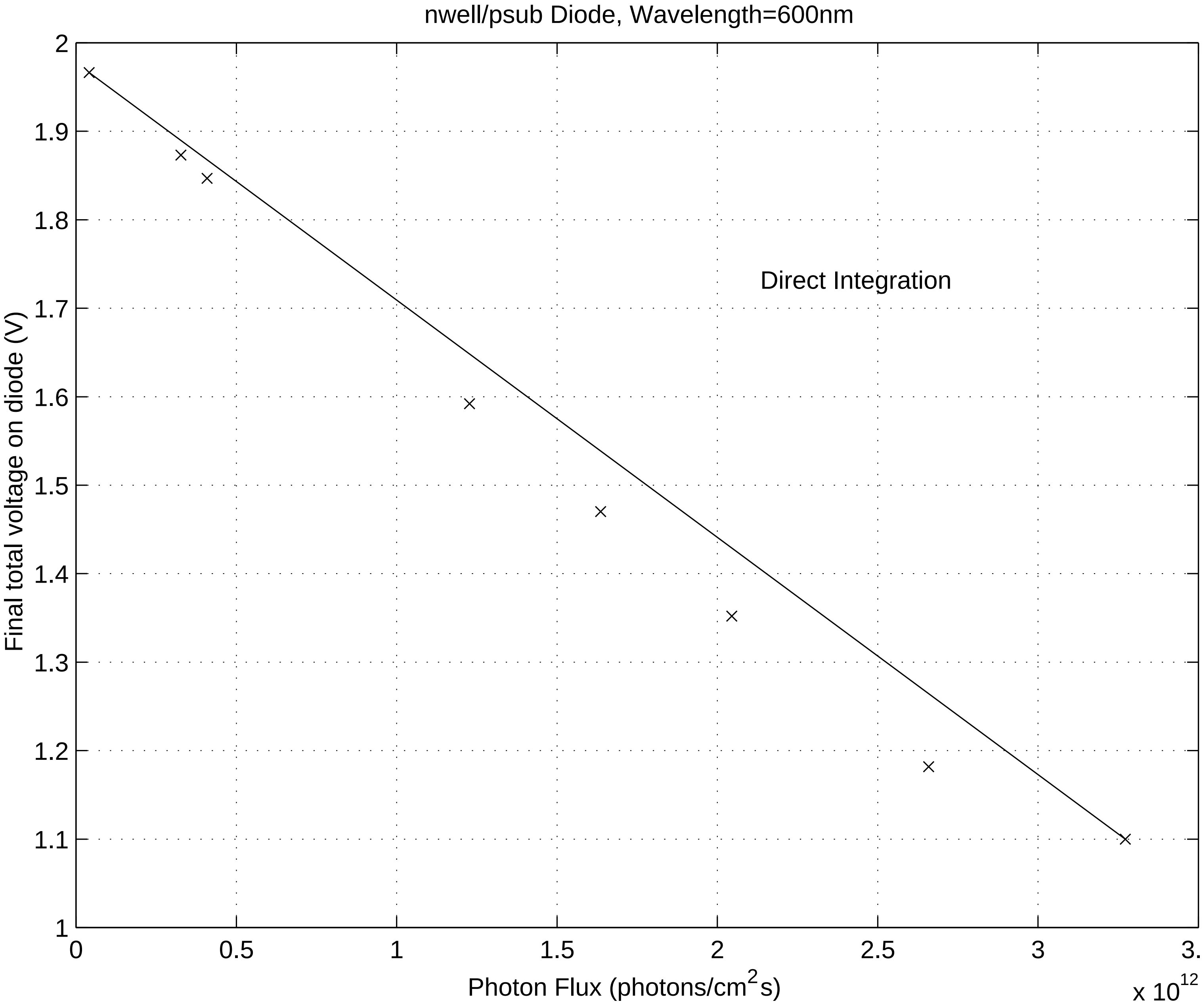
Finding $\mathbf{Q(t_{int})}$ and $\mathbf{v_o(t_{int})}$ Numerically

- Since the depletion region width changes with the reverse bias voltage, C_D, j_{ph}, and j_{dc} are not constant during integration
- The output charge and voltage can be found numerically
  - Set λ and F_0 to desired values
  - Set $v_o(0) = v_D$ and $Q(0) = 0$ and calculate $v_o(k\Delta t)$ and $Q(k\Delta t)$ iteratively beginning with $k = 1$ and ending with $k = \frac{t_{int}}{\Delta t}$
  - To calculate $v_o((k+1)\Delta t)$ and $Q((k+1)\Delta t)$:
    1. Calculate the depletion region width and C_D^k (using $v_o(k\Delta t)$)
    2. Calculate the current densities $j_{ph}(k\Delta t)$ and $j_{dc}(k\Delta t)$ and the charge accumulated $\Delta Q^k = (j_{ph}(k\Delta t) + j_{dc}(k\Delta t))\Delta t$
    3. Set $v_o((k+1)\Delta t) = v_o(k\Delta t) - \frac{\Delta Q^k}{C_D^k}$ and $Q((k+1)\Delta t) = Q(k\Delta t) + \Delta Q^k$
- The following graphs provide computed $Q(t_{int})$ and $v_o(t_{int})$ as a function of F_0 for $v_D = 2.2$V


nwell/psub Diode, Wavelength=600nm
x 10^{-9}
7
6
5
4
3
2
1
0
Photo Charge (C/cm^2)
Direct Integration
0
0.5
1
1.5
2
2.5
3
3.5
Photon Flux (photons/cm^{2}s)
x 10^{12}

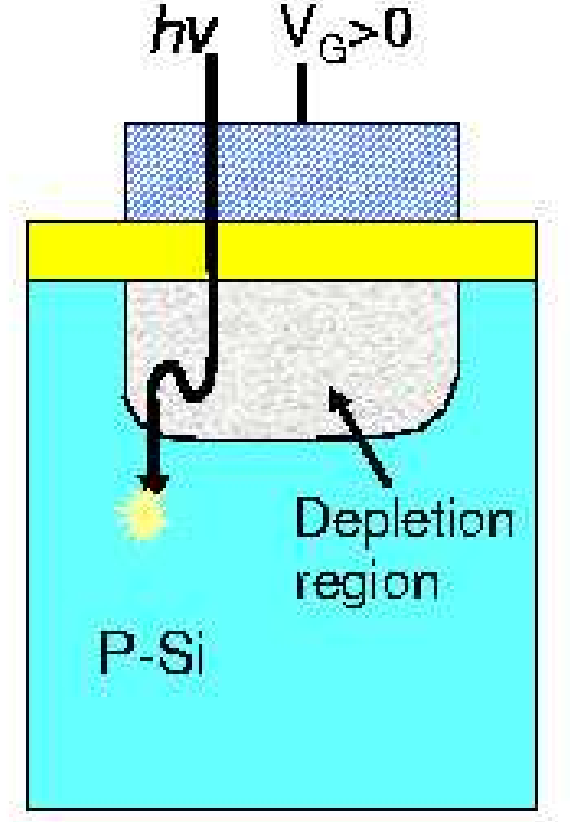

nwell/psub Diode, Wavelength=600nm
x 10^{-9}
6
5
4
3
2
1
0
Total Charge (C/cm^2)
Direct Integration
0
0.5
1
1.5
2
2.5
3
3.5
x 10^{12}
Photon Flux (photons/cm^{2}s)


nwell/psub Diode, Wavelength=600nm
Final voltage due to photocharge (V)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
0.5
1
1.5
2
2.5
3
3.5
Photon Flux (photons/cm^2 s)
x 10^{12}
Direct Integration


nwell/psub Diode, Wavelength=600nm
Final total voltage on diode (V)
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.5
1
1.5
2
2.5
3
3.5
x 10^{12}
Photon Flux (photons/cm^2s)
Direct Integration

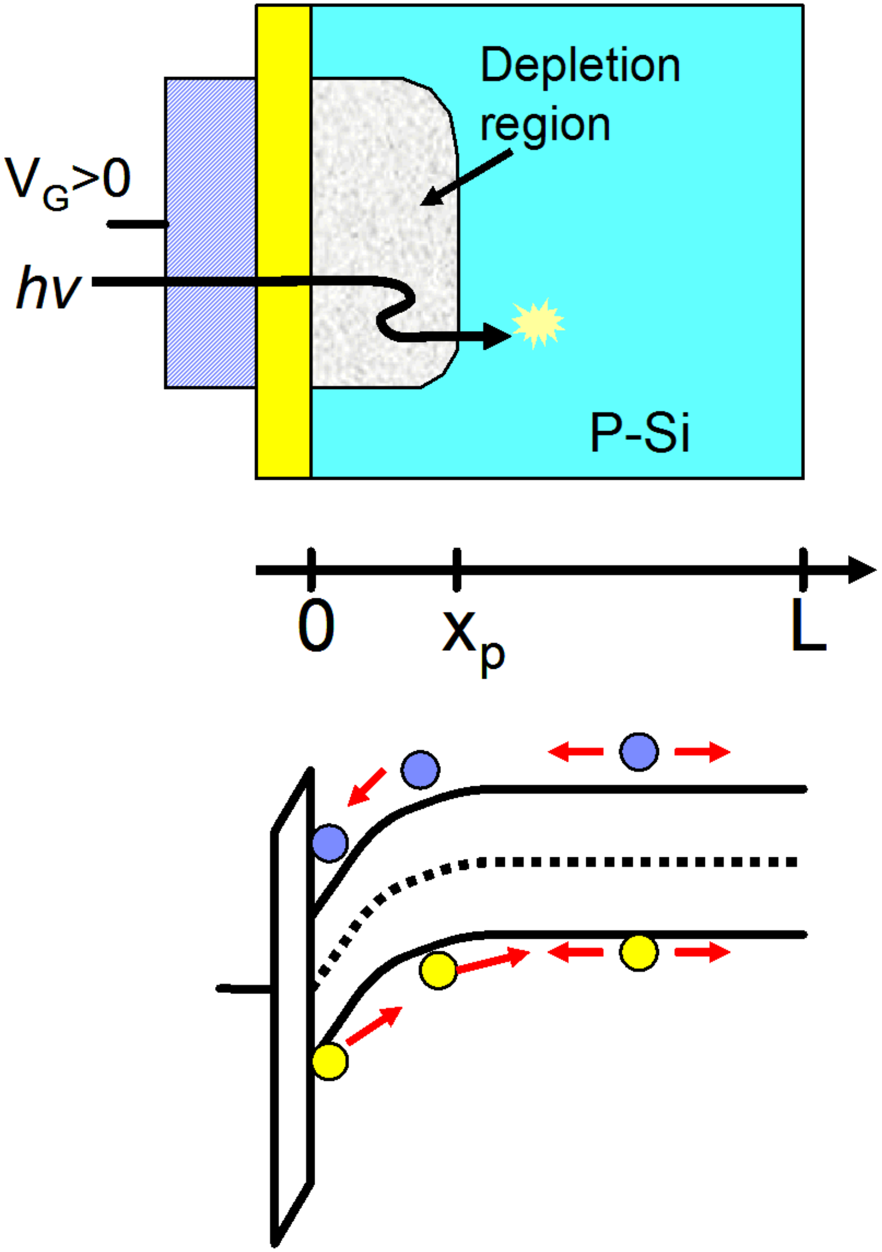
Photogate

- Photogate is used in CMOS sensors (CMOS APS), frame transfer CCD (FT-CCD) and time-delay-and-integration CDD (TDI-CCD)



Photogate Operation

- Gate voltage v_G is set high enough to bias the MOS capacitor into the *deep depletion regime* (this requires $v_G >> v_T$) (see Appendix)
- Electrons generated in the depletion region are collected in the potential well
- Electrons generated in the quasi-neutral region will
  - Recombine with holes
  - Diffuse to depletion region and get collected in the potential well if it is within the diffusion length of the minority carriers
- Holes will be collected in the substrate
- How many of the photo-generated carriers are collected depends on:
  - Diffusion length of minority carriers
  - Location and length of the depletion region


Depletion region
$V_G>0$
$h\nu$
P-Si
0
x_p
L

Quantum Efficiency of Photogate

- Photocurrent has two components
  - Current due to generation in the depletion region, i_{ph}^{sc}, again almost all carriers contribute to the current
  - Diffusion current due to generation in the quasi-neutral p-region, i_{ph}^{n}
- To calculate the current we make the depletion approximation and use the basic MOS capacitor equations to find the depletion region width (see Appendix III) (you will derive it in HW2)
- A disadvantage of the photogate is lower quantum efficiency, especially for shorter wavelengths (blue), due to absorption in the polysilicon gate (which has the same α as crystalline silicon)
- Photogate is also used in direct integration mode; charge accumulated on gate is transferred to another capacitor (as we shall see later)

Quantum Efficiency of Photogate

- It can be shown that QE for photogate, not including absorption in the polysilicon gate, is given by

$$\mathrm{QE}(\lambda) = 1 - e^{-\alpha x_d} + \frac{\alpha L_n^2}{\alpha^2 L_n^2 - 1}\left(\alpha e^{-\alpha x_d} + \frac{e^{-\alpha L} - e^{-\alpha x_d}\cosh\left((L - x_d)/L_n\right)}{L_n \sinh\left((L - x_d)/L_n\right)}\right)$$

- Limiting Cases:
  - Very long diffusion length ($L_n \to \infty$):

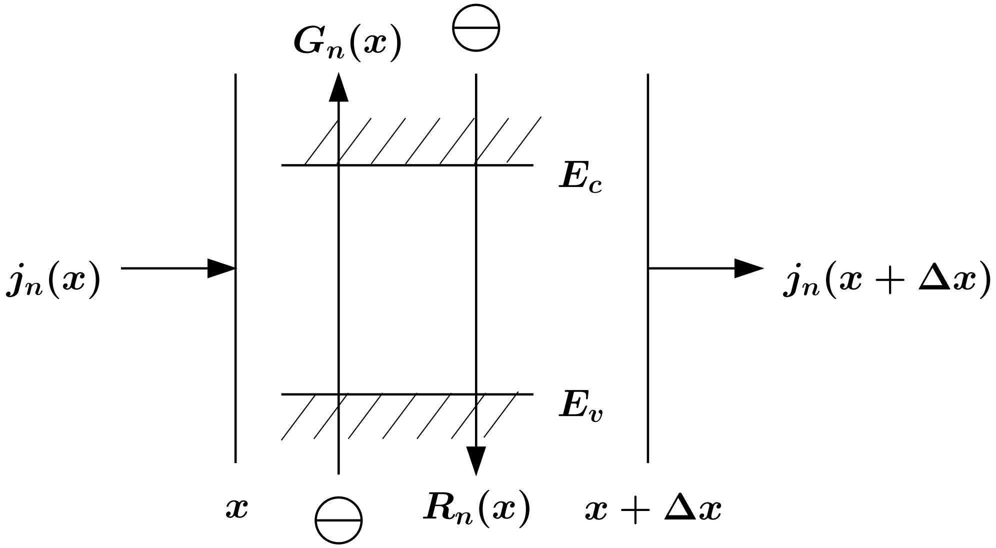
$$\mathrm{QE} = 1 - e^{-\alpha x_d} + e^{-\alpha x_d}\left(1 - \frac{1 - e^{-\alpha(L - x_d)}}{\alpha(L - x_d)}\right)$$

  - Very thick substrate ($L \to \infty$):

$$\mathrm{QE} = 1 - e^{-\alpha x_d} + \frac{\alpha L_n e^{-\alpha x_d}}{(\alpha L_n + 1)}$$

Appendix I – Derivation of 1-D Continuity Equation

- Consider minority carrier (electron) current flow in p-type silicon
- In a slab x to $x + \Delta x$



$j_n(x)$: electron current density at x
$G_n(x)$: generation rate (electrons/cm^3·s)
$R_n(x)$: recombination rate (electrons/cm^3·s)
$n(x)$: electron density at x (electrons/cm^3)

- The rate of electron density increase in the slab

$$\frac{\partial n(x)}{\partial t}\Delta x \approx -\frac{1}{q}(j_n(x) - j_n(x+\Delta x)) + (G_n(x) - R_n(x))\Delta x,$$

  which in the limit, gives

$$\frac{\partial n(x)}{\partial t} = \frac{1}{q}\frac{\partial j_n(x)}{\partial x} + (G_n(x) - R_n(x))$$

  assuming no electric field, the current is only due to diffusion and is given by

$$j_n(x) = qD_n\frac{\partial n(x)}{\partial x},$$

  where D_n is the diffusion constant for electrons in cm^2/s
  substituting, we get the continuity equation

$$\frac{\partial n(x)}{dt} = D_n\frac{\partial^2 n(x)}{\partial x^2} + (G_n(x) - R_n(x))$$

- Similarly for holes,

$$j_p(x) = -qD_p\frac{\partial p(x)}{\partial x}$$

  and the continuity equation is

$$\frac{\partial p(x)}{\partial t} = D_p\frac{\partial^2 p(x)}{\partial x^2} + (G_p(x) - R_p(x))$$

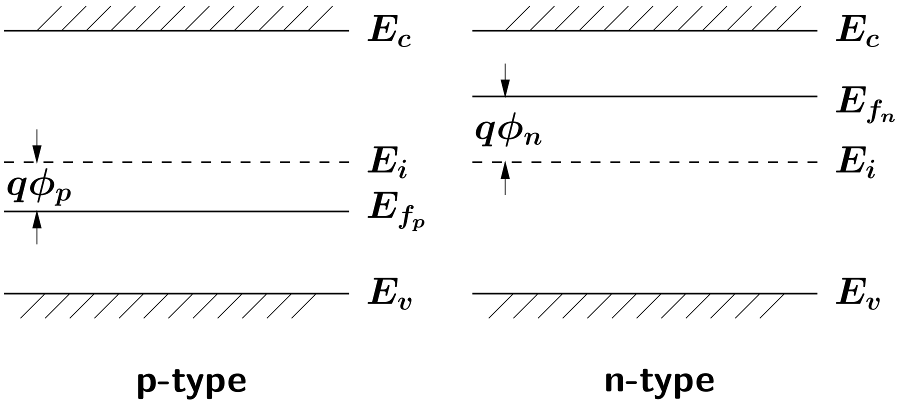
- Assuming low level injection, i.e., that excess carrier concentration $<<$ majority carrier concentration, we get that

$$R_n = \frac{n_p - n_{p0}}{\tau_n}$$

where n_{p0} is the intrinsic minority carrier concentration, and τ_n is the carrier lifetime

Appendix II – Depletion Width for PN Junction

- Energy band diagrams at thermal equilibrium



Here $\phi_n = \frac{kT}{q}\ln\frac{N_d}{n_i}$ and $\phi_p = \frac{kT}{q}\ln\frac{N_a}{n_i}$, where

$k = 8.62 \times 10^{-5}$eV K^{-1} is the Boltzman constant

T is the temperature in Kelvin

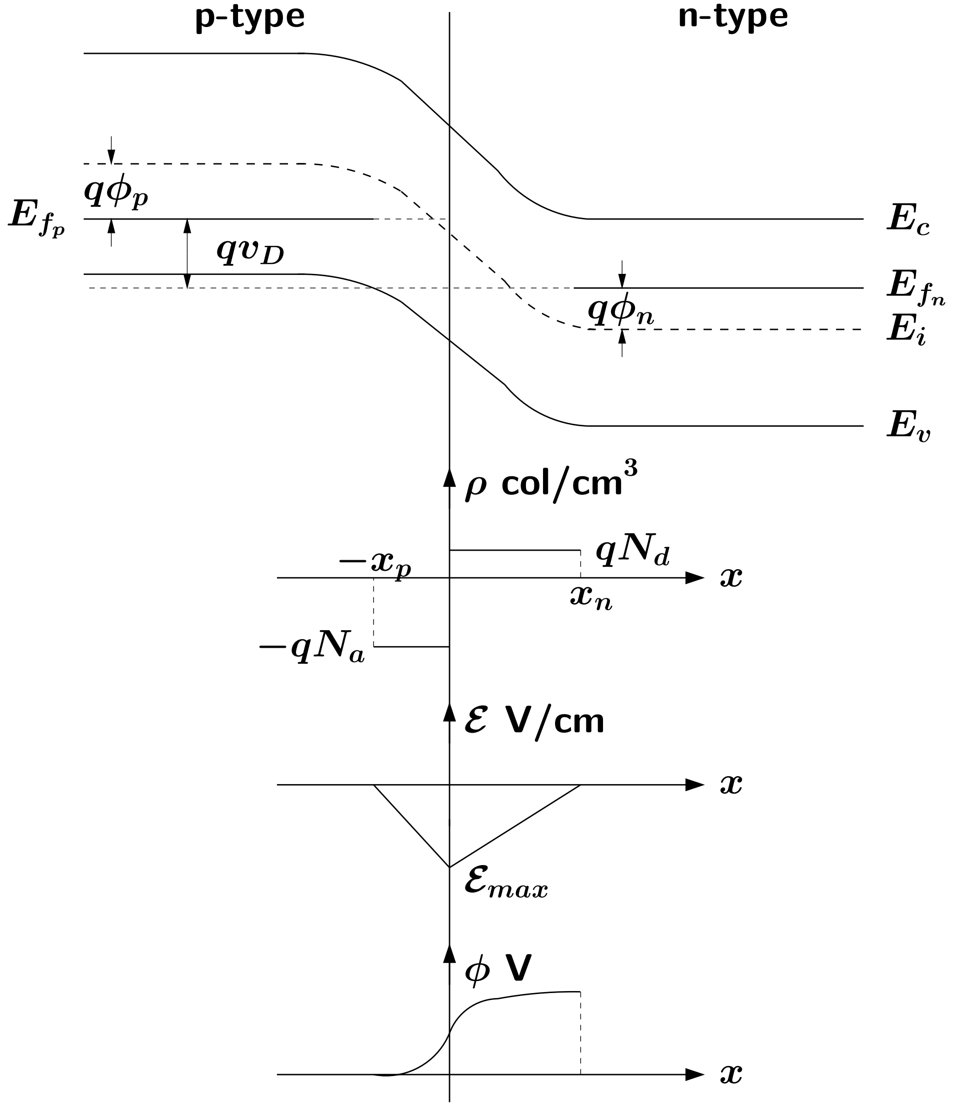
$q = 1.6 \times 10^{-19}$Col is the electron charge

n_i is the intrinsic carrier concentration $\approx 1.45 \times 10^{10}$cm^{-3} at room temperature

N_d and N_a are the donor and acceptor densities in cm^{-3}

PN Junction Energy Band Diagram

- The energy band diagram for reverse biased pn junction



$\mathcal{E}$ and ϕ are found by solving the Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{\rho(x)}{\epsilon_s},$$

where $\epsilon_s = 10.45 \times 10^{-13}$F/cm is the permittivity of Si

So in the p-type region, we obtain

$$\mathcal{E}(x) = -\frac{qN_a}{\epsilon_s}(x + x_p)$$

and

$$\mathcal{E}_{max} = -\frac{qN_a}{\epsilon_s}x_p$$

Similarly, in the n-type region we have

$$\mathcal{E}(x) = \frac{qN_d}{\epsilon_s}(x - x_n)$$

and

$$\mathcal{E}_{max} = -\frac{qN_d}{\epsilon_s}x_n$$

Thus

$$\frac{x_n}{x_p} = \frac{N_a}{N_d}$$

Now

$$\begin{aligned}\phi(x_n) &= -\int_{-x_p}^{x_n} \mathcal{E} dx \\ &= \frac{qN_d x_n^2}{2\epsilon_s} + \frac{qN_a x_p^2}{2\epsilon_s} \\ &= v_D + \phi_n + \phi_p\end{aligned}$$

Combining the last two equations, we obtain that the depletion width

$$x_d = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q}(v_D + \phi_n + \phi_p)(\frac{1}{N_a} + \frac{1}{N_d})}$$

- Example (nwell/psub diode): assuming $v_D = 2$V, $\phi_n = 0.3486$V, and $\phi_p = 0.289$V, we get $x_n = 0.176\mu$m and $x_p = 1.76\mu$m
- The (small signal) diode capacitance per unit area is defined as

$$C = \frac{dQ}{dv_D},$$

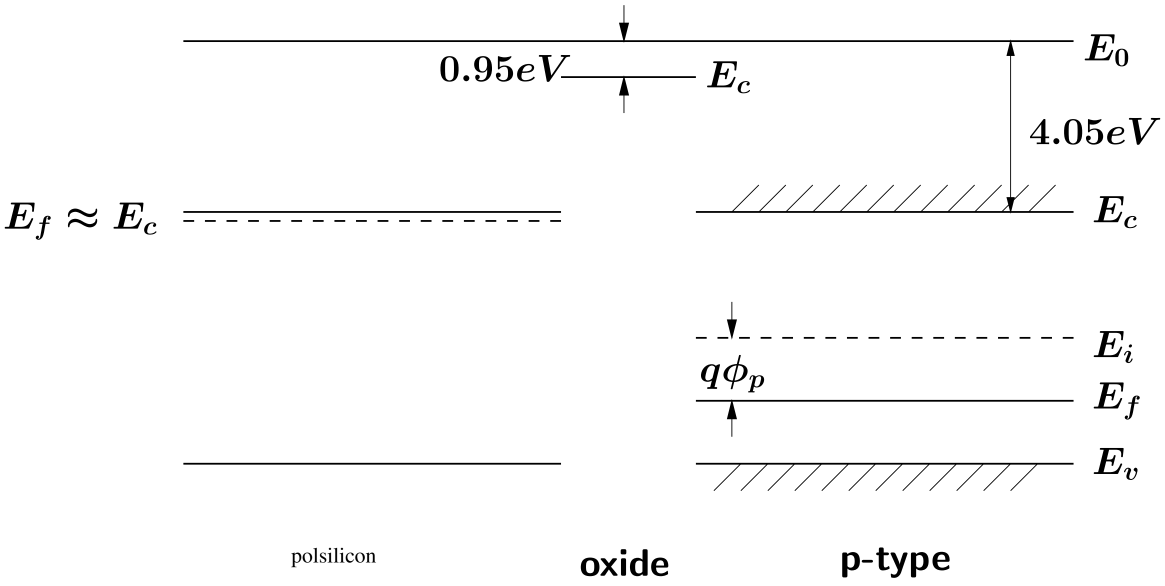
where the charge $Q = qN_d x_n = qN_a x_p$. Thus,

$$C = \frac{\epsilon_s}{x_n + x_p}\text{F/cm}^2$$

For the previous example $C = 5.4 \times 10^{-9}$F/cm^2

Appendix III – MOS Capacitor

- First consider the energy band diagrams under thermal equilibrium for polysilicon, oxide, and silicon

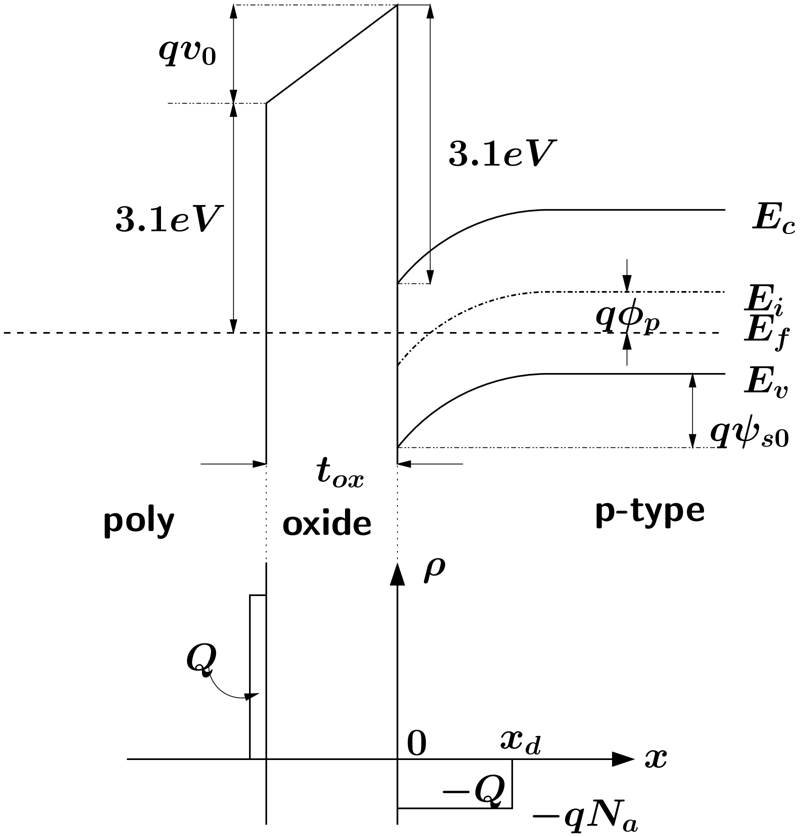


E_0 is the free electron energy

$E_0 - E_c = 4.05$eV is the semiconductor electron affinity

$E_0 - {E_c}^{\text{oxide}} = 0.95$eV is the oxide electron affinity

- The energy band diagram for the MOS system under thermal equilibrium assuming $v_G = 0$



We can find v_0, ψ_0, and x_d by writing the flat-band voltage in two ways and solving the Poisson equation

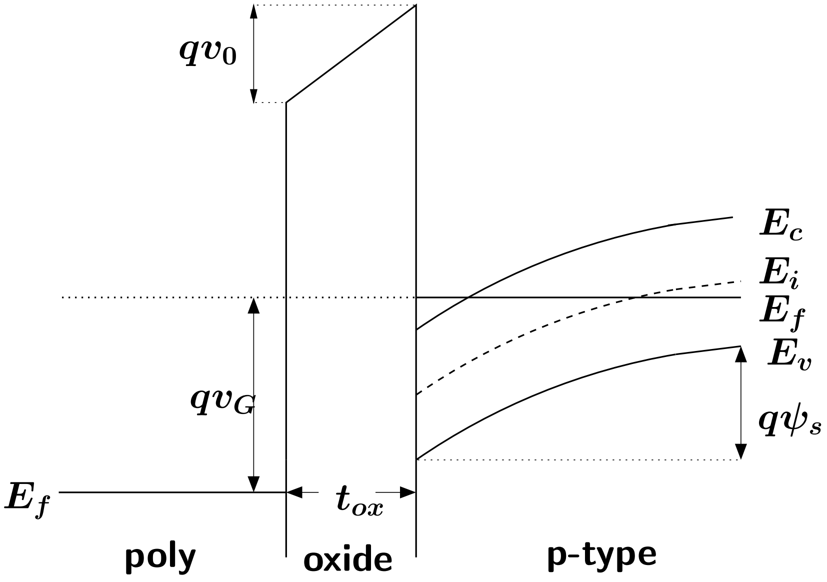
$$v_{FB} = \frac{E_g}{2q} + \phi_p = v_0 + \psi_{s0},$$

$$v_0 = \frac{qN_a x_d}{C_{ox}}, \text{ and}$$

$$\psi_{s0} = \frac{qN_a x_d^2}{2\epsilon_s}$$

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$F/cm^2, and $\epsilon_{ox} = 34.5 \times 10^{-14}$F/cm

- Energy band diagram in the deep depletion regime



The MOS system is in deep depletion when $\psi_s > 2\phi_p$ (this is the same condition as for strong inversion except that here we are interested in the transient response before the onset of strong inversion)

This gives that

$$v_G > 2\phi_p - v_{FB} + \frac{1}{C_{ox}}\sqrt{4qN_a\epsilon_s\phi_p} = v_T,$$

where v_T is the $threshold\ voltage$ (assuming no threshold adjust implant is used)

- To find the depletion region depth x_d, note that

$$v_0 + \psi_s = v_G + v_{FB},$$

where

$$\begin{aligned} \psi_s &= \frac{qN_a x_d^2}{2\epsilon_s}, \text{ and} \\ v_0 &= \frac{qN_a x_d}{C_{ox}} \end{aligned}$$

solving for ψ_s we get

$$\psi_s = v_1 + v_2 - \sqrt{v_2^2 + 2v_1v_2},$$

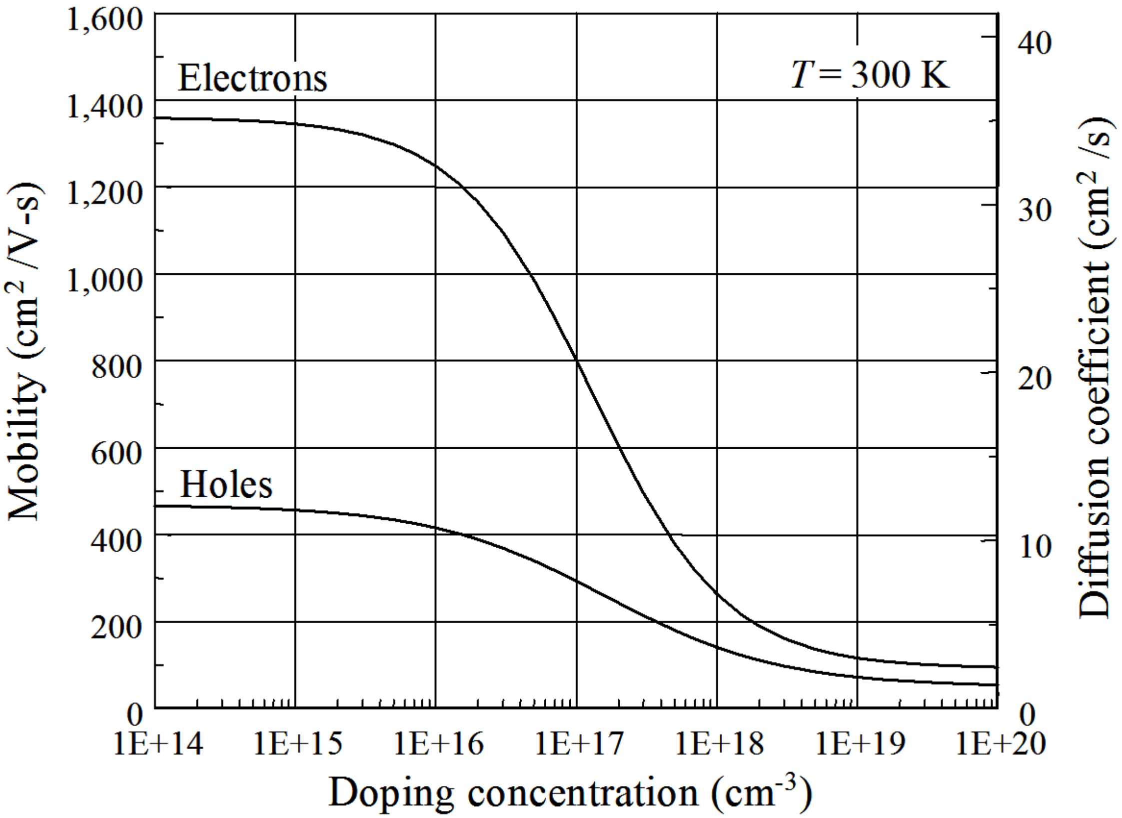
where

$$\begin{aligned} v_1 &= v_G + v_{FB}, \text{ and} \\ v_2 &= \frac{qN_a\epsilon_s}{C_{ox}^2} \end{aligned}$$

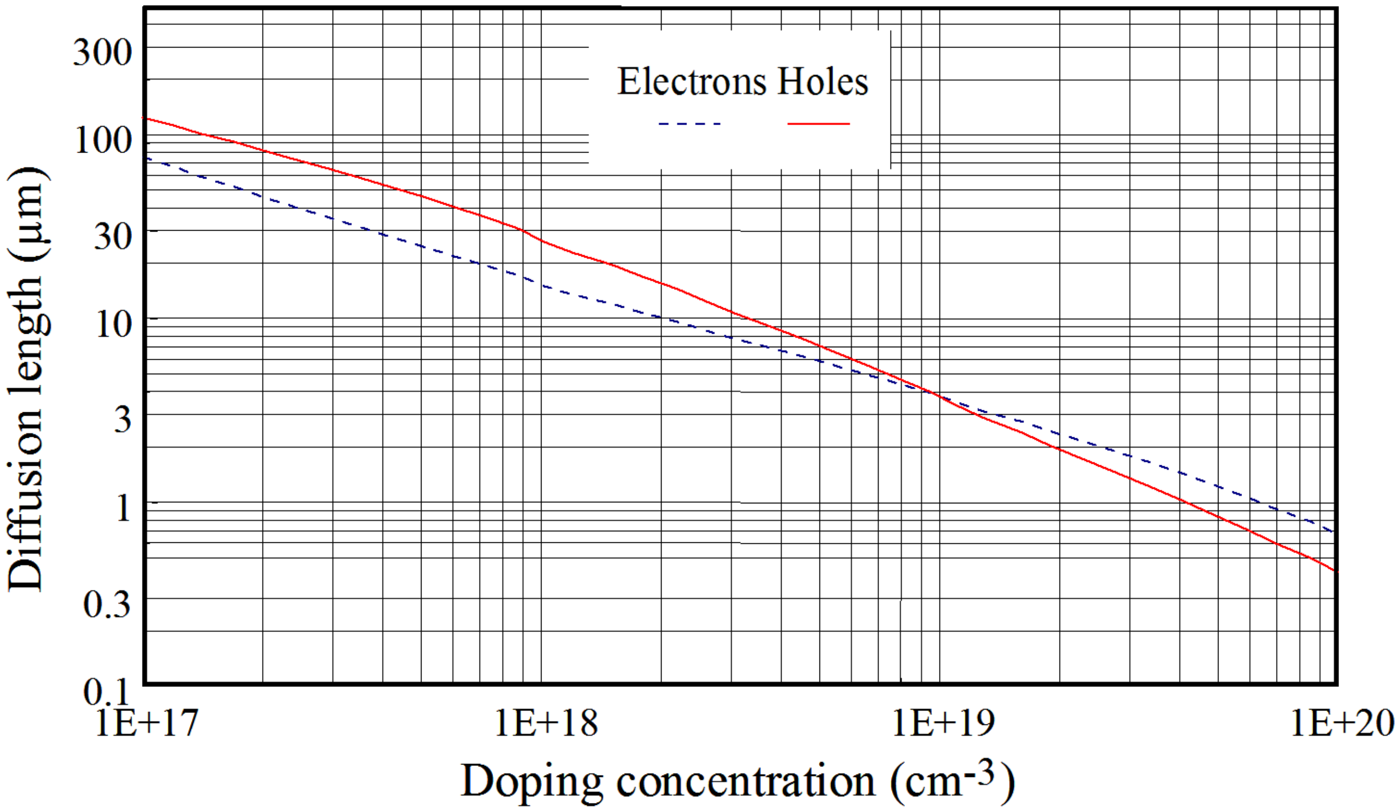
the depletion width can then be determined

Note: for the MOS capacitor to stay in the deep depletion we set $v_D = \psi_s$

Appendix IV
Bulk Mobility



Minority Carrier Diffusion Length



Minority Carrier Lifetime