# Lecture #1 Introduction

(Reading: NIT Preface, 1.1–1.4)

- Network information flow
- Graphical unicast networks
- Point-to-point information theory
- Network information theory
- Course overview

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### Networked information processing system



- System: Internet, peer-to-peer network, sensor network, ...
- Sources: data, video, sensor measurement, biochemical signals, ...
- Nodes: computers, handsets, sensor nodes, neurons, ...
- Network: wired, wireless, or hybrid of the two
- Task: Communicate sources or make decision based on them

# Network information flow questions



- What is the limit on the amount of communication needed?
- What are the coding schemes/techniques that achieve this limit?

### Graphical unicast networks



- Model for wired networks (Internet, distributed storage, ...)
- Directed weighted graph  $(\mathcal{N}, \mathcal{E})$  with link capacities  $C_{jk}$
- Source node 1 wishes to send *R*-bit message *M* to destination node *N*
- What is the network capacity C (highest achievable R)?

Max-flow min-cut theorem (Ford–Fulkerson 1956)



• Network capacity:

$$C = \min_{\mathcal{S} \in \mathcal{N}: 1 \in \mathcal{S}, N \in \mathcal{S}^c} C(\mathcal{S})$$

where  $C(S) = \sum_{j \in S, k \in S^c} C_{jk}$  is capacity of the cut  $(S, S^c)$ 

- Achieved error-free using simple forwarding (routing)
- Information treated as commodity flow

### Example



- *C* = 3
- Minimum cut:  $S = \{1, 2, 3, 5\}$
- Achieved by routing 1 bit along  $1 \rightarrow 2 \rightarrow 4 \rightarrow 6$  and 2 bits along  $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$

## Point-to-point information theory (Shannon 1948, 1959)



- Mathematical model for a communication system
- Probabilistic discrete memoryless models for source p(u) and channel p(y|x)
- Block coding scheme
- Asymptotic approach to analyzing performance
- Four fundamental theorems in terms of entropy and mutual information
  - Lossless source coding:  $R^* = H(U)$
  - Channel coding:  $C = \max_{p(x)} I(X; Y)$
  - ► Lossy source coding:  $R(D) = \min_{p(\hat{u}|u): E(d(U,\hat{U})) \le D} I(U; \hat{U})$
  - Source-channel separation

## Network information theory

- Multiple sources and destinations
- Function computation or collaborative decision making
- Wireless is shared broadcast medium
- Feedback and interactive communication
- Network security
- No source-channel separation
- Dynamic data arrival and network topology

# **Brief history**

• First paper (Shannon 1961): "two-way communication channels"



- Significant research activities in 70s and early 80s, but
  - Many basic problems open
  - Little interest from practice
- Wireless communications and the Internet revived interest in mid 90s
  - Some progress on old open problems and many new problems
  - Very large number of papers in ISIT, T-IT, T-COM, T-WC, …
  - Results starting to have impact on real-world networks

## About the course

- Provides broad coverage of key results, techniques, and open problems
- Emphasis is on coding schemes
- Proof techniques and some coding theorem proofs are presented in detail
- Other proofs are assigned as reading
- This course is intended for:
  - Students interested in research in network information theory
  - Students interested in research in communication (wireless, wired) and multimedia
  - Students generally interested in information theory

# About the course

- Text: El Gamal and Kim, Network Information Theory (NIT), Cambridge Univ. Press
- Course requirements:
  - 5 homework sets
  - ITA workshop report
  - Take-home midterm
  - Final projects: surveys of topics not covered, new research results, or new problems
- Make-up lectures on January 9, 16, 30, March 12 (Atkinson Hall 4010)

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# Basic information theory

• Lossless source coding (NIT 2.1, 2.4, 3.5):



Shannon's lossless source coding theorem (entropy, typicality)

• Channel coding (NIT 2.2, 2.3, 2.5, 3.1, 3.3, 3.4):



Shannon's channel coding theorem (mutual information, joint typicality) Random coding; joint typicality decoding; from DMC to Gaussian

## Single-hop networks

• Distributed lossless compression (NIT 10.1–10.3, 10.5):



Problem solved; Slepian–Wolf; Cover's random binning

• Multiple access channels (NIT 3.2, 4.1, 4.2, 4.4, 4.5):



Problem solved; successive cancellation; simultaneous decoding; time sharing

#### Single-hop networks

• Degraded broadcast channels (NIT 5.1–5.6):

$$(M_0, M_1, M_2) \longrightarrow X \longrightarrow p(y_1, y_2|x) \longrightarrow Y_1 \longrightarrow (\hat{M}_{01}, \hat{M}_1)$$
  
 
$$Y_2 \longrightarrow (\hat{M}_{02}, \hat{M}_2)$$

Problem solved; superposition coding; simultaneous nonunique decoding

• Interference channels (NIT 6.1–6.7):

Problem open; strong interference; Han-Kobayashi

# Single-hop networks

• Channels with state (NIT 3.7, 7.1, 7.2, 7.4–7.7):



Shannon strategy; Gelfand–Pinsker; multicoding; writing on dirty paper

• General broadcast channels (NIT 8.1–8.5, 9.5, 9.6):

$$(M_0, M_1, M_2) \longrightarrow X \longrightarrow p(y_1, y_2|x) \longrightarrow Y_1 \longrightarrow (\hat{M}_{01}, \hat{M}_1) \\ \searrow Y_2 \longrightarrow (\hat{M}_{02}, \hat{M}_2)$$

Problem open; degraded message sets; Marton coding; mutual covering; Gaussian vector (MIMO) broadcast channel; vector writing on dirty paper

## Multihop networks

• Graphical networks (NIT 15.1–15.3):



Max-flow min-cut; network coding

# Multihop networks

• Relay channels (NIT 16.1–16.7):



Problem open; cutset bound; decode-forward; compress-forward

## Multihop networks

• Interactive communication (NIT 17.1–17.4):



Iterative refinement; feedback can increase capacity of multiuser channels

# Multihop networks

• Discrete memoryless and Gaussian networks (NIT 18.1–18.3, 19.1):



Cutset outer bound; network decode-forward; noisy network coding

### Joint source-channel coding

• Joint source-channel coding (NIT 3.9):



Shannon's source-channel separation theorem

• Correlated sources over a MAC (NIT 14.1):



Separation theorem does not hold in general for networks; common part

## **Course topics**

- Not all material in sections listed is covered
- Lecture slides and reading assignments will clarify what is covered
- Topics not covered in the lectures:
  - Distributed lossy compression (NIT 3.6–8, 11, 12)
  - Multiple description coding (NIT 13)
  - Compression over graphical networks (NIT 20)
  - Communication for computing (NIT 21)
  - Information theoretic secrecy (NIT 22)
  - Wireless fading channels (NIT 23)
  - Networking and information theory (NIT 24)

#### Tentative course schedule

- Jan 5 Introduction; Review of IT
- Jan 7 Review of IT; HW1
- $(Jan 9)^*$  Review of IT
- Jan 12 Distributed source coding
- Jan 14 Multiple access channels; HW1 due; HW2
- (Jan 16) Multiple access channels
- Jan 21 Degraded broadcast channels; project guidelines; HW2 due; HW3
- Jan 26 Degraded broadcast channels; interference channels
- Jan 28 Interference channels; HW3 due; HW4
- (Jan 30) Channels with states

\* Make-up lectures: Time/location TBD

# Tentative course schedule (contd.)

Feb 2-6 ITA workshop

- Feb 9 Channels with state; ITA assignment & final project proposal due
- Feb 11 General broadcast channel; HW4 due; HW5
- Feb 18 Graphical networks; HW5 due; take-home midterm
- Feb 23 Relay channels; midterm due
- Feb 25 Relay channels; discrete memoryless networks
- Mar 2 No lecture
- Mar 4 No lecture
- Mar 9 Discrete memoryless networks; Gaussian networks
- Mar 11 Joint source-channel coding
- (Mar 13) Final project presentations

#### References

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