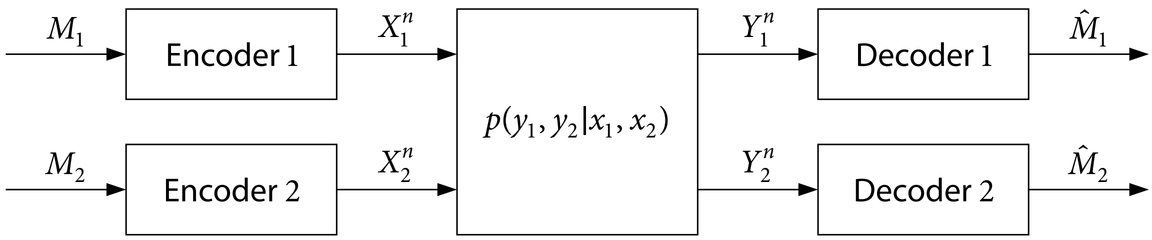
Lecture #6 Interference Channels

(Reading: NIT 6.1–6.7)

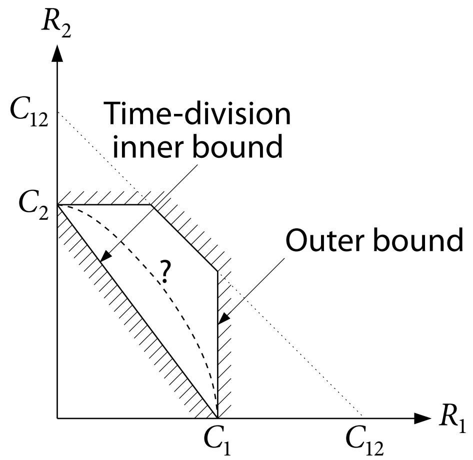
- Discrete memoryless interference channel
- Coding schemes using point-to-point codes
- Gaussian interference channel
- Han–Kobayashi inner bound
- Half-bit theorem for the Gaussian IC
- IC with more than 2 user pairs

Two sender–receiver pair communication system



- DM interference channel (IC) $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$
- $(2^{nR_1}, 2^{nR_2}, n)$ code, $P_e^{(n)}$, achievability: Similar to MAC and BC
- Capacity region $\mathscr{C}$: Closure of the set of achievable (R_1, R_2)
- As for BC, $\mathscr{C}$ depends on $p(y_1, y_2|x_1, x_2)$ only through $p(y_j|x_1, x_2)$, $j = 1, 2$
- Capacity region is not known in general
- We study coding schemes that are optimal in some special cases

Simple bounds on the capacity region



- Individual capacities:

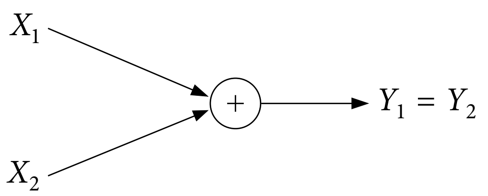
$$C_1 = \max_{p(x_1),\, x_2} I(X_1; Y_1 | X_2 = x_2), \quad C_2 = \max_{p(x_2),\, x_1} I(X_2; Y_2 | X_1 = x_1)$$

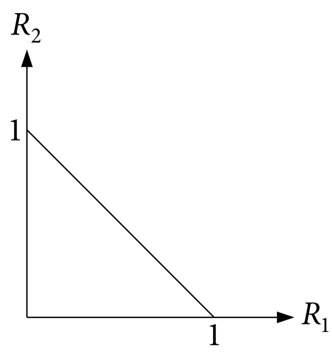
- Upper bound on the sum-rate:

$$R_1 + R_2 \le C_{12} = \max_{p(x_1)p(x_2)} \min_{\tilde{p}(y_1, y_2 | x_1, x_2)} I(X_1, X_2; Y_1, Y_2)$$

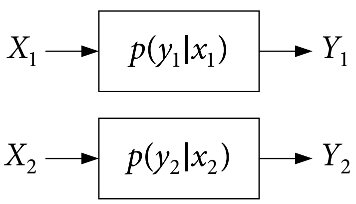
Examples

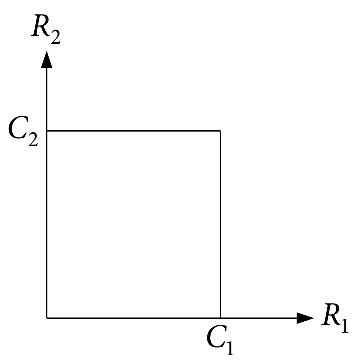
- Modulo-2 sum IC: $Y_1 = Y_2 = X_1 \oplus X_2$





- No interference: $p(y_1, y_2 | x_1, x_2) = p(y_1|x_1) p(y_2|x_2)$





Coding schemes using point-to-point codes

- Codebook generation (as for the MAC):
  - Fix $p(q)p(x_1|q)p(x_2|q)$
  - Randomly generate a time-sharing sequence $q^n \sim \prod_{i=1}^n p_Q(q_i)$
  - Conditionally independently generate $x_1^n(m_1) \sim \prod_{i=1}^n p_{X_1|Q}(x_{1i}|q_i)$, $m_1 \in [1:2^{nR_1}]$
  - Conditionally independently generate $x_2^n(m_2) \sim \prod_{i=1}^n p_{X_2|Q}(x_{2i}|q_i)$, $m_2 \in [1:2^{nR_2}]$
- Encoding:
  - To send message m_1, encoder 1 transmits $x_1^n(m_1)$
  - To send message m_2, encoder 2 transmits $x_2^n(m_2)$

Treating interference as noise

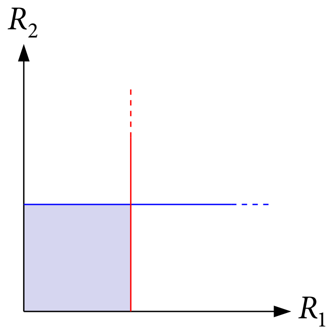
- Decoding: Find the unique $\hat{m}_j$ such that

$$(q^n, x_j^n(\hat{m}_j), y_j^n) \in \mathcal{T}_\epsilon^{(n)}, \quad j = 1, 2$$

- Interference-as-noise inner bound (IAN): (R_1, R_2) is achievable if

$$R_1 < I(X_1; Y_1|Q),$$
$$R_2 < I(X_2; Y_2|Q)$$

  for some $p(q)p(x_1|q)p(x_2|q)$



- Tight for no interference (and for the modulo-2 sum IC)
- Includes time division as a special case

Simultaneous decoding

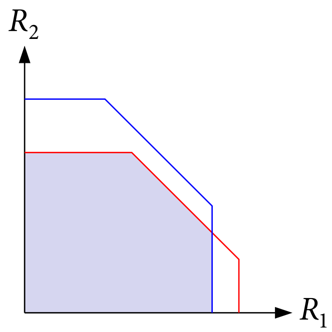
- Decoding: Find the unique $(\hat{m}_1, \hat{m}_2)$ such that

$$(q^n, x_1^n(\hat{m}_1), x_2^n(\hat{m}_2), y_j^n) \in \mathcal{T}_\epsilon^{(n)}, \quad j = 1, 2$$

- Simultaneous-decoding inner bound: (R_1, R_2) is achievable if

$$\begin{aligned} R_1 &< \min\{I(X_1; Y_1 | X_2, Q), I(X_1; Y_2 | X_2, Q)\}, \\ R_2 &< \min\{I(X_2; Y_1 | X_1, Q), I(X_2; Y_2 | X_1, Q)\}, \\ R_1 + R_2 &< \min\{I(X_1, X_2; Y_1 | Q), I(X_1, X_2; Y_2 | Q)\} \end{aligned}$$

  for some $p(q)p(x_1|q)p(x_2|q)$



- Can be strictly larger than the region using (uncoded) time sharing
- Tight for the symmetric DM-IC $p_{Y_1|X_1,X_2}(y|x_1, x_2) = p_{Y_2|X_1,X_2}(y|x_1, x_2)$

Simultaneous nonunique decoding

- Decoding: Find the unique $\hat{m}_1$ such that

$$(q^n, x_1^n(\hat{m}_1), x_2^n(m_2), y_1^n) \in \mathcal{T}_\epsilon^{(n)} \quad \text{for some } m_2$$

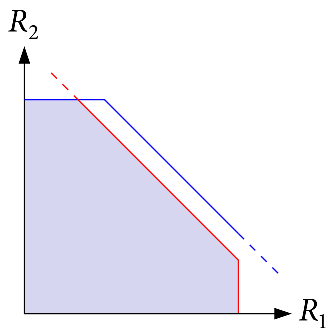
  Find the unique $\hat{m}_2$ such that

$$(q^n, x_1^n(m_1), x_2^n(\hat{m}_2), y_2^n) \in \mathcal{T}_\epsilon^{(n)} \quad \text{for some } m_1$$

- Simultaneous-nonunique-decoding inner bound: (R_1, R_2) is achievable if

$$\begin{aligned} R_1 &< I(X_1; Y_1 | X_2, Q), \\ R_2 &< I(X_2; Y_2 | X_1, Q), \\ R_1 + R_2 &< \min\{I(X_1, X_2; Y_1 | Q), I(X_1, X_2; Y_2 | Q)\} \end{aligned}$$

  for some $p(q)p(x_1|q)p(x_2|q)$



Analysis of the probability of error

- Consider $\mathsf{P}(\mathcal{E})$ conditioned on $(M_1, M_2) = (1, 1)$
- Error events for decoder 1:

$$\mathcal{E}_1 = \{(Q^n, X_1^n(1), X_2^n(1), Y_1^n) \notin \mathcal{T}_\epsilon^{(n)}\},$$
$$\mathcal{E}_2 = \{(Q^n, X_1^n(m_1), X_2^n(m_2), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1, m_2\}$$

- By the LLN, $\mathsf{P}(\mathcal{E}_1) \to 0$
- Decompose $\mathcal{E}_2$ into

$$\mathcal{E}_{21} = \{(Q^n, X_1^n(m_1), X_2^n(1), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1\},$$
$$\mathcal{E}_{22} = \{(Q^n, X_1^n(m_1), X_2^n(m_2), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1, m_2 \neq 1\}$$

- By the packing lemma,

$$\mathsf{P}(\mathcal{E}_{21}) \to 0 \text{ if } R_1 < I(X_1; Y_1 | X_2, Q),$$
$$\mathsf{P}(\mathcal{E}_{22}) \to 0 \text{ if } R_1 + R_2 < I(X_1, X_2; Y_1 | Q)$$

Strong interference

- Very strong interference if

$$I(X_1; Y_1 | X_2) \leq I(X_1; Y_2),$$
$$I(X_2; Y_2 | X_1) \leq I(X_2; Y_1)$$

  for all $p(x_1)p(x_2)$

- Strong interference if

$$I(X_1; Y_1 | X_2) \leq I(X_1; Y_2 | X_2),$$
$$I(X_2; Y_2 | X_1) \leq I(X_2; Y_1 | X_1)$$

  for all $p(x_1)p(x_2)$
  - Note similarity to more capable notion for BC

- Very strong $\Rightarrow$ strong, but converse does not hold in general
- Example: $X_1, X_2 \in \{0, 1\}$ and $Y_1 = Y_2 = X_1 + X_2 \in \{0, 1, 2\}$
  This channel satisfies strong interference, but not very strong interference

Strong interference

Theorem 6.1 (Costa–El Gamal 1987)

The capacity region with strong interference is the set of (R_1, R_2) such as

$$\begin{aligned} R_1 &\le I(X_1; Y_1 | X_2, Q), \\ R_2 &\le I(X_2; Y_2 | X_1, Q), \\ R_1 + R_2 &\le \min\{I(X_1, X_2; Y_1 | Q),\ I(X_1, X_2; Y_2 | Q)\} \end{aligned}$$

for some $p(q)p(x_1|q)p(x_2|q)$, where $|\mathcal{Q}| \le 4$

- Achievability: simultaneous nonunique decoding
- Converse: $I(X_1^n; Y_1^n | X_2^n) \le I(X_1^n; Y_2^n | X_2^n)$ for all $(X_1^n, X_2^n) \sim p(x_1^n)p(x_2^n)$ and all $n \ge 1$
- Capacity region with very strong interference: Set of (R_1, R_2) such that

$$\begin{aligned} R_1 &\le I(X_1; Y_1 | X_2, Q), \\ R_2 &\le I(X_2; Y_2 | X_1, Q) \end{aligned}$$

for some $p(q)p(x_1|q)p(x_2|q)$, where $|\mathcal{Q}| \le 2$

More on simultaneous nonunique decoding

- SND: Find the unique $\hat{m}_1$ such that $(q^n, x_1^n(\hat{m}_1), x_2^n(m_2), y_1^n) \in \mathcal{T}_\epsilon^{(n)}$ for some m_2
- Recall the error events for decoder 1 (when $(M_1, M_2) = (1, 1)$):

$$\begin{aligned} \mathcal{E}_1 &= \{(Q^n, X_1^n(1), X_2^n(1), Y_1^n) \notin \mathcal{T}_\epsilon^{(n)}\}, \\ \mathcal{E}_2 &= \{(Q^n, X_1^n(m_1), X_2^n(m_2), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \ne 1, m_2\} \end{aligned}$$

- Since $(Q^n, X_1^n(m_1), X_2^n(m_2), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \Rightarrow (Q^n, X_1^n(m_1), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}$ for every m_2,

$$\mathcal{E}_2 \subseteq \mathcal{E}_2' = \{(Q^n, X_1^n(m_1), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \ne 1\}$$

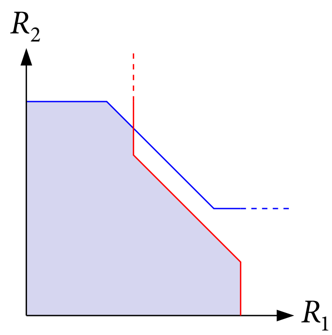
- By the packing lemma, $\mathsf{P}(\mathcal{E}_2') \to 0$ if $R_1 < I(X_1; Y_1 | Q)$ (SND includes IAN)
- Strengthened inequalities (Bandemer–El Gamal–Kim 2012):

$$R_1 < I(X_1; Y_1 | X_2, Q), \quad R_1 + R_2 < I(X_1, X_2; Y_1 | Q)$$

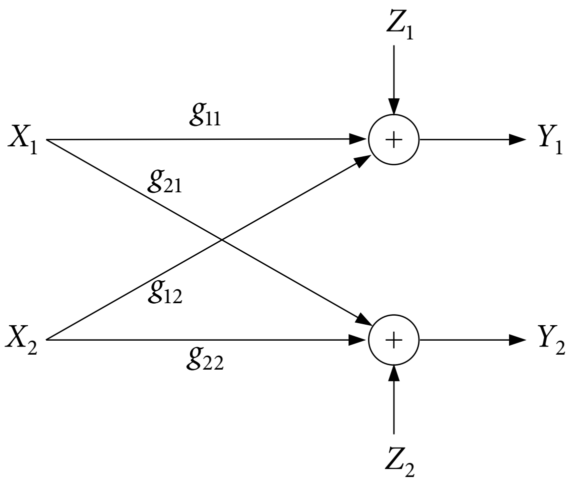
or

$$R_1 < I(X_1; Y_1 | Q)$$

- Similar inequalities for decoder 2



Gaussian interference channel



- $Z_j \sim \mathrm{N}(0,1)$ for $j = 1,2$
- Power constraint P of X_1 and on X_2
- Signal-to-noise ratios (SNRs) $S_1 = g_{11}^2 P$, $S_2 = g_{22}^2 P$
- Interference-to-noise ratios (INRs) $I_1 = g_{12}^2 P$ and $I_2 = g_{21}^2 P$

Coding schemes using point-to-point codes

- Treating interference as noise
  - Very difficult to evaluate (Gaussian is not optimal)
- Consider two special cases:
  - Time division with power control: $\mathrm{P}\{Q=1\} = \alpha$, $\mathrm{P}\{Q=2\} = \bar{\alpha}$,
    $X_1|\{Q=1\} \sim \mathrm{N}(0, P/\alpha)$, $X_2|\{Q=2\} = 0$,
    $X_2|\{Q=1\} = 0$, $X_2|\{Q=2\} \sim \mathrm{N}(0, P/\bar{\alpha})$

    Achieves set of (R_1, R_2) such that

    $$R_1 < \alpha\, \mathsf{C}(S_1/\alpha),$$
    $$R_2 < \bar{\alpha}\, \mathsf{C}(S_2/\bar{\alpha}) \quad \text{for some } \alpha \in [0,1]$$

  - Treat interference as (Gaussian) noise: $Q = \emptyset$, $X_j \sim \mathrm{N}(0,P)$, $j = 1,2$
    Achieves set of (R_1, R_2) such that

    $$R_1 < \mathsf{C}\left(S_1/(1+I_1)\right),$$
    $$R_2 < \mathsf{C}\left(S_2/(1+I_2)\right)$$

    Sum-rate optimal under weak interference (see NIT 6.4.3)

Coding schemes using point-to-point codes

- Simultaneous nonunique decoding: (R_1, R_2) achievable if

$$\begin{aligned} R_1 &< \mathsf{C}(S_1), \\ R_2 &< \mathsf{C}(S_2), \\ R_1 + R_2 &< \min\{\mathsf{C}(S_1 + I_1), \mathsf{C}(S_2 + I_2)\} \end{aligned}$$

- Theorem 6.2 (Sato 1981): Optimal under strong interference: $I_2 \geq S_1$ and $I_1 \geq S_2$
  - Need to show:

    $I_2 \geq S_1$ and $I_1 \geq S_2$ iff $I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2)$ and $I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1)$

- Optimal under very strong interference: $S_2 \leq I_1/(1 + S_1)$ and $S_1 \leq I_2/(1 + S_2)$

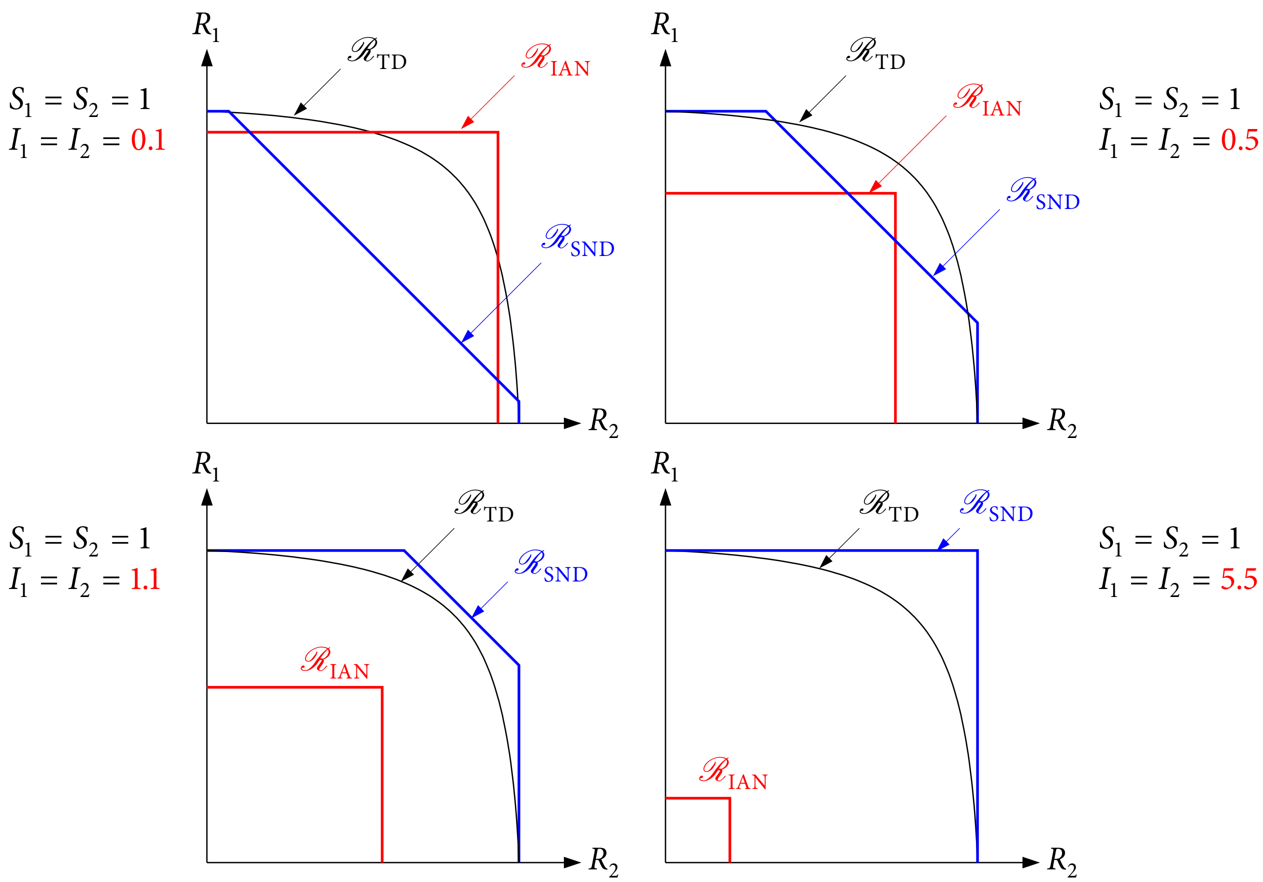
  Capacity region is set of (R_1, R_2) such that

$$\begin{aligned} R_1 &< \mathsf{C}(S_1), \\ R_2 &< \mathsf{C}(S_2) \end{aligned}$$

  - Interference does not impair communication!

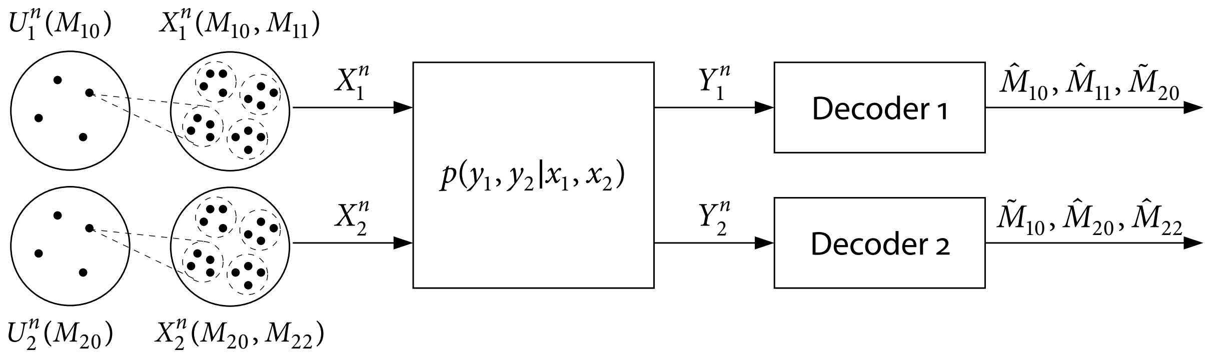
- Remark: As for the DM case, SND region can be improved to include IAN

Comparison of coding schemes



Han–Kobayashi coding scheme

- Point-to-point codes are good for two extreme cases:
  - Strong interference: Decode interference
  - Weak interference: Treat interference as noise
- H–K coding scheme: Decode part of interference, treat rest as noise
- Key new idea: Rate splitting ($M_1 = (M_{10}, M_{11})$ and $M_2 = (M_{20}, M_{22})$)
- Superposition coding of public message M_{j0} and private message M_{jj}



Han–Kobayashi inner bound

Theorem 6.4 (Han–Kobayashi 1981)

(R_1, R_2) is achievable if

$$\begin{aligned}
R_1 &< I(X_1; Y_1 | U_2, Q),\\
R_2 &< I(X_2; Y_2 | U_1, Q),\\
R_1 + R_2 &< I(X_1, U_2; Y_1 | Q) + I(X_2; Y_2 | U_1, U_2, Q),\\
R_1 + R_2 &< I(X_1; Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | Q),\\
R_1 + R_2 &< I(X_1, U_2; Y_1 | U_1, Q) + I(X_2, U_1; Y_2 | U_2, Q),\\
2R_1 + R_2 &< I(X_1, U_2; Y_1 | Q) + I(X_1; Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | U_2, Q),\\
R_1 + 2R_2 &< I(X_2, U_1; Y_2 | Q) + I(X_2; Y_2 | U_1, U_2, Q) + I(X_1, U_2; Y_1 | U_1, Q)
\end{aligned}$$

for some $p(q)p(u_1, x_1|q)p(u_2, x_2|q)$, where $|\mathcal{U}_1| \le |\mathcal{X}_1| + 4$, $|\mathcal{U}_2| \le |\mathcal{X}_2| + 4$, $|\mathcal{Q}| \le 7$

- Interference-as-noise inner bound: Set $U_j = \emptyset$
- Simultaneous-nonunique-decoding inner bound: Set $U_j = X_j$

Proof of the Han–Kobayashi inner bound

- Codebook generation: Rate splitting, superposition coding, coded time-sharing
- Encoding: Encoder $j = 1, 2$ transmits $x_j^n(m_{j0}, m_{jj})$
- Decoding: Use simultaneous nonunique decoding:
  - Decoder 1 finds unique $(\hat{m}_{10}, \hat{m}_{11})$ such that

$$(q^n, u_1^n(\hat{m}_{10}), u_2^n(m_{20}), x_1^n(\hat{m}_{10}, \hat{m}_{11}), y_1^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_{20} \in [1:2^{nR_{20}}]$$

  - Decoder 2 finds unique $(\hat{m}_{20}, \hat{m}_{22})$ such that

$$(q^n, u_1^n(m_{10}), u_2^n(\hat{m}_{20}), x_2^n(\hat{m}_{20}, \hat{m}_{22}), y_2^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_{10} \in [1:2^{nR_{20}}]$$

Analysis of the probability of error

- Assume message pair $((1,1),(1,1))$ is sent
- Consider the average probability of error for decoder 1:

	m_{10}	m_{20}	m_{11}	Joint pmf
1	1	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n, u_2^n)$
2	1	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n, u_2^n)$
3	$*$	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
4	$*$	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
5	1	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n)$
6	$*$	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
7	$*$	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
8	1	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n)$

- Cases 3,4 and 6,7 share same pmf, and case 8 does not cause an error

Proof of the Han–Kobayashi inner bound

- The probability of error $\to 0$ as $n \to \infty$ for decoder 1 if:

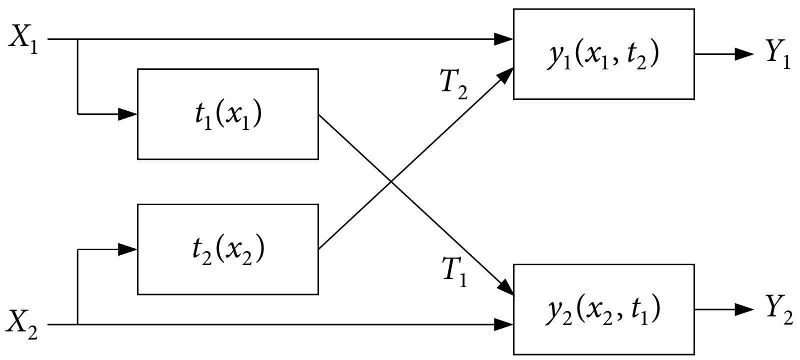
$$\begin{aligned} R_{11} &< I(X_1; Y_1 | U_1, U_2, Q), \\ R_{11} + R_{10} &< I(X_1; Y_1 | U_2, Q), \\ R_{11} + R_{20} &< I(X_1, U_2; Y_1 | U_1, Q), \\ R_{11} + R_{10} + R_{20} &< I(X_1, U_2; Y_1 | Q) \end{aligned}$$

- Following similar steps for decoder 2, its probability of error $\to 0$ as $n \to \infty$ if:

$$\begin{aligned} R_{22} &< I(X_2; Y_2 | U_1, U_2, Q), \\ R_{22} + R_{20} &< I(X_2; Y_2 | U_1, Q), \\ R_{22} + R_{10} &< I(X_2, U_1; Y_2 | U_2, Q), \\ R_{22} + R_{20} + R_{10} &< I(X_2, U_1; Y_2 | Q) \end{aligned}$$

- Fourier–Motzkin elimination finds the set of $(R_1, R_2) = (R_{10} + R_{11}, R_{20} + R_{22})$ (Read NIT Appendix D)

Injective deterministic IC (El Gamal–Costa 1982)



- For every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- For every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1
- In other words, $H(Y_1|X_1) = H(T_2)$ and $H(Y_2|X_2) = H(T_1)$ for all $p(x_1)p(x_2)$

Injective deterministic IC (El Gamal–Costa 1982)

Theorem 6.5

The capacity region of the injective deterministic IC is the set of (R_1, R_2) such that

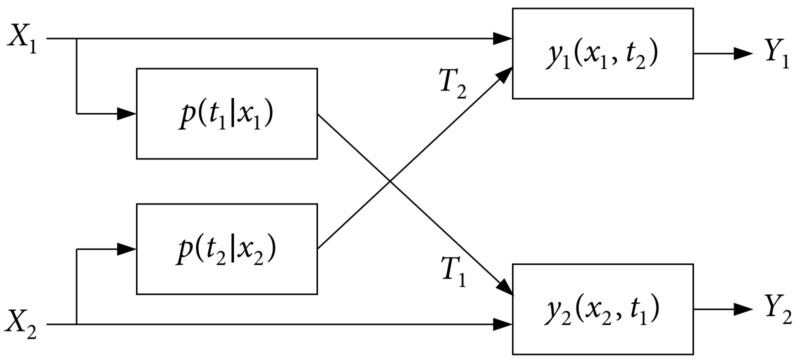
$$\begin{aligned} R_1 &\le H(Y_1|T_2, Q), \\ R_2 &\le H(Y_2|T_1, Q), \\ R_1 + R_2 &\le H(Y_1|Q) + H(Y_2|T_1, T_2, Q), \\ R_1 + R_2 &\le H(Y_1|T_1, T_2, Q) + H(Y_2|Q), \\ R_1 + R_2 &\le H(Y_1|T_1, Q) + H(Y_2|T_2, Q), \\ 2R_1 + R_2 &\le H(Y_1|Q) + H(Y_1|T_1, T_2, Q) + H(Y_2|T_2, Q), \\ R_1 + 2R_2 &\le H(Y_2|Q) + H(Y_2|T_1, T_2, Q) + H(Y_1|T_1, Q) \end{aligned}$$

for some $p(q)p(x_1|q)p(x_2|q)$

- Achievability: Han–Kobayashi ($U_j = T_j, j = 1, 2$)
- Converse: Genie argument ($T_j \to Y_j, j = 1, 2$)
- A special case turned out to be good model for Gaussian IC in high SNR (Avestimehr–Diggavi–Tse 2011)

Half-bit theorem for the Gaussian IC

- Han–Kobayashi inner bound within 1/2 bit per dimension from capacity region
- To prove this, consider the injective semideterministic IC (Telatar–Tse 2007)



  - For every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
  - For every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

- Gaussian IC is a special case:

$$T_1 = g_{21}X_1 + Z_2, \quad T_2 = g_{12}X_2 + Z_1,$$
$$Y_1 = g_{11}X_1 + T_2, \quad Y_2 = g_{22}X_2 + T_1$$

Injective semideterministic IC: Outer bound $\mathscr{R}_o$

Proposition 6.1 (Telatar–Tse 2007)

If a rate pair (R_1, R_2) is achievable, then

$$\begin{aligned}
R_1 &\le H(Y_1|X_2, Q) - H(T_2|X_2, Q),\\
R_2 &\le H(Y_2|X_1, Q) - H(T_1|X_1, Q),\\
R_1 + R_2 &\le H(Y_1|Q) + H(Y_2|X_1, U_2, Q) - H(T_1|X_1, Q) - H(T_2|X_2, Q),\\
R_1 + R_2 &\le H(Y_1|U_1, X_2, Q) + H(Y_2|Q) - H(T_1|X_1, Q) - H(T_2|X_2, Q),\\
R_1 + R_2 &\le H(Y_1|U_1, Q) + H(Y_2|U_2, Q) - H(T_1|X_1, Q) - H(T_2|X_2, Q),\\
2R_1 + R_2 &\le H(Y_1|Q) + H(Y_1|U_1, X_2, Q) + H(Y_2|U_2, Q)\\
&\quad - H(T_1|X_1, Q) - 2H(T_2|X_2, Q),\\
R_1 + 2R_2 &\le H(Y_2|Q) + H(Y_2|U_2, X_1, Q) + H(Y_1|U_1, Q)\\
&\quad - 2H(T_1|X_1, Q) - H(T_2|X_2, Q)
\end{aligned}$$

for some $p(q)p(x_1|q)p(x_2|q)p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$

- Proof: Genie argument $(U_j \to Y_j)$

Injective semideterministic IC: Inner bound $\mathscr{R}_i$

Proposition 6.2 (Telatar–Tse 2007)

A rate pair (R_1, R_2) is achievable if

$$\begin{aligned}
R_1 &< H(Y_1|U_2, Q) - H(T_2|U_2, Q),\\
R_2 &< H(Y_2|U_1, Q) - H(T_1|U_1, Q),\\
R_1 + R_2 &< H(Y_1|Q) + H(Y_2|U_1, U_2, Q) - H(T_1|U_1, Q) - H(T_2|U_2, Q),\\
R_1 + R_2 &< H(Y_1|U_1, U_2, Q) + H(Y_2|Q) - H(T_1|U_1, Q) - H(T_2|U_2, Q),\\
R_1 + R_2 &< H(Y_1|U_1, Q) + H(Y_2|U_2, Q) - H(T_1|U_1, Q) - H(T_2|U_2, Q),\\
2R_1 + R_2 &< H(Y_1|Q) + H(Y_1|U_1, U_2, Q) + H(Y_2|U_2, Q)\\
&\quad - H(T_1|U_1, Q) - 2H(T_2|U_2, Q),\\
R_1 + 2R_2 &< H(Y_2|Q) + H(Y_2|U_1, U_2, Q) + H(Y_1|U_1, Q)\\
&\quad - 2H(T_1|U_1, Q) - H(T_2|U_2, Q)
\end{aligned}$$

for some $p(q)p(x_1|q)p(x_2|q)p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$

- Proof: Han–Kobayashi with $p(u_j|x_j, q) = p_{T_j|X_j}(u_j|x_j)$

Half-bit theorem for the Gaussian IC

Lemma 6.3 (Telatar–Tse 2007)

If $(R_1, R_2) \in \mathscr{R}_o(Q, X_1, X_2)$, then

$$(R_1 - I(X_2; T_2 | U_2, Q), R_2 - I(X_1; T_1 | U_1, Q)) \in \mathscr{R}_i(Q, X_1, X_2)$$

Theorem 6.6 (Etkin–Tse–Wang 2008)

If $(R_1, R_2) \in \mathscr{R}_o(Q, X_1, X_2)$, then

$$(R_1 - 1/2, R_2 - 1/2) \in \mathscr{R}_i(Q, X_1, X_2)$$

- Proof: We have

$$T_1 = g_{21} X_1 + Z_2, \quad T_2 = g_{12} X_2 + Z_1,$$
$$U_1 = g_{21} X_1 + Z_2', \quad U_2 = g_{12} X_2 + Z_1'$$

$Z_j' \sim \mathrm{N}(0, 1)$ and $Z_j \sim \mathrm{N}(0, 1)$, $j = 1, 2$ are independent. Then

$$I(X_j; T_j | U_j, Q) = h(T_j | U_j, Q) - h(T_j | U_j, X_j, Q) \le h(T_j - U_j) - h(Z_j) = \frac{1}{2}$$

Interference channels with more than two user pairs

- IC with > 2 user pairs much less understood, e.g.,
  - We don't know how to extend strong interference

- Straightforward extension of H–K for > 2 user pairs can be improved:
  - Decoding combined interference instead of individual interfering signals (Bresler–Parekh–Tse 2010, Bandemer–El Gamal 2011)
  - Designing code so that combined interference is aligned (Cadambe–Jafar 2008)

Summary

- Discrete memoryless interference channel (DM-IC)
- Simultaneous nonunique decoding is optimal under strong interference
- Han–Kobayashi coding scheme:
  - Rate splitting and superposition coding
  - Fourier–Motzkin elimination
  - Optimal for injective deterministic ICs
- Gaussian interference channel:
  - Capacity region under strong interference achieved via simultaneous decoding
  - Sum-capacity under weak interference achieved by treating interference as noise
  - Han–Kobayashi coding scheme achieves within half a bit of the capacity region

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