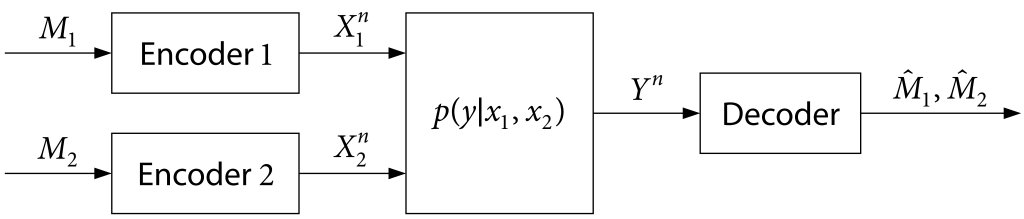
Lecture #4 Multiple access channels

(Reading: NIT 4.1, 4.2, 4.5)

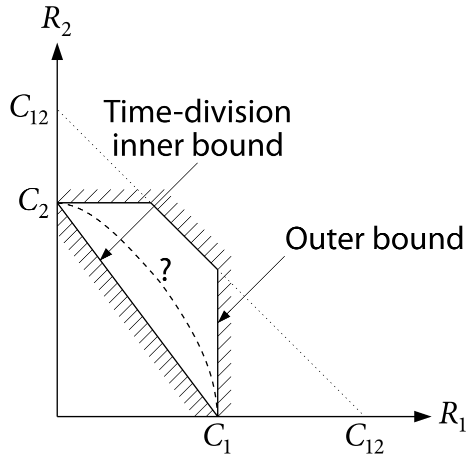
- Discrete memoryless multiple access channel
- Simple bounds on the capacity region
- Capacity region
- Gaussian multiple access channel
- Extension to more than two senders

Multiple access communication system



- DM multiple access channel (MAC) $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2), \mathcal{Y})$
- A $(2^{nR_1}, 2^{nR_2}, n)$ code:
  - Message sets: $[1:2^{nR_1}]$ and $[1:2^{nR_2}]$
  - Encoder $j = 1, 2$: $x_j^n(m_j)$
  - Decoder: $(\hat{m}_1(y^n), \hat{m}_2(y^n))$
- $(M_1, M_2) \sim \text{Unif}([1:2^{nR_1}] \times [1:2^{nR_2}])$: $x_1^n(M_1)$ and $x_2^n(M_2)$ independent
- Average probability of error: $P_e^{(n)} = \mathsf{P}\{(\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\}$
- (R_1, R_2) achievable if $\exists$ $(2^{nR_1}, 2^{nR_2}, n)$ codes such that $\lim_{n\to\infty} P_e^{(n)} = 0$
- Capacity region $\mathscr{C}$: Closure of the set of achievable rate pairs (R_1, R_2)

Simple bounds on the capacity region



- Maximum achievable individual rates:

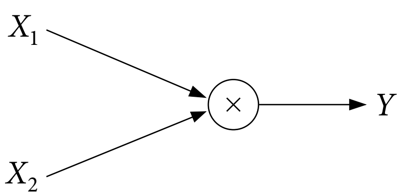
$$C_1 = \max_{x_2,\, p(x_1)} I(X_1; Y | X_2 = x_2), \quad C_2 = \max_{x_1,\, p(x_2)} I(X_2; Y | X_1 = x_1)$$

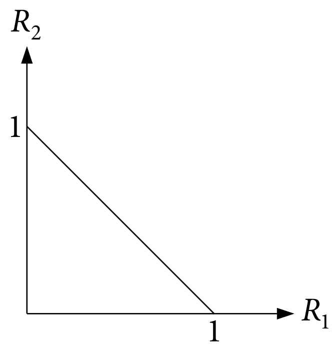
- Upper bound on the sum-rate:

$$R_1 + R_2 \le C_{12} = \max_{p(x_1)p(x_2)} I(X_1, X_2; Y)$$

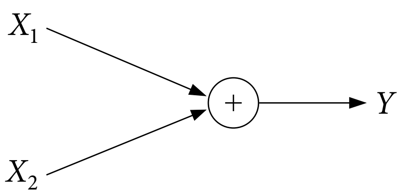
Examples

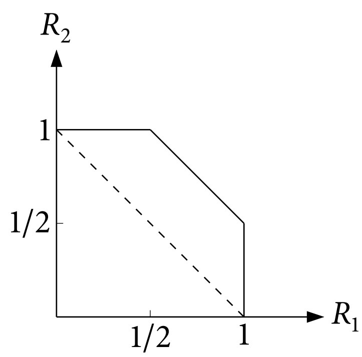
- Binary multiplier MAC: $X_1, X_2 \in \{0,1\}$, $Y = X_1 \cdot X_2 \in \{0,1\}$





- Binary erasure MAC: $X_1, X_2 \in \{0,1\}$, $Y = X_1 + X_2 \in \{0,1,2\}$

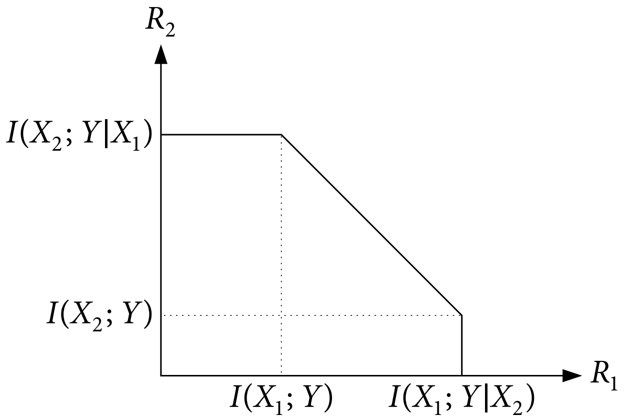




$\mathscr{R}(X_1, X_2)$ region

- For $(X_1, X_2) \sim p(x_1)p(x_2)$, let $\mathscr{R}(X_1, X_2)$ be the set of (R_1, R_2) such that

$$R_1 \le I(X_1; Y|X_2),$$
$$R_2 \le I(X_2; Y|X_1),$$
$$R_1 + R_2 \le I(X_1, X_2; Y)$$

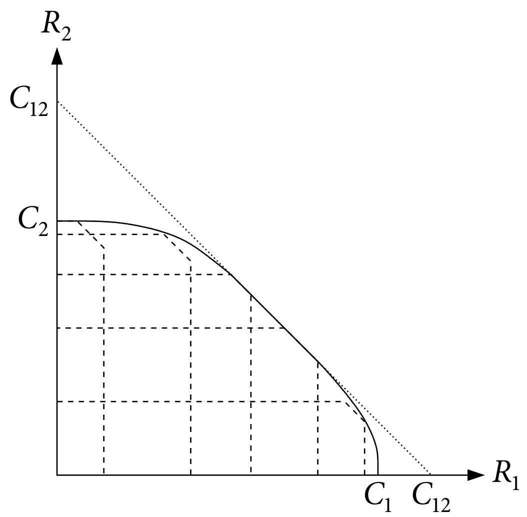


- This region is always a pentagon since

$$\max\{I(X_1; Y|X_2), I(X_2; Y|X_1)\} \le I(X_1, X_2; Y) \le I(X_1; Y|X_2) + I(X_2; Y|X_1)$$

Capacity region (Ahlswede 1971, Liao 1972)

Theorem 4.2.
$\mathscr{C}$ is the convex closure of the set $\bigcup_{p(x_1)p(x_2)} \mathscr{R}(X_1, X_2)$



- For binary erasure MAC example, outer bound is tight

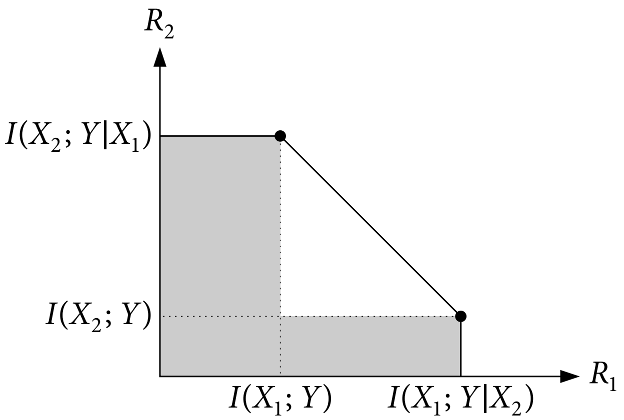
Proof of achievability

- We show that $\mathscr{R}(X_1, X_2)$ is achievable for every $(X_1, X_2) \sim p(x_1)p(x_2)$
- The rest of the proof follows by time sharing

- Codebook generation:
  - Independently generate 2^{nR_1} sequences $x_1^n(m_1) \sim \prod_{i=1}^n p_{X_1}(x_{1i})$, $m_1 \in [1:2^{nR_1}]$
  - Independently generate 2^{nR_2} sequences $x_2^n(m_2) \sim \prod_{i=1}^n p_{X_2}(x_{2i})$, $m_2 \in [1:2^{nR_2}]$
- Encoding:
  - To send message m_1, encoder 1 transmits $x_1^n(m_1)$
  - To send message m_2, encoder 2 transmits $x_2^n(m_2)$
- Decoding:
  - Successive cancellation decoding
  - Simultaneous decoding

Successive cancellation decoding

- We show that corner point $(I(X_1; Y), I(X_2; Y|X_1))$ is achievable
- Corner point $(I(X_1; Y|X_2), I(X_2; Y))$ is similarly achievable
- Rest of $\mathscr{R}(X_1, X_2)$ is achievable via time sharing



- Decoding for first corner point:
  - Find unique $\hat{m}_1$ such that $(x_1^n(\hat{m}_1), y^n) \in \mathcal{T}_\epsilon^{(n)}$
  - If such $\hat{m}_1$ is found, find unique $\hat{m}_2$ such that $(x_1^n(\hat{m}_1), x_2^n(\hat{m}_2), y^n) \in \mathcal{T}_\epsilon^{(n)}$

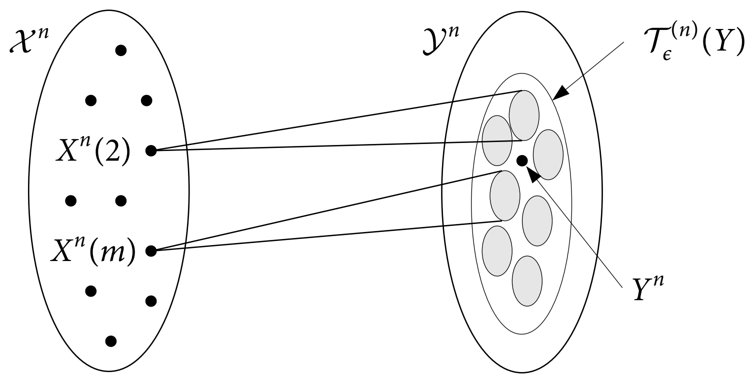
"Little" packing lemma

- Let $(X, Y) \sim p(x, y)$
- Let $\tilde{Y}^n \sim \prod_{i=1}^n p_Y(\tilde{y}_i)$
- Let $X^n(m) \sim \prod_{i=1}^n p_X(x_i)$, $m \in \mathcal{A}$, $|\mathcal{A}| \le 2^{nR}$, be pairwise independent of $\tilde{Y}^n$

"Little" packing lemma

There exists $\delta(\epsilon) \to 0$ as $\epsilon \to 0$ such that

$$\lim_{n\to\infty} \mathrm{P}\{(X^n(m), \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m \in \mathcal{A}\} = 0, \qquad \text{if } R < I(X;Y) - \delta(\epsilon)$$



Analysis of the probability of error

- Consider $\mathrm{P}(\mathcal{E})$ conditioned on $(M_1, M_2) = (1, 1)$
- Error events:

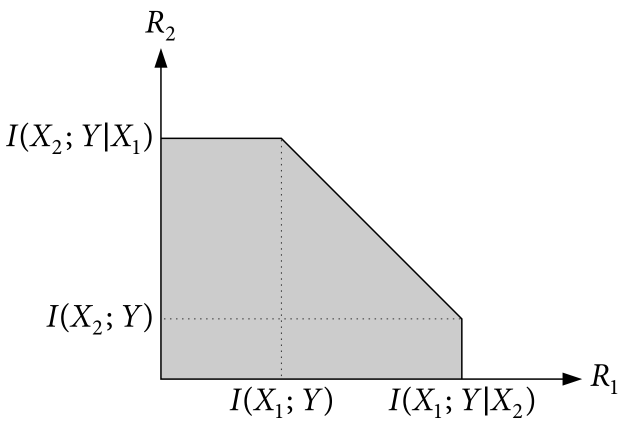
$$\begin{aligned}
\mathcal{E}_1 &= \{(X_1^n(1), X_2^n(1), Y^n) \notin \mathcal{T}_\epsilon^{(n)}\}, \\
\mathcal{E}_2 &= \{(X_1^n(m_1), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \ne 1\}, \\
\mathcal{E}_3 &= \{(X_1^n(1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_2 \ne 1\}
\end{aligned}$$

Thus, by the union of events bound

$$\mathrm{P}(\mathcal{E}) \le \mathrm{P}(\mathcal{E}_1) + \mathrm{P}(\mathcal{E}_2) + \mathrm{P}(\mathcal{E}_3)$$

- By the LLN, $\mathrm{P}(\mathcal{E}_1) \to 0$
- By the "little" packing lemma ($|\mathcal{A}| = 2^{nR_1} - 1$, $X \leftarrow X_1$),
  $\mathrm{P}(\mathcal{E}_2) \to 0$ if $R_1 < I(X_1; Y) - \delta(\epsilon)$
- By the "little" packing lemma ($|\mathcal{A}| = 2^{nR_2} - 1$, $X \leftarrow X_2$, $Y \leftarrow (X_1, Y)$),
  $\mathrm{P}(\mathcal{E}_3) \to 0$ if $R_2 < I(X_2; Y, X_1) - \delta(\epsilon) = I(X_2; Y|X_1) - \delta(\epsilon)$

Simultaneous decoding



- Find unique $(\hat{m}_1, \hat{m}_2)$ such that $(x_1^n(\hat{m}_1), x_2^n(\hat{m}_2), y^n) \in \mathcal{T}_\epsilon^{(n)}$

Analysis of the probability of error

- Consider $\mathrm{P}(\mathcal{E})$ conditioned on $(M_1, M_2) = (1, 1)$

m_1	m_2	Joint pmf
1	1	$p(x_1^n)p(x_2^n)p(y^n \vert x_1^n, x_2^n)$
$*$	1	$p(x_1^n)p(x_2^n)p(y^n \vert x_2^n)$
1	$*$	$p(x_1^n)p(x_2^n)p(y^n \vert x_1^n)$
$*$	$*$	$p(x_1^n)p(x_2^n)p(y^n)$

- Error events:

$$\begin{aligned}
\mathcal{E}_1 &= \{(X_1^n(1), X_2^n(1), Y^n) \notin \mathcal{T}_\epsilon^{(n)}\}, \\
\mathcal{E}_2 &= \{(X_1^n(m_1), X_2^n(1), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1\}, \\
\mathcal{E}_3 &= \{(X_1^n(1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_2 \neq 1\}, \\
\mathcal{E}_4 &= \{(X_1^n(m_1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1, m_2 \neq 1\}
\end{aligned}$$

Analysis of the probability of error

- Error events:

$$\begin{aligned}
\mathcal{E}_1 &= \{(X_1^n(1), X_2^n(1), Y^n) \notin \mathcal{T}_\epsilon^{(n)}\}, \\
\mathcal{E}_2 &= \{(X_1^n(m_1), X_2^n(1), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1\}, \\
\mathcal{E}_3 &= \{(X_1^n(1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_2 \neq 1\}, \\
\mathcal{E}_4 &= \{(X_1^n(m_1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1, m_2 \neq 1\}
\end{aligned}$$

- By the LLN, $\mathsf{P}(\mathcal{E}_1) \to 0$
- By the "little" packing lemma,

$$\begin{aligned}
&\mathsf{P}(\mathcal{E}_2) \to 0 \text{ if } R_1 < I(X_1; Y|X_2) - \delta(\epsilon), \\
&\mathsf{P}(\mathcal{E}_3) \to 0 \text{ if } R_2 < I(X_2; Y|X_1) - \delta(\epsilon), \\
&\mathsf{P}(\mathcal{E}_4) \to 0 \text{ if } R_1 + R_2 < I(X_1, X_2; Y) - \delta(\epsilon)
\end{aligned}$$

Proof of the converse (Slepian–Wolf 1973)

- Show that: For any sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with $P_e^{(n)} \to 0$, $(R_1, R_2) \in \mathscr{C}$
- Code induces empirical pmf

$$(M_1, M_2, X_1^n, X_2^n, Y^n) \sim 2^{-n(R_1+R_2)} p(x_1^n|m_1) p(x_2^n|m_2) \prod_{i=1}^n p_{Y|X_1,X_2}(y_i|x_{1i}, x_{2i})$$

- By Fano's inequality,

$$H(M_1, M_2|Y^n) \leq n(R_1 + R_2)P_e^{(n)} + 1 = n\epsilon_n$$

- Bounds on rates:

$$\begin{aligned}
R_1 &\leq \frac{1}{n}\sum_{i=1}^n I(X_{1i}; Y_i|X_{2i}) + \epsilon_n, \\
R_2 &\leq \frac{1}{n}\sum_{i=1}^n I(X_{2i}; Y_i|X_{1i}) + \epsilon_n, \\
R_1 + R_2 &\leq \frac{1}{n}\sum_{i=1}^n I(X_{1i}, X_{2i}; Y_i) + \epsilon_n
\end{aligned}$$

Proof of the converse

- Time-sharing random variable: $Q \sim \mathrm{Unif}[1:n]$ be independent of (X_1^n, X_2^n, Y^n)
- Define $X_1 = X_{1Q}$, $X_2 = X_{2Q}$, $Y = Y_Q$, then $p(y_q|x_{1q}, x_{2q}) = p_{Y|X_1,X_2}(y_q|x_{1q}, x_{2q})$
- The bounds can be expressed as

$$\begin{aligned} R_1 &\le I(X_1; Y|X_2, Q) + \epsilon_n, \\ R_2 &\le I(X_2; Y|X_1, Q) + \epsilon_n, \\ R_1 + R_2 &\le I(X_1, X_2; Y|Q) + \epsilon_n \end{aligned}$$

  for some joint pmf $p(x_1|q)p(x_2|q)$
- Thus (R_1, R_2) must be in the closure of the set of (R_1, R_2) such that

$$\begin{aligned} R_1 &\le I(X_1; Y|X_2, Q), \\ R_2 &\le I(X_2; Y|X_1, Q), \\ R_1 + R_2 &\le I(X_1, X_2; Y|Q) \end{aligned}$$

  for some joint pmf $p(q)p(x_1|q)p(x_2|q)$
- Denote this region by $\mathscr{C}'$

Proof of the converse

- Clearly the capacity region $\mathscr{C} \subseteq \mathscr{C}'$
- Can show that $\mathscr{C}' \subseteq \mathscr{C}$
- But neither $\mathscr{C}$ nor $\mathscr{C}'$ seems computable—cardinality of Q?
- Can show: $|Q| \le 2$; obtain the equivalent characterization of the capacity region

Theorem 4.3

The capacity region $\mathscr{C}$ is the set of (R_1, R_2) such that

$$\begin{aligned} R_1 &\le I(X_1; Y|X_2, Q), \\ R_2 &\le I(X_2; Y|X_1, Q), \\ R_1 + R_2 &\le I(X_1, X_2; Y|Q) \end{aligned}$$

for some pmf $p(q)p(x_1|q)p(x_2|q)$ with $|\mathcal{Q}| \le 2$

- Can we achieve the region in Theorem 4.3 directly?

Proof of achievability of the alternative characterization

- Key idea: Coded time sharing (Han–Kobayashi 1981)
- Codebook generation:
  - Fix $p(q)p(x_1|q)p(x_2|q)$
  - Randomly generate a time-sharing sequence $q^n \sim \prod_{i=1}^n p_Q(q_i)$
  - Conditionally independently generate $x_1^n(m_1) \sim \prod_{i=1}^n p_{X_1|Q}(x_{1i}|q_i)$, $m_1 \in [1:2^{nR_1}]$
  - Conditionally independently generate $x_2^n(m_2) \sim \prod_{i=1}^n p_{X_2|Q}(x_{2i}|q_i)$, $m_2 \in [1:2^{nR_2}]$
- Encoding:
  - To send message m_1, encoder 1 transmits $x_1^n(m_1)$
  - To send message m_2, encoder 2 transmits $x_2^n(m_2)$
- Decoding:
  - Find the unique $(\hat{m}_1, \hat{m}_2)$ such that $(q^n, x_1^n(\hat{m}_1), x_2^n(\hat{m}_2), y^n) \in \mathcal{T}_\epsilon^{(n)}$

Analysis of the probability of error

- Consider $\mathsf{P}(\mathcal{E})$ conditioned on $(M_1, M_2) = (1, 1)$

m_1	m_2	Joint pmf
1	1	$p(q^n)p(x_1^n\|q^n)p(x_2^n\|q^n)p(y^n\|x_1^n, x_2^n)$
$*$	1	$p(q^n)p(x_1^n\|q^n)p(x_2^n\|q^n)p(y^n\|x_2^n, q^n)$
1	$*$	$p(q^n)p(x_1^n\|q^n)p(x_2^n\|q^n)p(y^n\|x_1^n, q^n)$
$*$	$*$	$p(q^n)p(x_1^n\|q^n)p(x_2^n\|q^n)p(y^n\|q^n)$

- Error events:

$$
\begin{aligned}
\mathcal{E}_1 &= \{(Q^n, X_1^n(1), X_2^n(1), Y^n) \notin \mathcal{T}_\epsilon^{(n)}\},\\
\mathcal{E}_2 &= \{(Q^n, X_1^n(m_1), X_2^n(1), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1\},\\
\mathcal{E}_3 &= \{(Q^n, X_1^n(1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_2 \neq 1\},\\
\mathcal{E}_4 &= \{(Q^n, X_1^n(m_1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1, m_2 \neq 1\}
\end{aligned}
$$

Packing lemma

- Let $(U, X, Y) \sim p(u, x, y)$
- Let $(\tilde{U}^n, \tilde{Y}^n) \sim p(\tilde{u}^n, \tilde{y}^n)$ be arbitrarily distributed
- Let $X^n(m) \sim \prod_{i=1}^n p_{X|U}(x_i|\tilde{u}_i)$, $m \in \mathcal{A}$, $|\mathcal{A}| \le 2^{nR}$,

  be pairwise conditionally independent of $\tilde{Y}^n$ given $\tilde{U}^n$

Packing lemma

There exists $\delta(\epsilon) \to 0$ as $\epsilon \to 0$ such that

$$\lim_{n\to\infty} \mathsf{P}\{(\tilde{U}^n, X^n(m), \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m \in \mathcal{A}\} = 0,$$

if $R < I(X; Y|U) - \delta(\epsilon)$

- Generalizes "little" packing lemma in two says:
  - $\tilde{Y}^n$ conditionally independent of $X^n(m)$ given $\tilde{U}^n$
  - $(\tilde{U}^n, \tilde{Y}^n)$ can have an arbitrary pmf

Analysis of the probability of error

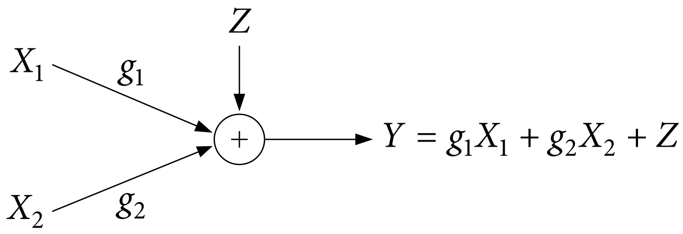
- Error events:

$$\mathcal{E}_1 = \{(Q^n, X_1^n(1), X_2^n(1), Y^n) \notin \mathcal{T}_\epsilon^{(n)}\},$$
$$\mathcal{E}_2 = \{(Q^n, X_1^n(m_1), X_2^n(1), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \ne 1\},$$
$$\mathcal{E}_3 = \{(Q^n, X_1^n(1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_2 \ne 1\},$$
$$\mathcal{E}_4 = \{(Q^n, X_1^n(m_1), X_2^n(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \ne 1, m_2 \ne 1\}$$

- By the LLN, $\mathsf{P}(\mathcal{E}_1) \to 0$
- By the packing lemma,

$$\mathsf{P}(\mathcal{E}_2) \to 0 \text{ if } R_1 < I(X_1; Y|X_2, Q) - \delta(\epsilon),$$
$$\mathsf{P}(\mathcal{E}_3) \to 0 \text{ if } R_2 < I(X_2; Y|X_1, Q) - \delta(\epsilon),$$
$$\mathsf{P}(\mathcal{E}_4) \to 0 \text{ if } R_1 + R_2 < I(X_1, X_2; Y|Q) - \delta(\epsilon)$$

Gaussian multiple access channel



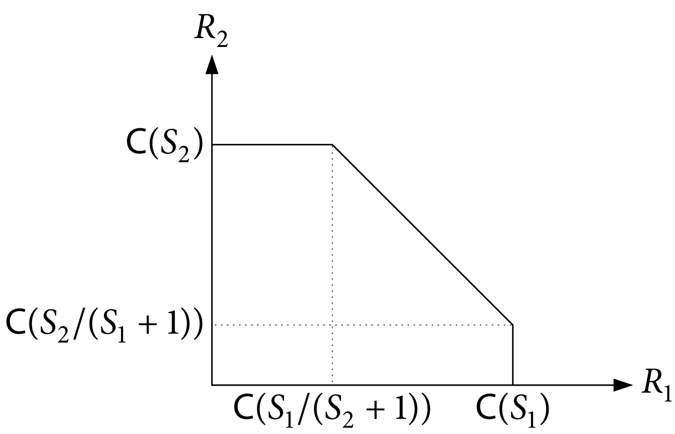
- g_1, g_2: channel gains
- $Z \sim \mathrm{N}(0, 1)$ independent of the messages
- Average power constraints: $\sum_{i=1}^n x_{ji}^2(m_j) \le nP$, $m_j \in [1:2^{nR_j}]$, $j = 1, 2$
- Define received powers (SNRs) as $S_j = g_j^2 P$, $j = 1, 2$

Capacity region (Cover 1975, Wyner 1974)

Theorem 4.4

The capacity region of the Gaussian MAC is the set of (R_1, R_2) such that

$$\begin{aligned} R_1 &\le \mathsf{C}(S_1), \\ R_2 &\le \mathsf{C}(S_2), \\ R_1 + R_2 &\le \mathsf{C}(S_1 + S_2) \end{aligned}$$



Proof sketch

- Let $X_1 \sim \mathrm{N}(0, P)$ and $X_2 \sim \mathrm{N}(0, P)$ be independent
- Then $\mathscr{R}(X_1, X_2)$ is the set of (R_1, R_2) such that

$$R_1 \le \mathsf{C}(S_1),$$
$$R_2 \le \mathsf{C}(S_2),$$
$$R_1 + R_2 \le \mathsf{C}(S_1 + S_2)$$

- Thus, $\mathscr{C} = \mathscr{R}(X_1, X_2)$ and no time sharing is needed
- Proof of achievability:
  - Achievability for the DM-MAC with input costs (see NIT Problem 4.8)
  - Discretization procedure (NIT 3.4.1)
- Proof of the converse: By the maximum differential entropy lemma,

$$h(Y|X_2, Q) \le (1/2)\log(2\pi e(S_1 + 1)),$$
$$h(Y|X_1, Q) \le (1/2)\log(2\pi e(S_2 + 1)),$$
$$h(Y|Q) \le (1/2)\log(2\pi e(S_1 + S_2 + 1))$$

Comparison to point-to-point coding schemes

- Use point-to-point Gaussian codes
- Treating other codeword as noise:

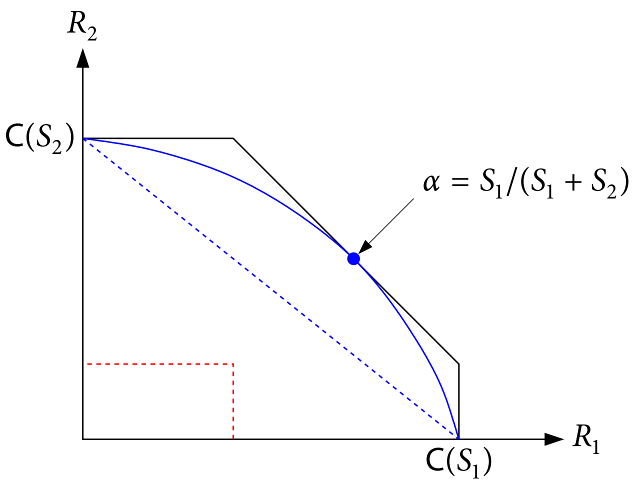
$$R_1 < \mathsf{C}(S_1/(S_2 + 1)),$$
$$R_2 < \mathsf{C}(S_2/(S_1 + 1))$$

- Time division: (R_1, R_2) such that

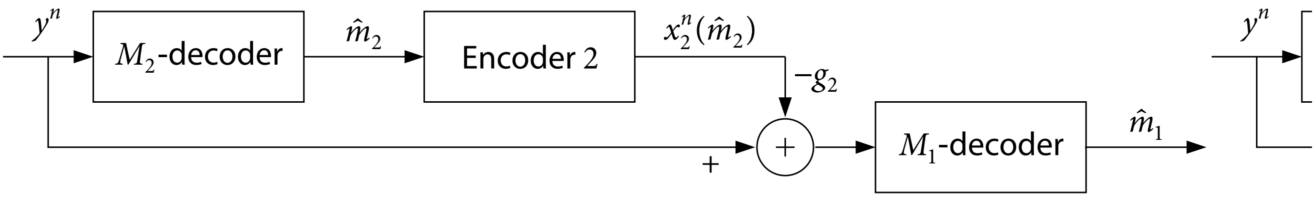
$$R_1 < \alpha\, \mathsf{C}(S_1),$$
$$R_2 < \bar{\alpha}\, \mathsf{C}(S_2) \quad \text{for some } \alpha \in [0, 1]$$

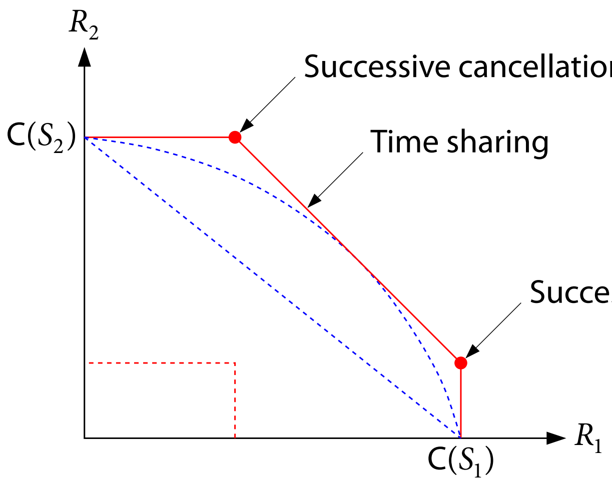
- Time division with power control:

$$R_1 < \alpha\, \mathsf{C}(S_1/\alpha),$$
$$R_2 < \bar{\alpha}\, \mathsf{C}(S_2/\bar{\alpha}) \quad \text{for some } \alpha \in [0, 1]$$



Successive cancellation decoding





Extension to more than two senders

Theorem 4.5

The capacity region of the k-sender DM-MAC is the set of $(R_1, \ldots, R_k)$ such that

$$\sum_{j \in \mathcal{J}} R_j \le I(X(\mathcal{J}); Y | X(\mathcal{J}^c), Q) \quad \text{for every } \mathcal{J} \subseteq [1:k]$$

for some pmf $p(q) \prod_{j=1}^{k} p(x_j|q)$ with $|\mathcal{Q}| \le k$

- For $k = 3$, the capacity region is the set of (R_1, R_2, R_3) such that

$$\begin{aligned} R_1 &\le I(X_1; Y | X_2, X_3, Q), \\ R_2 &\le I(X_2; Y | X_1, X_3, Q), \\ R_3 &\le I(X_3; Y | X_1, X_2, Q), \\ R_1 + R_2 &\le I(X_1, X_2; Y | X_3, Q), \\ R_1 + R_3 &\le I(X_1, X_3; Y | X_2, Q), \\ R_2 + R_3 &\le I(X_2, X_3; Y | X_1, Q), \\ R_1 + R_2 + R_3 &\le I(X_1, X_2, X_3; Y | Q) \end{aligned}$$

  for some $p(q)p(x_1|q)p(x_2|q)p(x_3|q)$

Extension to more than two senders

- The capacity region of the k-sender G-MAC is the set of $(R_1, \ldots, R_k)$ such that

$$\sum_{j \in \mathcal{J}} R_j \leq \mathsf{C}\Big(\sum_{j \in \mathcal{J}} S_j\Big) \quad \text{for every } \mathcal{J} \subseteq [1:k]$$

- For $k = 3$, the capacity region is the set of (R_1, R_2, R_3) such that

$$\begin{aligned} R_1 &\leq \mathsf{C}(S_1), \\ R_2 &\leq \mathsf{C}(S_2), \\ R_3 &\leq \mathsf{C}(S_3), \\ R_1 + R_2 &\leq \mathsf{C}(S_1 + S_2), \\ R_1 + R_3 &\leq \mathsf{C}(S_1 + S_3), \\ R_2 + R_3 &\leq \mathsf{C}(S_2 + S_3), \\ R_1 + R_2 + R_3 &\leq \mathsf{C}(S_1 + S_2 + S_3) \end{aligned}$$

- When $S_1 = S_2 = \cdots = S_k$, the sum-capacity $C_{\text{sum}} = O(\log k)$

Summary

- Discrete memoryless multiple access channel (DM-MAC)
- Capacity region
- Successive cancellation decoding
- Simultaneous decoding is more powerful than successive cancellation
- Time-sharing random variable
- Coded time sharing is more powerful than time sharing
- Packing lemma
- Gaussian multiple access channel
  - Time division with power control achieves the sum-capacity
  - Capacity region achieved via ptp codes, successive cancellation decoding, time sharing

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