PERFORMANCE TRADE-OFFS IN LARGE WIRELESS NETWORKS

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James Mammen
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I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Abbas El Gamal (Principal Adviser)

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Balaji Prabhakar

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Thomas M. Cover

Approved for the University Committee on Graduate Studies.

Preface

Recent advances in electronics and wireless communications have made it possible to incorporate wireless communication capability into various electronic devices such as laptops, PDAs, domestic appliances, etc. It has also enabled the development of tiny and inexpensive sensors that can sense physical quantities (such as temperature, moisture, or light) and have the capability for wireless communication. This has enabled the exciting possibility of forming large wireless networks of such devices for a variety of applications.

The wireless nature of communication creates interference among simultaneously communicating nodes in the network, resulting in performance degradation as the number of nodes in the network increases. Useful performance measures are data-rate, delay, and energy consumption. Performance can be improved if nodes cooperate by relaying data for each other, instead of each node directly transmitting to its desired destination. For example, in a crowded room, instead of shouting to communicate with another person at the other end of the room, one could whisper to a neighboring person, who would then whisper it to his or her neighbor, and so on, until the message reaches the intended person. Several simultaneous conversations could take place with this form of cooperation.

In order to develop algorithms with good performance in such networks, it is necessary to first understand the optimal or best possible performance. This thesis provides a mathematical study of the scaling of optimal performance with the number of nodes in the network. For this study, we develop models and prove scaling laws for performance metrics such as throughput (data communication rate), delay, and energy consumption, and the trade-offs between them. These scaling laws characterize optimal performance when the network has sufficiently many nodes and they provide guidelines for designing real-world networks. The techniques used to establish these results belong to several areas such as analysis of algorithms, probabilistic analysis, Markov chains, queuing theory, and random walks.

The results regarding optimal performance are proved by a two-pronged approach, which consists of showing achievability, and then proving its optimality, that is, a converse. The

part dealing with achievability consists of proposing a communication scheme and evaluating its performance. The converse consists of showing that no other scheme can perform better, thus establishing the optimality of the proposed scheme.

Our approach is to treat the wireless network as a packet data network. The network consists of nodes that can transmit and receive packets and have buffers for temporary storage (for later forwarding) of packets. Each transmitted packet is either successfully received or lost depending on the amount of interference at that time. This approach models the way in which current technology operates. Using this approach, we establish the fundamental trade-offs between throughput, delay, and energy consumption, both when the nodes are static and when they move around. Node mobility arises in several applications, for example, when automobiles or cellular phones or robots form a wireless network.

The deployment of a wireless network for any application would also require practical and efficient algorithms for a variety of tasks such as initialization, maintenance, information processing, data storage, and the like, in addition to data communication. Such algorithms are not within the scope of this work. However, our work does provide guidelines for the design of efficient routing and scheduling algorithms for data communication.

We believe that this thesis, along with work by other researchers in this area, enhances our understanding of large wireless networks, by revealing the effect of interference and the role of cooperation between nodes for combating it, so as to obtain optimal performance.

Acknowledgments

I am grateful to my adviser, Abbas El Gamal, for his guidance and support throughout the course of this work. His manifold interests, clarity of thought, and intensity of effort, have been a constant source of motivation. Following the work in this dissertation, I was fortunate to be able to work with him on other information theoretic problems related to channel capacity.

This work began as a class project with Devavrat Shah in a course (EE478) offered by Abbas El Gamal. It was my good fortune to learn first-hand from Devavrat about how to do theoretical research. His insights, immense knowledge, and friendship have been crucial to the development of this work.

I would like to thank Balaji Prabhakar for his advice and encouragement throughout my stay at Stanford. My learning experience has been significantly enriched by the opportunity to do research with Dave Donoho.

It has been my privilege to take many excellent courses, whose variety have been the spice of my student life. These include courses by Stephen Boyd, Persi Diaconis, Amir Dembo, Rajiv Motwani, Tom Cover, Abbas El Gamal, Balaji Prabhakar, Tim Roughgarden, George Papanicolaou, and Peter Glynn. Their passion for the subjects they taught and their skill at teaching have been a great inspiration to me.

I would like to thank members of the El Gamal research group – Sam, Hossein, Sina, Olivier, Helmy, Kunal, Khaled, Keith, Mingjie, and Ali, for the pleasure of their company. A little joint conspiracy with some of them will perhaps be instrumental in submitting this dissertation! I would also like to thank Arpita, Vivek, Young-Han, Lu Yi, Mohsen, Neda, Taesup, Daemon, Stephanie, and other friends in the Information Systems Laboratory, and Denise and Kelly, for making ISL a great place.

I owe a huge debt of gratitude to friends for the wonderful time I have had with them. Dua has been a patient ear and adviser in a variety of matters for the last two years as apartment-mate. Georgie and Sudeshna have sweetened my stay at Stanford by their friendship and ready help. Chandra introduced me to the pleasures of biking and inspired

me by his intensity of spirit. I feel fortunate to have enjoyed a steadfast friendship with Atul and Juhul, in spite of their being far away.

Special thanks are due to many authors and poets, who have filled in the vacuum in my experience and enterprise. A fortuitous purchase from a clearance sale at a local library provided me with countless hour of pleasure. And also weight-lifting exercise at each of my four moves during these five years. A large chunk of both is due to Will Durant's classic: The Story of Civilization.

My late father taught me to leave nothing unexamined. My mother's strength of character and discipline have been a source of faith and hope. I also realize that my parents and my sister shouldered many responsibilities while I was too far from home to share it with them. Their love and admiration have been my greatest nourishment. This dissertation is dedicated to them.

Contents

Chapter 1

Introduction

A wireless network consists of a collection of nodes, each capable of transmitting to or receiving from other nodes. When a node transmits to another node, it creates interference for other nodes in its vicinity. When several nodes transmit simultaneously, a receiver can successfully receive the data sent by the desired transmitter only if the interference from the other nodes is sufficiently small. An important characteristic of wireless networks is that the topology of the nodes may not be known. The wireless nodes could also be mobile, in which case the topology could be continuously changing.

The most familiar and pervasive wireless network today is the cellular network for communication via mobile phones. In the cellular network, the entire area of coverage is divided into cells with a base-station in each cell. When a source mobile phone communicates with another a destination mobile phone, the data is first sent from the sender to its nearest base-station. The data is then routed to the base-station in which the destination mobile phone is located through a base-station network. Finally this base-station deliver the data to the destination. The cellular network relies heavily on the infrastructure provided by the base-station network. Setting up and maintaining this infrastructure is expensive, but it makes the network reliable and increases ease of operation.

Technological developments in recent years have made it possible to incorporate wireless communication capability into several electronic devices such as laptops, PDAs, domestic appliances, and so on. It has also become possible to fabricate tiny and inexpensive sensors with wireless communication capability. Typical sensed quantities include temperature, humidity, pressure, light, position, speed, and motion. Such devices with capability for wireless communication could form a wireless network such as a network of laptops, a home network of domestic appliances, or a network of sensors. In many applications, it is desirable to set up and operate the network without any fixed infrastructure like that of the cellular

network. We shall refer to such networks as ad hoc wireless networks. Such ad hoc wireless networks have a variety of applications in areas such as health care, agriculture, forestry, construction engineering, industrial automation, traffic control, and surveillance [43], [2]. The importance of such networks is increasing rapidly with further advances in technology that result in smaller, cheaper, and energy-efficient devices.

Clearly there are many challenges in successfully deploying such networks [18], [5]. Due to lack of infrastructure it is essential that the network be able to self-organize and maintain itself since individual nodes may get disconnected from the network or new nodes may join. The purpose of most networks is not just communication but to perform a particular task. For example in a surveillance system the goal would be to detect intrusion and raise an alarm. This motivates collaborative signal processing among nodes for performing the required task. Many applications involve hundreds or thousands of nodes and hence scalability of algorithms is a major issue. In many applications the nodes are powered by batteries, which requires the algorithms used in the network to be energy-efficient. From the data communication perspective, there is a need for scalable and energy-efficient algorithms for routing and scheduling.

Data communication is a key aspect regardless of the particular application and our work studies only this aspect. As the complexity of wireless networks increases, there is a need to develop better understanding of the fundamental trade-offs that govern their behavior. How much does interference limit the throughput (bit-rate at which each source can communicate with its destination)? How much does cooperation between the users help combat such interference? How does mobility affect network performance? Attempting to answer such questions by studying instances of wireless networks is not likely to lead to answers applicable to most of them.

1.1 Random network framework

The groundbreaking work by Gupta and Kumar [23] set the stage for answering the above questions. They introduced the idea of using a random network model instead of specific instances to obtain results that apply to most large networks. They showed that for almost all large network realizations, the total throughput scales as the square root of the number of users.

In many network applications, communication delay and energy consumption, in addition to throughput, are key measures of network performance. This thesis extends the

Gupta-Kumar random network framework to include models for delay and energy consumption. Several other researchers have also been inspired by the work of Gupta and Kumar and this has led to significant research using random network models. In what follows, we review research in this area, including our work.

1.1.1 Throughput in static networks

In their seminal paper [23], Gupta and Kumar introduced a random network model for studying throughput scaling in a static wireless network, i.e., when the nodes do not move. They defined a random network to consist of n nodes distributed independently and uniformly on a unit disk. These n nodes are split into $n/2$ source-destination pairs at random. Each node can transmit at a constant rate provided that the interference at the receiver is sufficiently small. They showed that for almost all realizations of the random network, the maximum throughput per source-destination pair scales as $1/\sqrt{n \log n}$. Their result also showed that cooperation among users is essential to combating the adverse effects of interference. Gupta and Kumar later extended their work to networks in three dimensions [22].

In [29], Kulkarni and Viswanath studied throughput scaling in static networks and obtained results similar to that of [23] using an elementary deterministic approach.

In very recent work, Jelenkovic, Momcilovic, and Squillante [26] studied wireless networks with finite buffers at each node. They showed throughput scaling of $1/\sqrt{n \log n}$ can be achieved with small buffers at each node.

In [19], Franceschetti, Dousse, Tse, and Thiran showed using a slightly different model (with exponential attenuation of signal with distance) that throughput of order $1/\sqrt{n}$ can be achieved. Their result relies on arguments from percolation theory.

In another variation of the random network model, Gowaikar, Hochwald, and Hassibi [20] use a model in which the strength of the channel between any two nodes is chosen at random from some common distribution. They show that the throughput scaling can be very different depending on the choice of the common distribution.

We would like to note that several authors use the notion of transport capacity instead of throughput. Transport capacity is measured in bit-meters per second. It also takes into account the distance traveled by bits instead of just the bit-rate. Although they are not the same they are both equivalent in the sense that both quantities are determined by each other for the random network model. We use throughput as our performance metric throughout this work.

Information theoretic approaches

The initial approach by Gupta and Kumar [23] may be called network theoretic in the sense that the wireless network is modeled as a packet network where a packet is either received or lost depending on the level of interference. From the point of view of the physical layer of data communication, such an approach is clearly suboptimal since there is the possibility of employing sophisticated error-correction and multi-user communication schemes. This has inspired research regarding the information theoretic notion of capacity of such networks. Another attraction of this approach is its continuity with the elegant and well-established multi-user information theory [6]. However, this approach does not lend itself to studying delay. Further, due its generality, it does not provide practical schemes for achieving capacity. Nevertheless, it does provide a fundamental upper bound on throughput.

This approach was initiated by Gupta and Kumar in [24] by studying achievable regions. Xie and Kumar [45] studied several network scenarios and establish a dichotomy between the cases of relatively high and relatively low attenuation. In [46], Xie, Xue, and Kumar, studied the effect of fading in wireless networks for the case of high attenuation.

Jovicic, Viswanath, and Kulkarni [27] consider a model with fading (that is, the channel gains vary over time). Signal is assumed to attenuate as $(1+r)^{-\alpha}$ with distance r. For $\alpha > 5/2$, they obtain an upper bound of order $\log n/\sqrt{n}$ on throughput scaling. Note that this upper bound is loose only by a logarithmic factor from the network theoretic approach.

Leveque and Telatar [32] assumed signal attenuation of $r^{-\alpha}$ with distance r and obtained an upper bound of order $n^{-\beta}\log n$, where $\beta = \frac{1}{d} - \frac{1}{\alpha}$. For one-dimensional networks, Leveque and Preissman [30] obtained an upper bound that is loose only by a logarithmic factor from the network theoretic approach. In [31], Leveque, Telatar, and Tse, obtain an upper bound of order $(\log n)^3/\sqrt{n}$ for networks with a regular placement of nodes on a two-dimensional grid.

In conclusion, the information theoretic framework provides upper bounds on throughput that are off by logarithmic factors from the network theoretic framework, which vindicates the simplifying assumptions of the network theoretic approach.

Two related models are worth mentioning – many-to-one data gathering and relay networks. The case of many nodes sending data to a common destination arises in some applications. For related literature using the random network framework, see [17] and the references therein. Relay networks consist of a network with a single source-destination pair in which all other nodes can act as relays. See [45], [4] and the references therein for related literature.

1.1.2 Throughput in mobile networks

In [21], Grossglauser and Tse showed that by allowing the nodes to move, throughput scaling changes dramatically. They showed that if node motion is independent across nodes and has a uniform stationary distribution, a constant throughput scaling ($\Theta(1)$) per S-D pair is achievable. Later, Diggavi, Grossglauser, and Tse [10] showed that a constant throughput per S-D pair is feasible even with a more restricted mobility model.

1.1.3 Delay

In most networking applications, delay is also a key performance metric along with throughput. Further, throughput that can be obtained from a network at the cost of increase in delay may not be useful. In this context, the understanding of throughput-delay trade-off is key to achieving the quality of service required by the application. The first study to appear in literature, in this direction, was by Bansal and Liu [3]. They considered a random network model with both mobile and static nodes. The static nodes are split into sources and destinations, whereas the mobile nodes have no data of their own and can act as relays. The authors propose a routing algorithm that is almost optimal in terms of throughput and study its delay. The optimal throughput-delay trade-off for static networks was studied in [12] by El Gamal, Mammen, Shah, and Prabhakar. It was later generalized by the authors in [14]. Neely and Modiano [38] studied the throughput-delay trade-off in mobile networks using an i.i.d. mobility model. In this model, each node is equally likely to be in any part of the network at each time instant, independent of the past. Throughput-delay trade-off for mobile networks using a random walk mobility model was studied in [15] by El Gamal, Mammen, Prabhakar, and Shah. See [42] for a recent study of throughput-delay trade-offs with various mobility models. However, this study ignores queuing delay, which is a crucial component of packet delay, under the assumption that it does not affect the scaling behavior. In [36], Mammen and Shah showed that even with the mobility restriction as in [10] (mentioned above), delay scaling remains unchanged.

1.1.4 Energy

As mentioned earlier, energy consumption is a key concern in several applications since wireless nodes run on batteries. Dana and Hassibi [8] studied the energy efficiency of random wireless networks. The focus of their work was on energy efficiency without regard to throughput or delay and without considering interference and transceiver circuit energy. In [13], El Gamal, Mammen, Prabhakar, and Shah, conducted a preliminary study of the

trade-off between throughput, delay and transmission energy for random wireless networks with constant area. Both these works reached the conclusion that energy efficiency increases with hopping. In [11], El Gamal and Mammen used a model with variable network area that also takes into account the energy spent in the transceiver circuits and obtained the optimal trade-offs between throughput, delay, and energy.

Thus the random network framework has spawned a significant amount of research in the past few years. Among the several references mentioned above, our work consisting of [15], [14], [11], and [36], will be presented in detail in this thesis.

1.2 Contributions

This thesis extends the random network framework to study delay and energy, in addition to throughput, and the trade-offs between them and the role of mobility in large wireless networks. The following are the main contributions of this work:

1. We establish the optimal throughput-delay trade-off for static random networks.
2. We establish the optimal throughput-delay-energy trade-off for static random networks.
3. We show that both the above results hold for random networks with packets of constant size. Previous work, including [23], had implicitly assumed that the packet size could be made arbitrarily small.
4. We establish the throughput-delay trade-off for mobile networks using a random walk mobility model. An extreme point of the trade-off shows that constant throughput scaling is achievable and that the corresponding delay is of order $n \log n$.
5. We show that even with restricted mobility, constant throughput scaling is achievable at a delay of order $n \log n$.

In addition to the above, our work provides a unified framework for understanding the seemingly disparate results for static and mobile networks. Further, in the course of establishing the optimal throughput-delay trade-off, we provide simpler proofs of previous throughput results. Our results also have several implications for the architecture of wireless networks. These are mentioned in the concluding section of each of the following chapters.

1.3 Outline of the thesis

Chapter 2 studies throughput-delay trade-off for the static random network by assuming a fluid model, which means that the packet size is allowed to be arbitrarily small. This chapter is based on [15] and the proofs use elementary probabilistic analysis. Chapter 3 shows that the optimal trade-off remains unchanged even if the packets are constrained to have constant size. However, without the fluid assumption, a sophisticated scheduling algorithm is required to achieve the optimal trade-off. This chapter is based on [15] and relies on some classical results in queuing theory regarding product form equilibrium in networks. Chapter 4 develops models for taking energy consumption into account and establishes optimal throughput-delay-energy trade-offs. This chapter is based on [11] and the proofs use probabilistic analysis. Chapter 5 studies the throughput-delay trade-off for mobile random networks by assuming a random walk model for node mobility. This chapter is based on [15] and the proofs rely on analysis of algorithms, theory of Markov chains, queuing theory, and analysis of random walks. Chapter 6 deals with a random network on a sphere, in which nodes are restricted to move on great circles of the sphere. In spite of this restriction on mobility, we show that a constant throughput scaling can be achieved at the same delay scaling as for the model without any restriction of mobility, which is studied in Chapter 5. This chapter is based on [36] and the techniques used are similar to those in Chapter 5. Finally we conclude in Chapter 7 with some overall implications of our work.

The general structure of each of the following five chapters is as follows. We first introduce the needed models and definitions and then present the main results. These results are proved in the sections following it. Each chapter ends with a discussion about the implications of the results contained in the chapter.

Throughout this work, we use c_i to denote constants that do not depend on n.

Chapter 2

Throughput-Delay Trade-off in Static Networks: The Fluid Model

Delay is an important performance metric in several applications along with throughput. Hence the maximum throughput that can be obtained under a delay constraint is more useful than throughput itself for describing network capability. This leads to the notion of the optimal throughput-delay trade-off, which is studied in this chapter for wireless networks in which nodes do not move. In this chapter, we use a fluid model, which means that the packet size is allowed to be arbitrarily small.

2.1 Model, definitions, and main result

We begin by reminding the reader of the order notation: (i) $f(n) = O(g(n))$ means that there exists a constant c and integer N such that $f(n) \leq cg(n)$ for $n > N$. (ii) $f(n) = o(g(n))$ means that $\lim_{n\to\infty} f(n)/g(n) = 0$. (iii) $f(n) = \Omega(g(n))$ means that $g(n) = O(f(n))$, (iv) $f(n) = \omega(g(n))$ means that $g(n) = o(f(n))$. (v) $f(n) = \Theta(f(n))$ means that $f(n) = O(g(n));\ g(n) = O(f(n))$.

We now present the static random network model and the model for successful wireless transmission and then provide definitions of throughput, delay, and their trade-off.

Definition 1 (Random network model). The random network consists of n nodes distributed uniformly at random in a unit torus. The nodes are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for packetized transmission. For simplicity, we assume that the time-slots are of unit length. In a *static network* nodes do not move.

Definition 2 (Relaxed Protocol Model). Under the Relaxed Protocol model, a transmission from node i to node j in a time-slot is successful, if for any other node k that is transmitting simultaneously,

$$d(k,j) \geq (1+\Delta)d(i,j) \quad \text{for} \quad \Delta > 0,$$

where $d(i,j)$ is the distance between nodes i and j. During a successful transmission, nodes send data at a constant rate of W bits per second.

Our model for successful transmission is very similar to the Protocol model defined in [23]. The Protocol model imposes the additional constraint that all nodes have the same transmission range. Our results would also hold with the Protocol model, however, we use the relaxed version above since the additional constraint is not required for our results.

In the other commonly used model of successful transmission, namely the Physical model (see Chapter 4), a transmission is successful if the signal to interference and noise ratio (SINR) is greater than some constant. It is well known [23] that if signal decays with distance r as $r^{-\delta}$ for $\delta > 1$, the Protocol model is equivalent to the Physical model, where each transmitter uses the same power. In the rest of this chapter, we shall assume the Relaxed Protocol model.

To establish our results without being encumbered by issues related to scheduling packets in the network, we allow the packet size to be arbitrarily small. We refer to this as the *fluid model*. In this model, the data sent in a time-slot could correspond to multiple packets. Thus the time taken for a packet transmission may only be a small fraction of the time-slot itself. However, a packet received by a node in some time-slot cannot be transmitted by the node until the next time-slot. The scheduling problem is avoided by using the fluid model so that the packet size to be small enough depending on the number of nodes in the network.

Definition 3 (Scheme). A scheme Π for a random network is a sequence of communication policies, $\{\Pi_n\}$, where policy Π_n determines how communication takes place in a network of n nodes.

Definition 4 (Throughput of a scheme). Let $B_{\Pi_n}(i,t)$ be the number of bits of S-D pair i transferred in t time-slots under policy Π_n, for $1 \leq i \leq n/2$,. Note that this could be a random quantity for a given realization of the network. Scheme Π is said to have throughput $T_\Pi(n)$ if $\exists$ a sequence of events $A_\Pi(n)$ such that

$$A_\Pi(n) = \left\{ \min_{1\leq i\leq n/2} \liminf_{t\to\infty} \frac{1}{t} B_{\Pi_n}(i,t) \geq T_\Pi(n) \right\},$$

and $P\left(A_{\Pi}(n)\right) \to 1$ as $n \to \infty$.

We allow randomness in policies and as a result the set $A_{\Pi}(n)$ above is in the joint probability space including both the random network of size n and the policy. Randomness will be used in our schemes for mobile networks in later chapters. We say that an event A_n occurs with high probability (*whp*) if $P\left(A_n\right) \to 1$ as $n \to \infty$.

Definition 5 (Delay of a scheme). The delay of a packet is the time it takes for the packet to reach its destination after it leaves the source. Let $D^i_{\Pi_n}(j)$ denote the delay of packet j of S-D pair i under policy Π_n, then the sample mean of delay (over packets that reach their destinations) for S-D pair i is

$$\bar{D}^i_{\Pi_n} = \limsup_{k \to \infty} \frac{1}{k} \sum_{j=1}^{k} D^i_{\Pi_n}(j).$$

The average delay over all S-D pairs for a particular realization of the random network is then

$$\bar{D}_{\Pi_n} = \frac{2}{n} \sum_{i=1}^{n/2} \bar{D}^i_{\Pi_n}.$$

The delay for a scheme Π is the expectation of the average delay over all S-D pairs, i.e.,

$$D_{\Pi}(n) = E[\bar{D}_{\Pi_n}] = \frac{2}{n} \sum_{i=1}^{n/2} E[\bar{D}^i_{\Pi_n}].$$

Note that since the fluid model allows us to scale down the size of the packets, packet delay, as defined above, is not equivalent to the delay per bit. To measure delay per bit one would need to keep the packet size constant; but then this would require scheduling in the network. In Chapter 3, we propose a scheduling scheme to deal with constant size packets.

Definition 6 (Throughput-delay optimality). A pair $(T(n), D(n))$ is said to be T-D (throughput-delay) optimal if there exists a scheme Π with $T_{\Pi}(n) = \Theta(T(n))$ and $D_{\Pi}(n) = \Theta(D(n))$ and $\forall$ scheme Π' with $T_{\Pi'}(n) = \Omega(T(n))$, $D(\Pi')(n) = \Omega(D(n))$.

Definition 7 (Optimal throughput-delay trade-off). The optimal throughput-delay trade-off consists of all optimal T-D pairs.

Although we have introduced detailed notation in order to unambiguously define the above quantities, in the rest of this work we shall avoid the use of subscripts to indicate the scheme and policy since the scheme or policy under consideration will be clear from the

context. Also when describing a scheme Π, we shall just describe the policy Π_n for arbitrary n.

Main result:

In Section 2.2, we introduce a cellular scheme (Scheme 1) for static networks, which can trade-off throughput for delay by varying the cell size. We then prove that the scheme is optimal leading to the following result.

Theorem 2.1. *The optimal throughput-delay trade-off in a static random network is given by*

$$T(n) = \Theta\left(\frac{D(n)}{n}\right), \tag{2.1}$$

when $T(n) = O\left(1/\sqrt{n \log n}\right)$.

The above result, which is illustrated in Figure 2.1, says that: (i) The highest throughput per node achievable in a static network is $\Theta\left(1/\sqrt{n \log n}\right)$, as Gupta and Kumar obtained. At this throughput the average delay $D(n) = \Theta\left(\sqrt{n/\log n}\right)$ (point Q in Figure 2.1). (ii) By increasing the cell size and hence the transmission radius, the average number of hops can be reduced. But because the interference is higher now, the throughput is lower. When throughput is $O\left(1/\sqrt{n \log n}\right)$, equation (2.1) shows the optimal delay-constrained throughput (segment PQ in Figure 2.1).

2.2 Throughput-delay trade-off

This section establishes the optimal throughput-delay trade-off in a static wireless network by providing a proof of Theorem 2.1. We first present Scheme 1 and compute the trade-off achievable using this scheme in Theorem 2.2. Theorem 2.3 provides a converse, which states that for a given delay scaling no scheme can provide a better throughput scaling than that of Scheme 1, thus establishing the optimality of Scheme 1 and also proving Theorem 2.1.

2.2.1 Achievability

Our trade-off scheme is a multi-hop, time-division-multiplexed (TDM), cellular scheme with square cells of area $a(n)$ so that the unit torus consists of $1/a(n)$ cells as shown in Figure 2.2. In the following analysis, we ignore the edge effects due to $1/a(n)$ not being a perfect square. Before presenting the trade-off scheme, we present three lemmas about the geometry of the n nodes on the torus divided into square cells of area $a(n)$.

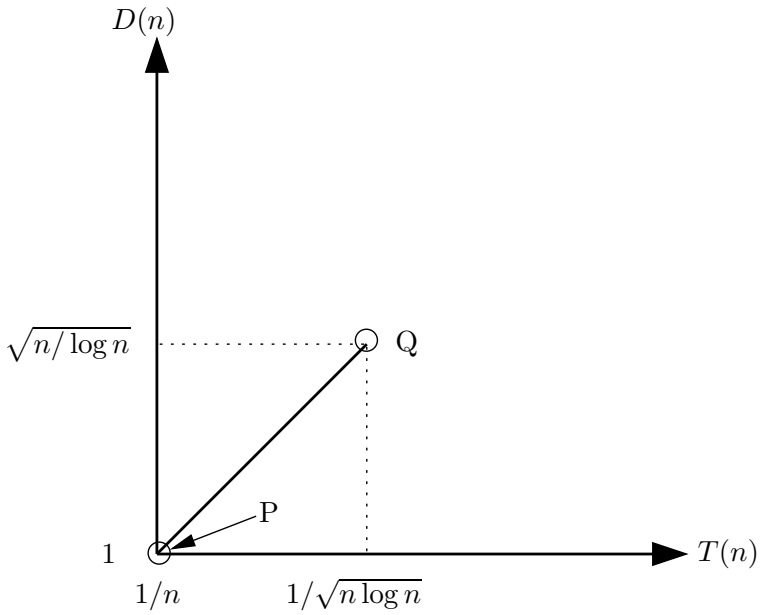

Figure 2.1: Throughput-delay trade-off for the static random network. The scales of the axes are in terms of the orders in n.

Lemma 2.1. *If $a(n) \geq 2\log n/n$, then each cell has at least one node* whp.

Proof. Let A_i be the event that cell i is not empty and let $m = 1/a(n)$ be the number of cells. Then

$$P(A_i) = 1 - (1 - 1/m)^n \to 1 - e^{-n/m}.$$

With $m \leq n/2\log n$, it follows that $P(A_i) \geq 1 - 1/n^2$ and hence an application of the union bound completes the proof. □

We say that cell B *interferes* with another cell A if a transmission by a node in cell B can affect the success of a simultaneous transmission by a node in cell A.

Lemma 2.2. *Under the Relaxed Protocol model, the number of cells that interfere with any given cell is bounded above by a constant c_1, independent of n.*

Proof. Consider a node in a cell transmitting to another node within the same cell or in one of its 8 neighboring cells. Since each cell has area $a(n)$, the distance between the transmitting and receiving nodes cannot be more than $r = \sqrt{8a(n)}$. Under the Relaxed Protocol model, data is successfully received if no node within distance $\bar{r} = (1 + \Delta)r$ of the receiver transmits at the same time. Therefore, the number of interfering cells, c_1, is at

most

$$c_1 \leq 2\frac{\bar{r}^2}{a(n)} = 16(1+\Delta)^2,$$

which, for a constant Δ, is a constant, independent of n (and $a(n)$). □

A consequence of Lemma 2.2 is that there exists an interference-free schedule such that each cell becomes active regularly once in $1 + c_1$ time-slots and it does not interfere with any other simultaneously transmitting cell.

We say that a cell is *active* in a time-slot if any of its nodes transmits in that time-slot. A consequence of Lemma 2.2 is that, there exists an interference-free schedule where each cell becomes active regularly, once in $1 + c_1$ time-slots and no cell interferes with any other simultaneously transmitting cell.

Let the straight line connecting a source S to its destination D be called an *S-D line*.

Lemma 2.3. *For $a(n) = \Omega(\log n/n)$, the number of S-D lines passing through each cell is $O\left(n\sqrt{a(n)}\right)$,* whp.

Proof. Let H_i be the number of hops taken by a packet for S-D pair i, $1 \leq i \leq n/2$ in traveling from S to D along the S-D line by hops along adjacent cells of area $a(n)$. For each S-D pair, H_i depends on the distance L_i between S and D and also the orientation of the S-D line. Now $E[L_i]$ is a constant and hence since the hops are along cells having side-length $1/\sqrt{a(n)}$, it can be shown that

$$E[H_i] = \Theta\left(E[L_i]/\sqrt{a(n)}\right). \tag{2.2}$$

There are $m = 1/a(n)$ cells. Fix a cell j and define Y_i^j to be the indicator of the event that the S-D line of S-D pair i passes through cell j. That is,

$$Y_i^j = \begin{cases} 1 & \text{if any hop of S-D pair } i \text{ is in cell } j \\ 0 & \text{otherwise} \end{cases}$$

for $1 \leq i \leq n/2$ and $1 \leq j \leq m$. Summing up the total number of hops in the cell in two different ways we obtain

$$\sum_{i=1}^{n/2}\sum_{j=1}^{m} Y_i^j = \sum_{i=1}^{n/2} H_i.$$

Taking expectations on both sides and noting that all the $E[Y_i^j]$ are equal due to symmetry on the torus, we obtain

$$\frac{nm}{2}E[Y_i^j] = \frac{n}{2}E[H_i]. \tag{2.3}$$

From (2.2) and (2.3) it follows that $P\{Y_i^j = 1\} = \Theta\left(\sqrt{a(n)}\right)$. Now for a fixed cell j, the total number of S-D lines passing through it is given by $Y = \sum_{i=1}^{n/2} Y_i^j$. This is the sum of i.i.d. Bernoulli random variables since the position of each node is independent of that of the others and Y_i^j depends only on the positions of the source and destination nodes of S-D pair i. Moreover $E[Y] = \Theta\left(n\sqrt{a(n)}\right)$, which is $\Omega\left(\sqrt{n \log n}\right)$ since $a(n) = \Omega(\log n/n)$ and hence the Chernoff bound for the sum of i.i.d. Bernoulli random variables (e.g. see [37]) yields

$$P\{Y > (1+\delta)E[Y]\} \leq \exp(-E[Y]\delta^2/4).$$

Choosing $\delta = 2\sqrt{2 \log n/E[Y]}$ results in

$$P\{Y > (1+\delta)E[Y]\} \leq 1/n^2.$$

Since $\delta = o(1)$, for $a(n) = \Omega(\log n/n)$ this means that $Y = O(E[Y])$ with probability $\geq 1 - 1/n^2$. Now using the union bound over $m = O(n/\log n)$ cells shows that the number of lines passing through each cell is $O(E[Y]) = O\left(n\sqrt{a(n)}\right)$ with probability $\geq 1 - 1/n$. □

The above lemma shows that the number of S-D lines passing through each cell is $\leq c_2 n\sqrt{a(n)}$ *whp*, for an appropriate choice of the constant c_2.

Now we are ready to describe Scheme 1, which is parameterized by the cell area $a(n)$, where $a(n) = \Omega(\log n/n)$ and $a(n) \leq 1$.

Scheme 1: Static networks

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1. Divide the unit torus using a square grid into square cells, each of area $a(n)$ (see Figure 2.2).

2. Verify whether the following conditions are satisfied for the given realization of the random network.

   - Condition 1: No cell is empty.
   - Condition 2: The number of S-D lines through each cell is at most $c_2 n\sqrt{a(n)}$.

3. If either of the above conditions is not satisfied then use a time-division policy, where each of the $n/2$ sources transmits directly to its destination in a round-robin fashion.

4. Otherwise, i.e., if both conditions are satisfied, use the following policy Σ_n:

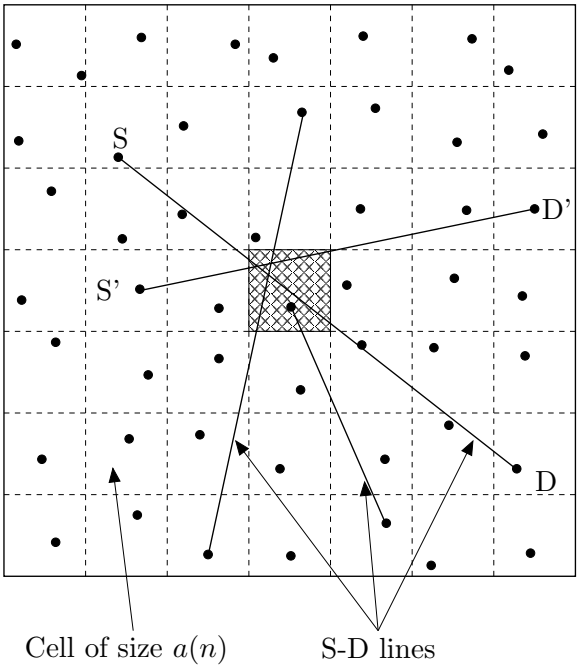


Figure 2.2: The unit torus is divided into cells of size $a(n)$ for Scheme 1. The S-D lines passing through the shaded cell in the center are shown.

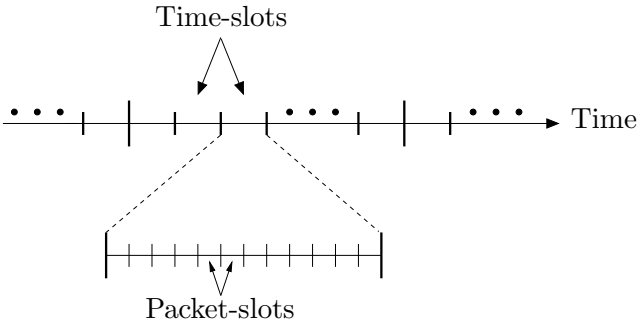


Figure 2.3: The TDM transmission schedule of Scheme 1. Each cell becomes active once in $1 + c_1$ time-slots and each active time-slot is divided into several packet-slots.

(a) Each cell becomes active at a regular interval of $1 + c_1$ time-slots (the constant c_1 comes from Lemma 2.2). Several cells which are sufficiently far apart become active simultaneously. Thus the scheme uses TDM between nearby cells.

(b) Let the straight line connecting a source S to its destination D be denoted as an S-D line. A source S transmits data to its destination D by hops along the adjacent cells lying on its S-D line as shown in Figure 2.2.

(c) When a cell becomes active, it transmits a single packet for each of the S-D lines passing through it. This is again performed using a TDM scheme that slots each cell time-slot into *packet time-slots* as shown in Figure 2.3.

The point of trade-off at which Scheme 1 operates is determined by the parameter $a(n)$ and the dependence is made precise in the following theorem.

Theorem 2.2. *For* $a(n) \geq 2\log n/n$,

$$T(n) = \Theta\left(\frac{1}{n\sqrt{a(n)}}\right) \quad \textit{and} \quad D(n) = \Theta\left(\frac{1}{\sqrt{a(n)}}\right),$$

i.e., the throughput-delay trade-off achieved by Scheme 1 is

$$T(n) = \Theta\left(\frac{D(n)}{n}\right) \quad \textit{for} \quad T(n) = O\left(\frac{1}{\sqrt{n\log n}}\right).$$

Proof. If the time-division policy with direct transmission is used, then the throughput is $2W/n$ and the delay is 1. But since it happens with a vanishingly low probability, as shown by Lemmas 2.1 and 2.3, the throughput and delay for Scheme 1 are determined by that of policy Σ_n.

First we analyze the throughput of Scheme 1. When policy Σ_n is used, since Condition 1 is satisfied, each cell has at least one node. This guarantees that each source can send data to its destination by hops along adjacent cells on its S-D line. From Lemma 2.2, it follows that each cell gets to transmit a packet every $1 + c_1$ time-slots, or equivalently, the cell throughput is $\Theta(1)$. The total traffic through each cell is that due to all the S-D lines passing through the cell, which is $O\left(n\sqrt{a(n)}\right)$ since Condition 2 is also satisfied. This shows that

$$T(n) = \Theta\left(1/\left(n\sqrt{a(n)}\right)\right).$$

Next we compute the average packet delay $D(n)$. As defined earlier, packet delay is the sum of the amount of time spent in each hop. We first bound the average number of hops then use the fact that the time spent at each hop is constant, independent of n.

Since each hop covers a distance of $\Theta\left(\sqrt{a(n)}\right)$, the number of hops per packet for S-D pair i is $\Theta\left(d_i/\sqrt{a(n)}\right)$, where d_i is the length of S-D line i. Thus the number of hops taken by a packet averaged over all S-D pairs is $\Theta\left(\frac{1}{n}\sum_{i=1}^{n} d_i/\sqrt{a(n)}\right)$. Since for large n, the average distance between S-D pairs is $\frac{1}{n}\sum_{i=1}^{n} d_i = \Theta(1)$, the average number of hops is $\Theta\left(1/\sqrt{a(n)}\right)$.

Now note that by Lemma 2.2 each cell can be active once every constant number of cell time-slots and by Lemma 2.3 each S-D line passing through a cell can have its own packet time-slot within that cell's time-slot. Since we assumed that packet size scales in proportion to the throughput $T(n)$, each packet arriving at a node in the cell departs in the next active time-slot of the cell. Thus the delay is at most c_1 times the number of hops. From the above discussion, we conclude that the delay $D(n) = \Theta\left(1/\sqrt{a(n)}\right)$. This concludes the proof of Theorem 2.2. □

2.2.2 Converse

Next we show that the throughput-delay trade-off provided by Scheme 1 is optimal for a static wireless network as far as the scaling is concerned.

Theorem 2.3. *If any scheme has throughput, $T(n)$, and delay, $D(n)$, then $D(n) = \Omega(nT(n))$.*

Proof. This proof uses techniques similar to those used in the proof of Theorem 2.1 in [23]. Consider a given fixed placement of $2n$ nodes in the unit torus. Let $\bar{L}$ be the sample mean of the lengths of the S-D lines for the given node placement and let the throughput of the scheme under consideration be λ. Consider a large enough time t, so that by definition, the total number of bits transported in the network is λnt. Let $h(b)$ be the number of hops taken by bit b, $1 \le b \le \lambda nt$ and let $r(b,h)$ denote the length of hop h of bit b. Therefore,

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} r(b,h) \ge \lambda nt\bar{L}. \tag{2.4}$$

Now, for two simultaneous transmissions from node i to node j and from node k to node l, consider

$$\begin{aligned} d(j,l) &\ge d(j,k) - d(l,k) \\ &\ge (1+\Delta)d(i,j) - d(l,k), \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} d(j,l) &\geq d(l,i) - d(i,j) \\ &\geq (1+\Delta)d(l,k) - d(i,j). \end{aligned} \tag{2.6}$$

Combining (2.5) and (2.6), we obtain

$$d(j,l) \geq \frac{\Delta}{2}\left(d(i,j) + d(k,l)\right).$$

This result implies that if we place a disk around each receiver of radius $\Delta/2$ times the length of the hop, the disks must be disjoint for successful transmission under the Protocol model. Since a node transmits at W bits per second, each bit transmission time is $1/W$ seconds. During each bit transmission, the total area covered by the disks surrounding the receivers must be less than the total unit area. Summing over the Wt bits transmitted in time t and accounting for edge effects, we obtain

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} \frac{\pi}{4}\left(\frac{\Delta}{2}r(b,h)\right)^2 \leq Wt. \tag{2.7}$$

Let the total number of hops taken by all bits be $H = \sum_{b=1}^{\lambda nt} h(b)$. Then by convexity, it follows that

$$\left(\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} \frac{1}{H}r(b,h)\right)^2 \leq \sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} \frac{1}{H}r(b,h)^2. \tag{2.8}$$

Using (2.4), (2.7) and (2.8), we obtain

$$\left(\lambda nt\bar{L}\right)^2 \leq \left(\frac{16Wt}{\pi\Delta^2}\right)H. \tag{2.9}$$

Now defining $\bar{h}$ to be the sample mean of the number of hops over λnt bits, i.e., $\bar{h} = \frac{1}{\lambda nt}H$. Using this to rewrite (2.9) and the obvious fact that $\lambda \leq W$, we obtain

$$\lambda n \leq \min\left\{\frac{16W}{\pi\Delta^2\bar{L}^2}\bar{h},\ nW\right\}. \tag{2.10}$$

By the law of large numbers, $\bar{L} = \Theta(1)$ *whp.* Moreover the rate of convergence is exponential in n. Let A_n be the set such that $\bar{L} = \Theta(1)$ and let $I(A_n)$ be the indicator of

set A. Then from (2.10), we have

$$\begin{aligned} nE[\lambda] &\leq E\left[\frac{16W}{\pi\Delta^2\bar{L}^2}\bar{h}I(A_n)\right] + E\left[nWI(A_n^c)\right] \\ &\leq c_5 E[\bar{h}] + o(1), \end{aligned} \tag{2.11}$$

where the last term is $o(1)$ since $P(A_n^c)$ converges to 0 exponentially.

By definition, if a scheme has throughput $T(n)$ then there exists a set B_n on which $\lambda \geq T(n)$ and $P(B_n)$ converges to 1. Therefore we have

$$\begin{aligned} E[\lambda] &= E[\lambda I(B_n)] + E[\lambda I(B_n^c)] \\ &\geq T(n)(1 - o(1)). \end{aligned} \tag{2.12}$$

From (2.11) and (2.12), it follows that $nT(n)(1-o(1)) \leq c_5 E[\bar{h}]+o(1)$, which is the same as $E[\bar{h}] = \Omega(nT(n))$. Now each packet spends at least one time-slot at each hop and hence the delay of each packet is at least as much as the number of hops it takes. As a result, if $D(n)$ is the delay of the scheme under consideration then by definition, $D(n) \geq E[\bar{h}]$. Thus we have shown that for any scheme, $D(n) = \Omega(nT(n))$. □

2.3 Discussion

We established the optimal throughput-delay trade-off in static random networks using a fluid model. Our scheme for achieving the optimal trade-off is a simple cellular scheme with two-level TDM (time-division-multiplexing). The entire network area is divided into cells and there is TDM between neighboring cells, as well as, TDM between nodes in the the same cell. Further, packets are routed on the shortest path along cells. This has the implication that such a simple cellular, TDM, shortest path routing scheme is optimal for throughput and delay scaling. Any scheduling and routing algorithm of this type can be expected to perform well, when the exact placement of nodes is not known.

Chapter 3

Throughput-Delay Trade-off in Static Networks: Constant-size Packets

In Chapter 2, the throughput-delay trade-off for static wireless networks was shown to be $D(n) = \Theta(nT(n))$, where $D(n)$ and $T(n)$ are packet delay and throughput in a network of n nodes, respectively. This trade-off was obtained using a fluid model, in which the packets are allowed to be arbitrarily small. In this fluid model, buffers are not required. Due to this packet scaling, $D(n)$ is not equivalent to delay per bit.

This raises the question whether the trade-off remains the same when the packet size is kept constant, which necessitates packet scheduling in the network. This is an important question, since in real networks, packet size does not change when more nodes are added to the network. Note that with the additional constraint that the packet size remains constant, the throughput-delay trade-off can be no better than that in the fluid model. However, a priori, it is not clear whether the same throughput-delay trade-off as in the fluid case can be achieved. This is because, packets of constant size necessitate the use of buffers in the network, due to which routing packets through the network also involves the additional task of scheduling in the network.

In this chapter, we extend our previous results to the case of static wireless networks with buffers and constant-size packets and show that the optimal trade-off is still $D(n) = \Theta(nT(n))$ (as shown in Figure 2.1), where now $D(n)$ is the delay per bit. The main contribution is a scheduling policy for which it is shown that the throughput-delay trade-off is the same as that for the fluid model. Analyzing the delay of any scheduling policy for

a wireless network corresponds to analyzing the delay of an induced discrete-time queuing network. It is natural to attempt to use a FIFO (First-In-First-Out) queue management in the wireless network, however, not much is known about delay with FIFO in discrete-time queuing networks. Thus the study of achievable throughput-delay trade-off with packets of constant size requires a scheduling policy with good performance that is amenable to analysis. We provide a solution by coupling the evolution of a discrete-time queuing network with that of a continuous-time queuing network. This leads to both a packet scheduling policy (see item 6 of Policy Σ_n in Section 3.1) for the wireless network and a method for analyzing the delay. The following is an outline of our solution. Packets in a wireless network have fixed routes depending on the source-destination pair to which they belong. The entire wireless network then corresponds to a discrete-time, open queuing network with general customer routes, in the terminology of queuing theory (e.g. see [28], [44]). In the case of continuous-time queuing networks, when some more conditions are satisfied (such as independent Poisson arrivals to each customer route and a symmetric queue at each server) these are known as Kelly or BCMP networks. For such networks, the equilibrium distribution is known to have a product form. We consider a continuous-time queuing network with general customer routes with the same topology as the discrete-time network we wish to study. Further this network is assumed to have Poisson arrivals, constant service time and Preemptive LIFO at each server so that it is a Kelly (BCMP) network. Then based on packet arrival times in this continuous-time queuing network at each server, we derive a scheduling policy for the discrete-time wireless network. Finally, using product form equilibrium results for continuous-time networks, we determine the exact order of queuing delay in the discrete-time wireless network.

We would like to note that Gupta and Kumar [23] implicitly used a fluid model for establishing throughput scaling. Later work by Kulkarni and Viswanath [29] consolidated the result with an explicit constant packet size model.

The model and definitions of the static random network, the relaxed Protocol model, throughput, delay, and the throughput-delay trade-off are the same as in Chapter 2. In this chapter the packet size is assumed to be constant – equal to W bits. Recall that we assume time-slots to be of unit length and that under the relaxed Protocol model, successful transmission occurs at W bits per second. Hence W is the maximum size of a packet. Note that in the definition of delay we used packet delay. However, since the packet size is constant, packets delay is equal to the delay per bit.

Our main result is as follows.

Theorem 3.1. *The optimal throughput-delay trade-off in the static random network model*

with packets of constant size is given by

$$T(n) = \Theta\left(D(n)/n\right),$$

for $T(n) = O\left(1/\sqrt{n \log n}\right)$.

The above result says that under a delay scaling constraint of $D(n)$ the optimal throughput scaling is $\Theta(D(n)/n)$. And this holds for $T(n) = O\left(1/\sqrt{n \log n}\right)$, that is, the entire range of achievable throughputs in the static random network model.

The rest of this chapter is organized as follows. In Section 3.1, we introduce Scheme Π and show that it achieves the throughput-delay trade-off stated in Theorem 3.1. Finally we present a converse that shows that no scheme can provide a better throughput-delay trade-off than Scheme Π, thus establishing Theorem 3.1.

3.1 Achievability and converse

Our trade-off scheme is a multi-hop, time-division-multiplexed (TDM), cellular scheme with square cells of area $a(n)$ so that the unit torus consists of $1/a(n)$ cells as shown in Figure 2.2. It is similar to Scheme 1 in Chapter 2.

Scheme Σ_n:

---

1. Divide the unit torus using a square grid into square cells, each of area $a(n)$ (see Figure 2.2).
2. Verify whether the following conditions are satisfied for the given realization of the random network.
   - Condition 1: No cell is empty.
   - Condition 2: The number of S-D lines through each cell is at most $c_2 n\sqrt{a(n)}$.
3. If either of the above conditions is not satisfied then use a time-division policy, where each of the $n/2$ sources transmits directly to its destination in a round-robin fashion.
4. Otherwise, i.e., if both conditions are satisfied, use the following policy Σ_n:
   (a) Each node generates packets according to a Poisson process of rate $T(n) = \Theta\left(1/n\sqrt{a(n)}\right)$. The random network is a discrete-time system whereas the packet generation is a continuous-time process. So if a packet is generated at time t, it is available for transmission from time-slot $\lceil t \rceil$ onwards.

(b) Each cell becomes active at a regular interval of $1+c_1$ time-slots (see Lemma 2.2). Several cells which are sufficiently far apart become active simultaneously. Thus the scheme uses TDM between nearby cells.

(c) A source S sends packets to its destination D by relaying or hopping along the adjacent cells lying on its S-D line as shown in Figure 2.2. Thus, in this scheme, direct transmission of packets is only between nodes in adjacent cells.

(d) One of the nodes in a cell acts as a relay by maintaining a buffer for the packets of all the S-D lines passing through that cell. In each time-slot only one packet can be transmitted. However, a relay node may receive up to four packets from its adjacent cells before it gets a chance to relay them. Moreover multiple packets may be generated within the cell which will be available for transmission in the next time-slot. Hence a virtual queue is formed in each cell which consists of packets generated within the cell as well as the packets to be relayed through the cell.

(e) When the cell becomes active, one packet from this virtual queue (if not empty) is transmitted to an adjacent cell according to a Last-In-First-Out (LIFO) type of queue service policy. However, the arrival times considered by this policy are not the actual arrival times of the packets, but the arrival times that would occur in a continuous-time network with the same arrivals and a PL (Preemptive LIFO) queue management at each server. This is elaborated later in this section during the analysis of delay.

Note that each cell has a single relay node and that it maintains a buffer for all packets of all S-D pairs passing through that cell except for the packets generated by source nodes within the cell. However, the virtual queue in the cell includes these latter type of packets, although, the source nodes do not transmit these packets to the relay node in the cell. We assume that there is coordination within the cell to allow this. As a result, the delay analysis only needs to consider this virtual queue.

The point of trade-off at which Scheme Π operates is determined by the parameter $a(n)$ and the dependence is made precise in the following theorem.

Theorem 3.2. *For* $a(n) = \Omega(\log n/n)$,

$$T(n) = \Theta\left(1/n\sqrt{a(n)}\right) \quad \textit{and} \quad D(n) = \Theta\left(1/\sqrt{a(n)}\right),$$

i.e., the throughput-delay trade-off achieved by Scheme Π *is*

$$T(n) = \Theta\left(D(n)/n\right).$$

Throughput analysis: If the time-division policy with direct transmission is used, then the throughput is $2W/n$ and delay of 1. But since it happens with a vanishingly low probability, as shown by Lemmas 2.1 and 2.3, the throughput and delay for Scheme Π are determined by that of policy Σ_n.

When policy Σ_n is used, since Condition A is satisfied, each cell has at least one node. This guarantees that each source can send data to its destination by hops along adjacent cells on its S-D line. From Lemma 2.2, it follows that each cell gets to transmit a packet every $1+c_1$ time-slots, or equivalently, the cell throughput is $\Theta(1)$. The total traffic through each cell is that due to all the S-D lines passing through the cell, which is $O\left(n\sqrt{a(n)}\right)$ since Condition B is also satisfied. This suggests that

$$T(n) = \Theta\left(1/n\sqrt{a(n)}\right),$$

is achievable, if the average delay is finite.

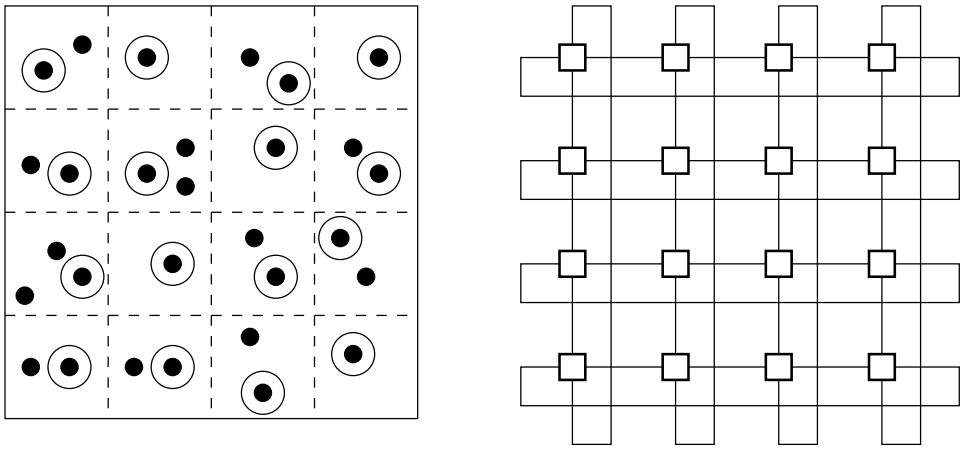


Figure 3.1: The torus on the left with has 16 cells and each cell contains at least one node. The circled node in each cell acts as a relay. The corresponding queuing network of 16 servers, with each server corresponding to a cell in the wireless network, is shown on the right.

Delay Analysis: Next we analyze the average packet delay in the wireless network for Scheme Π when Conditions A and B are satisfied, i.e, when policy Σ_n is used. Dividing the

unit torus into square cells of area $a(n)$ results in $1/a(n)$ cells. One of the nodes in each cell maintains a buffer and acts as a relay for all the S-D lines passing through that cell. These relay nodes are the circled nodes in Figure 3.1. The buffer in each cell corresponds to a queue and the cell itself corresponds to a server that can transmit one packet from this queue once in $1 + c_1$ time-slots. This is because each cell becomes active once in $1 + c_1$ time-slots as described earlier. Since Scheme Π restricts direct transmissions to be between adjacent cells, each cell can receive from or transmit to any four of its adjacent cells. This determines the connectivity between the servers so that the entire wireless network corresponds to a discrete-time queuing network of $1/a(n)$ servers, where each server is connected to four others as shown in Figure 3.1.

Note that the time-division-multiplexing between cells is such that in the c_1 slots before each cell becomes active again each of its neighbors becomes active exactly once. Hence we can ignore the effect of cells becoming active at regular intervals and instead consider a discrete-time network of queues $\mathcal{N}_D$ where D signifies the discrete time nature of this network. The actual delay in the wireless network would then be $1 + c_1$ times the delay in $\mathcal{N}_D$.

Queuing network $\mathcal{N}_D$: The discrete-time queuing network $\mathcal{N}_D$ consists of $1/a(n)$ servers, each of which can service one packet from its queue in a time-slot if it is not empty. Moreover, each server is connected to four others as explained above. In the wireless network, packets travel from their sources to their destinations by hops along adjacent cells on their S-D lines. Thus the route of a packet depends on the S-D pair to which it belongs. This means that in $\mathcal{N}_D$ there are $n/2$ customer routes corresponding to the $n/2$ S-D pairs. Recall that packets arrive in the wireless network at the sources according to independent Poisson processes of rate $T(n)$. These correspond to exogenous arrivals at the queues in $\mathcal{N}_D$. The remaining arrivals at the queues are due to the departures from other queues. In the terminology of queuing theory, $\mathcal{N}_D$ is a discrete-time, open network of queues with general customer routes (see Chapter 6.6 of [44]).

Delay analysis for such discrete-time networks with general customer routes is not known, which prevents us from using a simple First-In-First-Out (FIFO) order of service in $\mathcal{N}_D$. We leverage results known about continuous-time networks to obtain the queue management policy for $\mathcal{N}_D$ in such a way that the average delay can be computed.

Queuing network $\mathcal{N}_C$: Consider a continuous-time open network of $1/a(n)$ servers having the same connectivity structure as $\mathcal{N}_D$ and the same $n/2$ customer routes (see Figure 3.1). Let this network be called $\mathcal{N}_C$. Further, let the exogenous arrivals in both the networks $\mathcal{N}_C$ and $\mathcal{N}_D$ be the same. And let the service requirement of each packet at each server be

deterministically equal to unit time. From the description until now, it is clear that $\mathcal{N}_C$ is the continuous-time analog of the discrete-time network $\mathcal{N}_D$. A Preemptive LIFO (PL) queue management is used at each server in $\mathcal{N}_C$ (see Chapter 6.8 of [44] for more details).

The queue size distribution for the continuous time network $\mathcal{N}_C$ with PL queue management at each server has a product form in equilibrium as shown in [28] (see Theorems 3.7 and 3.8 of Chapter 3) provided that the following two conditions are satisfied. First, the service time distribution should be either phase-type (that is, a mixture of Gamma distributions) or the limit of a sequence of phase-type distributions. The second condition is that, the total traffic at each server is less than its capacity, which is one in our case.

In our case the service time is constant and equal to 1. Consider the sum of n exponential random variables each with mean $1/n$. This sum has a phase-type distribution and in the limit as n tends to infinity, its distribution converges to that of a constant random variable. Thus the first condition is satisfied.

In the wireless network the number of S-D lines passing through each cell is $O\left(n\sqrt{a(n)}\right)$ and the arrival process for each S-D pair is an independent Poisson process with rate $T(n) = \Theta\left(1/n\sqrt{a(n)}\right)$. Therefore an appropriate choice of constants guarantees that the total traffic at each server is less than 1, its service capacity, as Condition B (mentioned just before the description of policy Σ_n) is satisfied. Thus the second condition is also satisfied.

Using the product form for the queue size distribution in equilibrium, it follows that the average queue size at a queue with total traffic $\lambda < 1$ and unit mean service is of the form $c_3\lambda/(1-\lambda)$ where c_3 is some constant. By Little's law this implies that the average delay at each server is bounded above by a constant independent of n. We summarize the above discussion in the lemma below.

Lemma 3.1. *For the continuous-time open network $\mathcal{N}_C$ with $n/2$ customer routes as described above the average delay at each server is bounded above by a constant independent of n.*

Packet Scheduling in $\mathcal{N}_D$ using $\mathcal{N}_C$: However we cannot use this PL policy in the discrete time network $\mathcal{N}_D$ because of the following reasons:

1. Due to the discrete time nature of the network $\mathcal{N}_D$, a packet that is generated at time t becomes eligible for service (i.e. next hop transmission) only at time $\lceil t \rceil$.

2. A complete packet has to be transmitted in a time-slot, i.e. fractions of the packets cannot be transmitted. This means that a preemptive type of service like PL is not allowed.

To address these problems for $\mathcal{N}_D$, we present a centralized scheduling policy derived from emulating in parallel, the continuous-time network $\mathcal{N}_C$ with PL queue management at each server. The exogenous arrivals in both $\mathcal{N}_C$ and $\mathcal{N}_D$ are the same. Let a packet arrive in $\mathcal{N}_C$ at some server at time a_C and in $\mathcal{N}_D$ at the same server at time a_D. Then it is served in $\mathcal{N}_D$ using a LIFO policy with the arrival time considered to be $\lceil a_C \rceil$ instead of a_D.

Clearly such a scheduling policy can be implemented if and only if each packet arrives before its scheduled departure time. According to our scheduling policy, the scheduled departure time can be no earlier than $\lceil a_C \rceil$, whereas the actual arrival time is a_D. Hence for this scheduling policy to be feasible, it is sufficient to show that $a_D \leq \lceil a_C \rceil$ for every packet at each server. Let d_C and d_D be the departure times of a packet from some server in $\mathcal{N}_C$ and $\mathcal{N}_D$ respectively. Since the departure time at a server is the arrival time at the next server on the packet's route it is sufficient to show that $d_D \leq \lceil d_C \rceil$ for each packet in every busy cycle of each server in $\mathcal{N}_C$. In what follows, we show that for all packets in any busy cycle of any server, the departures in $\mathcal{N}_D$ occur at or before the departures in $\mathcal{N}_C$.

Lemma 3.2. *Let a packet depart in $\mathcal{N}_C$ from some server at time d_C and in $\mathcal{N}_D$ at time d_D, then $d_D \leq \lceil d_C \rceil$.*

Proof. Fix a server and a particular busy cycle of $\mathcal{N}_C$. Let it consist of packets numbered $1, \ldots, k$ with arrivals at times $a_1 \leq \ldots \leq a_k$ and departures at times $d_1, \ldots, d_k$. Let the arrival times of these packets in $\mathcal{N}_D$ be $A_1, \ldots, A_k$ and departures be at times $D_1, \ldots, D_k$. By assuming that $A_i \leq \lceil a_i \rceil$ for $i = 1, \ldots, k$, we need to show that $D_i \leq \lceil d_i \rceil$ for $i = 1, \ldots, k$.

Clearly this holds for $k = 1$ since $D_1 = \lceil A_1 \rceil + 1 \leq \lceil a_i \rceil + 1 = \lceil d_1 \rceil$. Now suppose it holds for all busy cycles of length k and consider any busy cycle of $k + 1$ packets.

If $\lceil a_1 \rceil < \lceil a_2 \rceil$, then because of the LIFO policy in $\mathcal{N}_D$ based on times a_i, we have $D_1 = \lceil a_1 \rceil + 1 \leq \lceil a_1 \rceil + k + 1 = \lceil d_1 \rceil$. The last equality holds since in $\mathcal{N}_C$, the PL service policy dictates that the first packet of the busy cycle is the last to depart. And the remaining packets would have departures times as for a busy cycle of length k.

Otherwise if $\lceil a_1 \rceil = \lceil a_2 \rceil$ then the LIFO policy in $\mathcal{N}_D$ based on arrival times a_i results in $D_1 = \lceil a_1 \rceil + k + 1 = \lceil d_1 \rceil$ and the packets numbered $2, \ldots, k$ depart exactly as if they belong to a busy cycle of length k. This completes the proof by induction. □

Thus we have shown that it is possible to use LIFO in $\mathcal{N}_D$ based on the arrival times in $\mathcal{N}_C$ instead of the actual arrival times in $\mathcal{N}_D$. We are now ready to prove Theorem 3.2.

Proof. (of Theorem 3.2) Packets reach their destination with finite average delay, which shows that the throughput is just the rate at which each source sends its data. This proves that the throughput $T(n) = \Theta\left(1/n\sqrt{a(n)}\right)$.

Next we compute the average packet delay $D(n)$. Lemma 3.2 also holds for the final departure of each packet from the network. Therefore if D_D^i is the delay of a packet of route i in $\mathcal{N}_D$ (i.e. S-D pair i in the wireless network) and D_C^i is the delay of the corresponding packet in $\mathcal{N}_C$ then $D_D^i \leq D_C^i + 1$. Hence taking expectations it follows that

$$E[D_D^i] \leq E[D_C^i] + 1, \quad 1 \leq i \leq n/2.$$

Therefore delay averaged over all $n/2$ routes is given by

$$D(n) = \frac{2}{n} \sum_{i=1}^{n/2} E[D_D^i] \leq \frac{2}{n} \sum_{i=1}^{n/2} E[D_C^i] + 1. \tag{3.1}$$

Since each hop in the wireless network covers a distance of $\Theta\left(\sqrt{a(n)}\right)$, the number of hops per packet for S-D pair i is $\Theta\left(d_i/\sqrt{a(n)}\right)$ where d_i is the length of S-D line i. Now D_C^i is the delay for a packet of route i, which is equal to the sum of the delays along all queues on its route. But from Lemma 3.1, the average delay at each server is bounded above by some constant independent of n. Therefore from (3.1), we obtain that

$$D(n) \leq \frac{2}{n} \sum_{i=1}^{n/2} c_2 \frac{E[d_i]}{\sqrt{a(n)}} + 1 = \Theta\left(1/\sqrt{a(n)}\right)$$

since $2\sum_{i=1}^{n/2} E[d_i]/n = \Theta(1)$. □

Finally to see that the trade-off provided by Scheme Π is optimal, consider Theorem 2.3 that was established for the fluid model. The constant packet size requirement is an additional constraint compared to the fluid model and hence its throughput-delay cannot be better than that for the fluid model. This proves that the throughput-delay scaling trade-off provided by Scheme Π is optimal for the static random network model with packets of constant size.

3.2 Discussion

The optimal throughput-delay trade-off for random wireless networks was determined in using a fluid model. In this chapter, we imposed the constraint that the packet size remains constant and showed that the throughput-delay trade-off remains unchanged. This provides a justification for the simplifying fluid assumption made earlier since it does not affect the

essential network dynamics.

It is worth noting that the techniques developed in this chapter cannot be applied directly to the mobile random network model. This is because, nodes cannot be identified with cells since they are moving around the network. As a result, it is not possible to associate a virtual queue with each cell as we did in this work.

In a related model, where the mobile network also has n static nodes along with n mobile nodes, the optimal trade-off can be obtained for sufficiently low throughputs. We can show that for any throughput $T(n) = \Theta(1/n^{1/2+\epsilon}), \epsilon > 0$, the trade-off given by $T(n) = \Theta(D(n)/n)$ can be achieved. This is the same as the trade-off for the fluid model. This establishes the optimal trade-off for this range of low throughputs. The scheme achieving this trade-off uses the scheduling scheme given in this chapter along with a randomization technique and chasing in a manner similar to Scheme 3(a) in Chapter 5. However the optimal trade-off for the mobile network with no static nodes is unknown.

Chapter 4

Throughput-Delay-Energy Trade-offs or Optimal Hopping

One of the insights provided by the work of Gupta and Kumar [23] on throughput is that the highest throughput scaling is obtained at the smallest transmission range, or equivalently, maximum amount of hopping is needed to mitigate interference in the network.

In Chapter 2, the optimal throughput-delay trade-off for a random wireless network was established to be $D(n) = \Theta(nT(n))$. The work in Chapter 2 also showed that at an optimal point of the trade-off, delay scales as the number of hops and both throughput and delay increase as the amount of hopping increases. Thus the amount of hopping determines the point of the optimal trade-off at which the network operates; more hops results in higher throughput but also higher delay.

Energy is another important metric and assumes great significance in the case of networks with limited energy resources, particularly sensor networks. With the assumption that signal attenuates with distance r as $r^{-\delta}$, it is easy to see that if relay nodes are placed uniformly between the source and destination, the total transmission energy decreases as the amount of hopping increases. This leads one to believe that hopping as much as possible is good for minimizing energy-per-bit. However, when transceiver circuit energy is also taken into account, this is no longer the case and the optimal number of hops depends on the topology and the size of the network [40]. A large body of work exists on optimizing one of throughput, delay and energy subject to a constraint on one of the others for a given placement of nodes (see [7] for a representative sample). These studies have focused on optimizing the performance metrics for a given realization of the network, which is quite different from the framework of random networks where the goal is to obtain general

guidelines for an entire class of networks.

In this chapter, we determine the optimal hopping for energy efficiency along with throughput and delay using the random network framework. We assume that n nodes are randomly placed in area $A(n)$ and are split into $n/2$ source-destination pairs. We allow the density of nodes in the network, $n/A(n)$, to be anywhere between 1, for a constant density network, where $A(n) = n$, and n for a constant area network, where $A(n) = 1$. The reason for characterizing networks according to their node densities, as we shall show later, is that node density determines the amount of optimal hopping in a network. We assume the Physical model for successful transmission [23], which captures the effect of interference in the network due to other simultaneously transmitting users. We consider both Radio-frequency (RF) transmission energy and transceiver circuit energy. Thus the energy used in communicating a bit from a source to a destination has two components – one due to transmission, which depends on the number of hops and the distance and power used at each hop, and the other due to transceiver circuit energy, which is proportional to the number of hops. Note that by increasing the number of hops, the transmission energy component decreases whereas the transceiver energy component increases. We establish the optimal trade-offs between throughput, delay and energy for this random network model. We find that even after the inclusion of energy consumption into the model, which allows for power control, hopping continues to determine the optimal trade-offs between throughput, delay and energy. This happens because the amount of hopping determines the amount of power to use for optimal energy scaling. As a by-product, we obtain the amount of hopping that results in the minimum energy scaling.

The outline of the rest of the chapter is as follows. Section 4.1 presents the random network model, required definitions and a preview of the main results. Section 4.2 states Theorem 4.1, which is the main result of this chapter and two corollaries. Section 4.3 presents a cellular scheme that achieves the trade-off that is stated to be optimal in Theorem 4.1. Section 4.4 shows that the trade-off achieved by this scheme is optimal, in that no scheme can outperform it in terms of scaling. Finally in Section 4.5, we discuss some of the implications of our results on the design of wireless ad hoc networks.

4.1 Model and main results

In this chapter we use a random network model in which the area is a parameter, rather than being fixed, as in earlier chapters. Moreover, the Physical model is used instead of the Relaxed Protocol model for determining successful transmission.

Definition 8 (Random network model). The random network consists of a torus of area $A(n)$ in which n nodes are distributed uniformly at random. These n nodes are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for packetized transmission. For simplicity, we assume that the time-slots are of unit length.

The network area $A(n)$ is allowed to be a function of the number of nodes n and the density of nodes in the network is $n/A(n)$. We refer to the case when $A(n) = 1$ as the *constant area* model. The other extreme is the *constant density* model in which $A(n) = n$, i.e., n nodes are placed in a torus of area n so that the density of nodes is 1.

Let the distance on the torus between two nodes i and j be denoted by r_{ij}. We assume that signal decays with distance r as $r^{-\delta}$ so that power decays as $r^{-2\delta}$, where $\delta > 1$. We assume the Physical model for successful transmission presented in [23] and also used by several others since.

Definition 9 (Physical model). A transmission from node i to j is successful if

$$SINR = \frac{Pr_{ij}^{-2\delta}}{N + \sum_{k\in\Gamma, k\neq i} r_{kj}^{-2\delta}} \geq \beta,$$

where Γ is the set of simultaneously transmitting nodes. When the transmission is successful, communication occurs at a constant rate W.

The basis of the Physical model is the AWGN channel with noise power N where interference from other transmitting users is treated as independent Gaussian noise. Thus when the transmission from node i to node j is successful under the Physical model, the energy-per-bit due to RF transmission is $P/W \geq \beta(N + I)r_{ij}^{2\delta}$, where I is the total interference power at node j due to other simultaneous transmissions.

In order to take the transceiver circuit power into account, we assume a constant amount of energy per bit, c_0, is also dissipated during each transmission/reception [40]. Thus the total energy-per-bit for a successful transmission from node i to j is $P/W + c_0 \geq \beta(N + I)r_{ij}^{2\delta} + c_0$.

Some authors assume the signal to attenuate with distance r as $(1 + r)^{-\delta}$ since the $r^{-\delta}$ attenuation is valid only in the far field. This results in transmission energy-per-bit approaching a constant strictly greater than 0 as the distance approaches 0. We note that the results in this chapter remain unchanged under this model for the constant density random network. This is because a constant amount of energy is anyway consumed at each hop due to the circuits and further the rate is constant when transmission is successful.

Scheme, throughput, and delay are the same as defined earlier in Chapter 2. Let the maximum power used by any node in a scheme be $P_{max}(n)$ and let the minimum power be $P_{min}(n)$. We impose the further condition that

$$P_{max}(n)/P_{min}(n) < \beta. \tag{4.1}$$

Thus the power used by a scheme can depend on n but all transmitters have more or less the same power.

Energy-per-bit is defined analogously to delay.

Definition 10 (Energy-per-bit). Let $\mathcal{E}^i_{\Pi_n}(j)$ be the energy spent to communicate bit j of S-D pair i, then the sample mean of energy-per-bit for S-D pair i is

$$\bar{\mathcal{E}}^i_{\Pi_n} = \limsup_{k\to\infty} \frac{1}{k} \sum_{j=1}^{k} \mathcal{E}^i_{\Pi_n}(j).$$

The average over all S-D pairs of the energy-per-bit for a particular realization is

$$\bar{\mathcal{E}}_{\Pi}(n) = \frac{2}{n} \sum_{i=1}^{n} \bar{\mathcal{E}}^i_{\Pi_n}.$$

Energy-per-bit of scheme Π is the expectation of the above average, i.e.,

$$\mathcal{E}_{\Pi}(n) = E[\bar{\mathcal{E}}_{\Pi}(n)].$$

We will use $\mathcal{E}_T(n)$ and $\mathcal{E}_C(n)$ to denote the components of $\mathcal{E}_{\Pi}(n)$ due to transmission and the circuit respectively.

The number of hops of a scheme Π is denoted by $H_{\Pi}(n)$ and is defined in the same way as the delay of a scheme. Since it is a repeat of the above with obvious modifications, the complete definition is omitted.

The T-D (throughput-delay) trade-off is defined as in 2. The D-E (delay-energy) and T-E (throughput-energy) trade-offs are defined similarly.

Definition 11 (T-D-E trade-off). A triple $(T(n), D(n), \mathcal{E}(n))$ is T-D-E optimal if there exists a scheme achieving it and for any scheme Π such that $T_{\Pi}(n) = \Omega(T(n))$, $D_{\Pi}(n) = \Omega(D(n))$ and for any scheme satisfying $T_{\Pi}(n) = \Omega(T(n))$ and $D_{\Pi}(n) = O(D(n))$, $\mathcal{E}_{\Pi}(n) = \Omega(\mathcal{E}(n))$. The T-D-E trade-off consists of all the T-D-E optimal triples.

We define T-E-D, D-T-E, D-E-T, E-T-D and E-D-T trade-offs similarly.

We use the shorthand notation, $\Omega(f(n)) = T(n) = O(g(n))$ to mean $T(n) = \Omega(f(n))$ and $T(n) = O(g(n))$.

Now that the model and the performance metrics have been defined, we summarize the T-D-E trade-offs that capture the essential elements of our results. The T-D-E and all other trade-offs are stated and proved in detail in subsequent sections.

For any value of $A(n)$ and $\delta > 1$, $D(n) = \Theta(H(n))$ at any optimal trade-off point and the T-D trade-off turns out to be $D(n) = \Theta(nT(n))$, which is the same as in [12]. The trade-offs involving delay and energy-per-bit depend on the value of $A(n)$ and are discussed below.

Constant Area Network (see Corollary 4.1(i))

The T-D-E trade-off for $A(n) = 1$ is given by

$$D(n) = \Theta\left(nT(n)\right) \quad \text{and} \quad \mathcal{E}(n) = \Theta\left(D(n)\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$.

In fact, $\mathcal{E}_T(n) = o(\mathcal{E}_C(n))$ and $\mathcal{E}_C(n) = \Theta(D(n))$. That is, the circuit energy, which is proportional to the number of hops, or equivalently the delay, dominates over the transmission energy as depicted in Figure 4.1(a). As a result, as long as $\delta > 1$, the exact value of δ does not affect the results.

The best $\mathcal{E}(n)$ that can be achieved in the constant area network is $\Theta(1)$ and is achieved when a constant number of hops (that does not increase with n) is used. The corresponding delay is $\Theta(1)$, which is the best possible. However, the highest energy efficiency and lowest delay come at the cost of the lowest throughput of $\Theta(1/n)$. As the amount of hopping in the network increases by decreasing the transmission range, throughput increases but at the cost of higher delay and higher energy-per-bit.

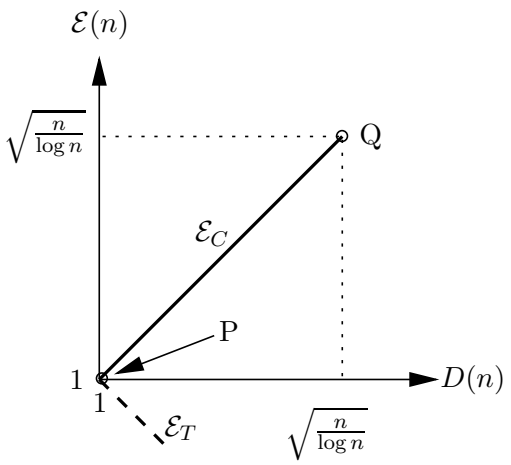
Constant Density Network (see Corollary 4.2(i))

The T-D-E trade-off for $A(n) = n$ is given by

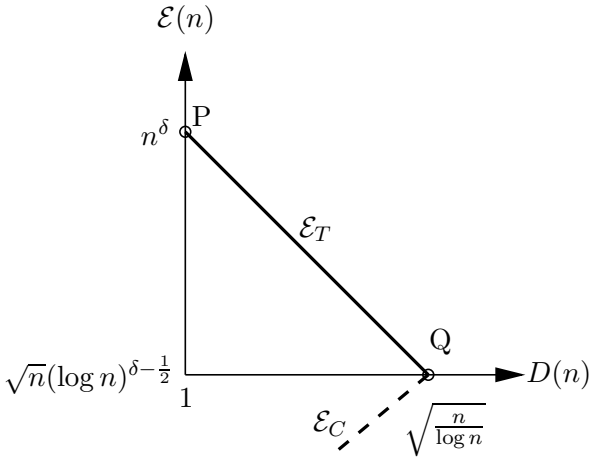
$$D(n) = \Theta\left(nT(n)\right) \quad \text{and}$$

$$\mathcal{E}(n) = \Theta\left(n^{\delta} D(n)^{1-2\delta}\right),$$

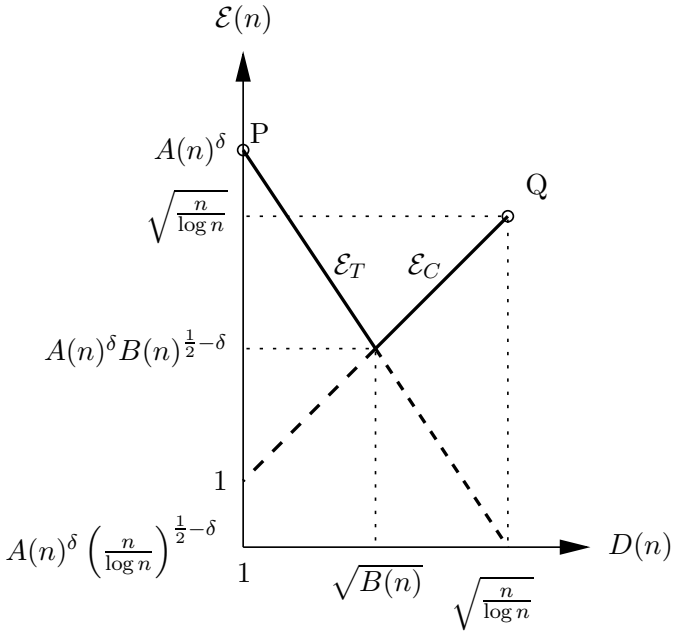
for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$.



(a) Constant area random network.



(b) Constant density random network.



(c) Random network with area $A(n)$.

Figure 4.1: The D-E trade-off in a random wireless network. The scale of each axis is in terms of the order in n. Point P corresponds to $\Theta(1)$ hops while point Q corresponds to $\Theta\left(\sqrt{n/\log n}\right)$ hops.

In fact, $\mathcal{E}_C(n) = \Theta(D(n)) = o(\mathcal{E}_T(n))$ and $\mathcal{E}_T(n) = \Theta\left(n^{\delta}(D(n))^{1-2\delta}\right)$ as shown in Figure 4.1(b). Contrary to the case of the constant area network, here the transmission energy dominates over the circuit energy. Therefore, the value of δ affects the energy consumption. The best $\mathcal{E}(n)$ that can be achieved in the constant density network is $\Theta\left(\sqrt{n}(\log n)^{\delta-\frac{1}{2}}\right)$. Thus even at best, the energy efficiency of the network decreases with increase in the size of the network. The best energy scaling is achieved when the maximum amount of hopping is used. This is because although the circuit energy consumption increases in proportion to the number of hops, the RF transmission energy, which is the dominant component, decreases by using more hops. The corresponding throughput is $\Theta\left(1/\sqrt{n\log n}\right)$, which is the highest possible. However, the highest energy efficiency and the highest throughput come at the cost of the highest delay of $\Theta\left(\sqrt{n/\log n}\right)$. As the amount of hopping in the network decreases, delay decreases but at the cost of lower throughput and higher energy-per-bit.

Intermediate Density Network (see Theorem 4.1(i))

For a random network with area $A(n)$, the T-D-E trade-off is given by

$$D(n) = \Theta\left(nT(n)\right), \quad \mathcal{E}_C(n) = \Theta(D(n))$$

$$\text{and} \quad \mathcal{E}_T(n) = \Theta\left(A(n)^{\delta}D(n)^{1-2\delta}\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n\log n}\right)$.

Thus for general $A(n)$, $\mathcal{E}_T$ dominates at $\Theta(1)$ hops and $\mathcal{E}_C$ dominates as hops increase as shown in Figure 4.1(c). The minimum energy-per-bit scaling is obtained when $\mathcal{E}_T = \Theta(\mathcal{E}_C)$. This happens at $\sqrt{B(n)}$ hops, where $B(n) = \min\{A(n), n/\log n\}$. The minimum energy-per-bit scaling is $\Theta\left(A(n)^{\delta}B(n)^{1-2\delta}\right)$. The trade-off between $D(n)$ and $\mathcal{E}(n)$ for general $A(n)$ is thus a combination of that for the extreme cases of $A(n) = 1$ and $A(n) = n$.

4.2 Optimal trade-offs

This section states the following main result of the chapter, which establishes the optimal trade-offs between throughput, delay and energy-per-bit. The corollaries following it are specializations to the case of the constant area network and the constant density network.

Theorem 4.1. *For the random network model with area $A(n)$, at any optimal trade-off point, $D(n) = \Theta(H(n))$. Further, let $B(n) = \min\{A(n), n/\log n\}$, then the following statements hold.*

(i) The T-D-E and D-T-E trade-off are given by

$$D(n) = \Theta\left(nT(n)\right), \quad \mathcal{E}_C(n) = \Theta(D(n))$$

$$\text{and} \quad \mathcal{E}_T(n) = \Theta\left(A(n)^{\delta} D(n)^{1-2\delta}\right), \tag{4.2}$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$.

(ii) The D-E-T and E-D-T trade-offs are given by (4.2) for $\Omega(1/n) = T(n) = O\left(\sqrt{B(n)}/n\right)$.
For $\Omega\left(\sqrt{B(n)}/n\right) = T(n) = O\left(1/\sqrt{n \log n}\right)$, *the D-E-T and E-D-T trade-offs are degenerate and are given by* $T(n) = \Theta\left(\sqrt{B(n)}/n\right)$, $D(n) = \Theta\left(\sqrt{B(n)}\right)$ *and* $\mathcal{E}(n) = \Theta\left(\sqrt{B(n)}\right)$.

(iii) The T-E-D and E-T-D trade-offs are given by (4.2) for $\Omega\left(\sqrt{B(n)}/n\right) = T(n) = O\left(1/\sqrt{n \log n}\right)$.
For $\Omega(1/n) = T(n) = O\left(\sqrt{B(n)}/n\right)$, *the T-E-D and E-T-D trade-offs are degenerate and are given by* $T(n) = \Theta\left(\sqrt{B(n)}/n\right)$, $D(n) = \Theta\left(\sqrt{B(n)}\right)$ *and* $\mathcal{E}(n) = \Theta\left(A(n)^{\delta} B(n)^{\frac{1}{2}-\delta}\right)$.

Substituting $A(n) = 1$ in Theorem 4.1 yields the following result for the constant area network.

Corollary 4.1. *For the random network with area* $A(n) = 1$, *at any optimal trade-off point,* $D(n) = \Theta(H(n))$. *Further the following statements hold.*

(i) The T-D-E trade-off is given by

$$D(n) = \Theta\left(nT(n)\right) \quad \text{and} \quad \mathcal{E}(n) = \Theta\left(D(n)\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$. *In fact,* $\mathcal{E}_T(n) = \Theta\left(D(n)^{1-2\delta}\right) = o(\mathcal{E}_C(n))$ *and* $\mathcal{E}_C(n) = \Theta(D(n))$.

(ii) The T-E-D, D-T-E and E-T-D trade-offs are identical to the T-D-E trade-off.

(iii) The D-E-T and E-D-T trade-offs are degenerate with $D(n) = \mathcal{E}(n) = \Theta(1)$ *and* $T(n) = \Theta(1/n)$.

The D-E-T and E-D-T trade-offs are degenerate, since both the lowest delay and lowest energy scaling come together when using a constant number of hops as shown by point

P in Figure 4.1(a). The other trade-offs are identical because as the amount of hopping increases, throughput increases but at the cost of higher energy-per-bit and higher delay.

Substituting $A(n) = n$ in Theorem 4.1 yields the following result for the constant density network.

Corollary 4.2. *For the constant density random network model, at any optimal trade-off point, $D(n) = \Theta(H(n))$. Further the following statements hold.*

(i) The T-D-E trade-off is given by

$$D(n) = \Theta\left(nT(n)\right) \quad \textit{and}$$

$$\mathcal{E}(n) = \Theta\left(n^{\delta} D(n)^{1-2\delta}\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$. *In fact,* $\mathcal{E}_C(n) = \Theta(D(n)) = o(\mathcal{E}_T(n))$ *and* $\mathcal{E}_T(n) = \Theta\left(n^{\delta}(D(n))^{1-2\delta}\right)$.

(ii) The D-T-E, D-E-T and E-D-T trade-offs are identical to the T-D-E trade-off.

(iii) The T-E-D and E-T-D trade-offs are degenerate with $T(n) = \Theta\left(1/\sqrt{n \log n}\right)$, $D(n) = \Theta\left(\sqrt{n/\log n}\right)$ *and* $\mathcal{E}(n) = \Theta\left(\sqrt{n}(\log n)^{\delta - \frac{1}{2}}\right)$.

The T-E-D and E-T-D trade-offs are degenerate, since both the highest throughput and the lowest energy-per-bit come together when using maximal hopping. The other trade-offs are identical because as the amount of hopping decreases, delay decreases but at the cost of lower throughput and higher energy-per-bit. This is clear from the T-D trade-off and the D-E trade-off shown in Figure 4.1(b).

4.3 Achievability using a cellular TDM scheme

In this section, we present Scheme Π that achieves a trade-off between throughput, delay and energy in a random network of area $A(n)$ with n nodes. We will show that the scaling trade-off provided by this scheme is of the same order as that claimed in Theorem 4.1. This will establish that the trade-off claimed in Theorem 4.1 is achievable.

Scheme Π is similar to Scheme 1 in Chapter 2 with modifications to account for the Physical model. It is a multi-hop, time-division-multiplexed (TDM), cellular scheme parameterized by $a(n)$, where $a(n) = \Omega(\log n/n)$ and $a(n) \leq 1$. The network area is divided into square cells, each of area $b(n) = a(n)A(n)$ so that the torus of area $A(n)$ consists of

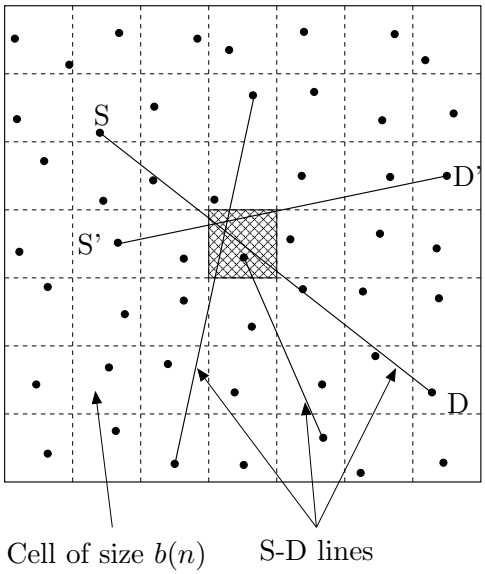


Figure 4.2: The torus of area $A(n)$ is divided into cells of area $b(n) = a(n)A(n)$ for Scheme 1. The S-D lines passing through the shaded cell in the center are shown.

$1/a(n)$ cells as shown in Figure 4.2. The parameter $a(n)$ is the fraction of the total network area that each cell occupies.

Let the straight line joining a source, S, and a its destination, D, be called an S-D line. Scheme Π is described below.

Scheme Π

---

1. Divide the unit torus using a square grid into square cells, each of area $b(n) = a(n)A(n)$ (see Figure 4.2). The packet size is $\Theta\left(1/n\sqrt{a(n)}\right)$.

2. Verify whether the following conditions are satisfied for the given realization of the random network.

   - Condition 1: No cell is empty.
   - Condition 2: The number of S-D lines through each cell is at most $c_2 n\sqrt{a(n)}$.

3. If either of the above conditions is not satisfied then use a time-division multiplexing (TDM) policy, where each of the $n/2$ sources transmits directly to its destination in a round-robin fashion.

4. Otherwise, i.e., if both conditions are satisfied, use the following policy Π_n:

(a) The cells are divided into k^2 groups, where k depends only on δ, β, and N, and is independent of n. Figure 4.3 illustrates this for the case of $k = 3$. All cells belonging to the same group become active simultaneously and each group becomes active at a regular interval of k^2 time-slots. Thus the scheme uses TDM between nearby cells.

(b) A source S transmits data to its destination D by hops along the adjacent cells lying on its S-D line as shown in Figure 2.2.

(c) When a cell becomes active, it transmits a single packet for each of the S-D lines passing through it. This is again performed using a TDM scheme that slots each cell time-slot into *packet time-slots* as shown in Figure 2.3.

(d) Each transmitting node transmits with power $P(n) = Pb(n)^{\delta}$, where P depends only on β, δ and N and not on n.

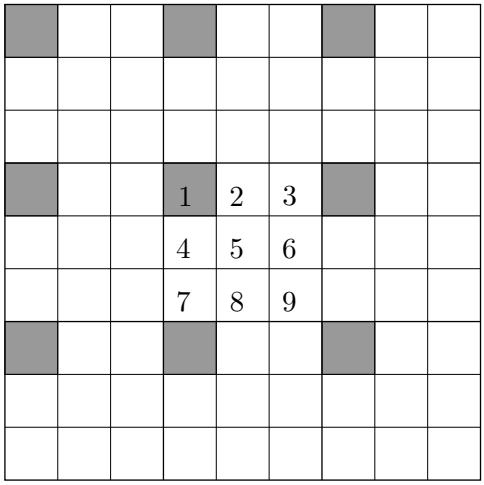


Figure 4.3: An illustration of the cells being divided into k^2 groups for the case of $k = 3$, i.e., 9 groups. All the shaded cells which are in group 1 transmit in the same time-slot. In the next time-slot all the cells in group 2 transmit and so on.

The point of trade-off at which Scheme Π operates is determined by the parameter $a(n)$ and this dependence is made precise in the following theorem.

Theorem 4.2. *With* $a(n) = \Omega(\log n/n)$, *Scheme* Π *has*

$$T(n) = \Omega\left(\frac{1}{n\sqrt{a(n)}}\right), \quad D(n) = \Theta\left(\frac{1}{\sqrt{a(n)}}\right),$$

$$\mathcal{E}(n) = O\left(\frac{1}{\sqrt{a(n)}} + A(n)^{\delta} a(n)^{\delta-\frac{1}{2}}\right).$$

Thus the trade-off achieved by this scheme is

$$D(n) = \Theta\left(nT(n)\right) \quad \textit{and}$$

$$\mathcal{E}(n) = \Theta\left(D(n) + A(n)^{\delta} D(n)^{1-2\delta}\right)$$

$$\textit{for} \quad \Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right).$$

Proof. As shown in Lemma 2.1 in Chapter 2, no cell is empty *whp*. Further, it follows from Lemma 2.3 in Chapter 2 that in our setting, the number of S-D lines passing through each cell is $O\left(n\sqrt{a(n)}\right)$ *whp*. This guarantees the existence of a constant c_2 so that Condition 2 is satisfied *whp*. Thus Conditions 1 and 2 are satisfied *whp*.

If the time-division policy with direct transmission is used, then the throughput is $2W/n$ with a delay of 1. But since it happens with a vanishingly low probability, the throughput and delay for Scheme Π are determined by that of policy Π_n. Hence we will only consider policy Π_n for the rest of the proof.

First we will establish that for an appropriate choice of $k = k(\beta, \delta, N)$ and $P = P(\beta, \delta, N)$, the SINR is greater than β at each receiver as required by the Physical model. This will be done by showing that the worst-case interference power $P_I(n)$ at any receiver is bounded above by a constant that decreases monotonically in k and does not depend on n for $\delta > 1$. It is easy to see that the placement of the receiver and the transmitters as shown in Figure 4.4 results in the the worst case interference at the receiver node. Let $I_{max}(n)$ be the total interference from all other transmitters when each transmitter uses power $Pb(n)^{\delta}$. This interference can be split into 3 components ($I_1(n)$, $I_2(n)$ and $I_3(n)$) based on the positions of the interfering transmitters as shown in Figure 4.4, so that

$$I_{max}(n) \leq I_1(n) + I_2(n) + 4I_3(n). \tag{4.3}$$

Using $m = 1/a(n)$ to denote the total number of cells, we have,

$$\begin{aligned} I_1(n) &= \sum_{i=1}^{\sqrt{m}/2} \frac{Pb(n)^{\delta}}{\left((ki-2)\sqrt{b(n)}\right)^{2\delta}} \\ &\leq \frac{P}{(k-2)^{2\delta}} + \frac{P}{(2k-2)^{2\delta}} + \sum_{i=2}^{\sqrt{m}/2} \frac{P}{(ki)^{2\delta}} \end{aligned}$$

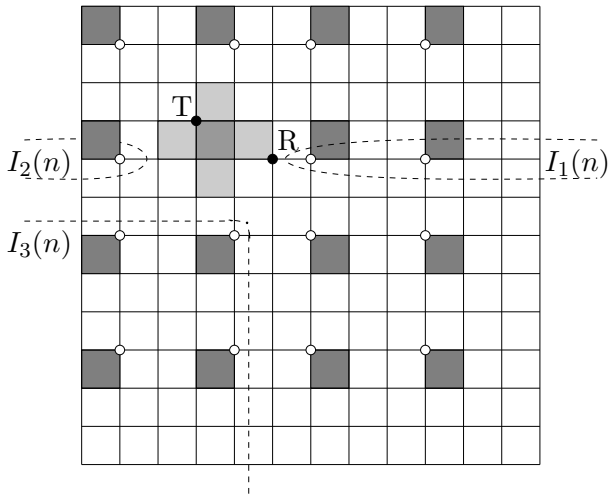


Figure 4.4: The node marked T transmits to the node marked R in an adjacent cell. The hollow nodes are the other nodes transmitting simultaneously.

$$\begin{aligned} &\leq \frac{P}{(k-2)^{2\delta}} + \frac{P}{(2k-2)^{2\delta}} \\ &\quad + \frac{P}{k^{2\delta}} \int_1^{\sqrt{m}/2-1} x^{-2\delta} dx \\ &\leq \frac{P}{(k-2)^{2\delta}} + \frac{P}{(2k-2)^{2\delta}} + \frac{P}{(2\delta-1)k^{2\delta}}, \end{aligned}$$

for $\delta > 1/2$. Similarly for $\delta > 1/2$, we obtain

$$\begin{aligned} I_2(n) &= \sum_{i=1}^{\sqrt{m}/2} \frac{Pb(n)^{\delta}}{\left((ki+1)\sqrt{b(n)}\right)^{2\delta}} \\ &\leq \frac{P}{(k+1)^{2\delta}} + \sum_{i=2}^{\sqrt{m}/2} \frac{P}{(ki)^{2\delta}} \\ &\leq P\left(\frac{1}{(k+1)^{2\delta}} + \frac{1}{(2\delta-1)k^{2\delta}}\right). \end{aligned}$$

Using similar but tedious manipulations, which we do not include here, it can be shown that for $\delta > 1$, $I_3(n)$ is less than P times a decreasing function of k. Thus from (4.3) it follows that $I_{max}(n) = Pf(k)$ where $f(k)$ is monotonically decreasing in k. Thus in the

worst case,

$$\begin{aligned} SINR &\geq \frac{Pb(n)^{\delta}\left(5b(n)\right)^{-2\delta}}{N + I_{max}(n)} \\ &= \frac{5^{-\delta}}{\frac{N}{P} + f(k)}. \end{aligned}$$

Since $f(k)$ decreases monotonically in k, we can choose k and P depending on β, δ and N so that even in the worst case, $SINR \geq \beta$.

We can use this to analyze the throughput of the trade-off scheme. When policy Π_n is used, since Condition 1 is satisfied, each cell has at least one node. This guarantees that each source can send data to its destination by hops along adjacent cells on its S-D line. In Scheme Π, each cell becomes active once in every k^2 time-slots and moreover the rate at each transmission is W according to the Physical model since at each receiver, $SINR \geq \beta$ as shown above. Hence the cell throughput is $\Theta(1)$. The total traffic through each cell is that due to all the S-D lines passing through the cell, which is $O\left(n\sqrt{a(n)}\right)$ since Condition 2 is also satisfied. This shows that

$$T(n) = \Omega\left(1/n\sqrt{a(n)}\right).$$

Next we compute the average packet delay $D(n)$. The delay of a packet is the time it takes to reach its destination after leaving its source. This is equal to the sum of the amounts of time spent at each hop and so we first bound the average number of hops over all $n/2$ S-D pairs.

Since each hop covers a distance of $\Theta\left(\sqrt{b(n)}\right)$, the number of hops per packet for S-D pair i is $\Theta\left(d_i/\sqrt{b(n)}\right)$, where d_i is the length of S-D line i. Thus the number of hops taken by a packet averaged over all S-D pairs is $\Theta\left(\frac{2}{n}\sum_{i=1}^{n/2} d_i/\sqrt{b(n)}\right)$. Now the expectation of the average distance between S-D pairs, $E[\frac{2}{n}\sum_{i=1}^{n/2} d_i] = \Theta\left(\sqrt{A(n)}\right)$ and so the expectation of the average number of hops is $\Theta\left(1/\sqrt{a(n)}\right)$, since $b(n) = a(n)A(n)$.

Recall that each cell is active once every k^2 time-slots and since Condition 2 is satisfied, each S-D line passing through a cell can have its own packet time-slot within that cell's time-slot. Since we allow the packet size to scale in proportion to the throughput $T(n)$, each packet arriving at a node in the cell departs in the next active time-slot of the cell. Thus the delay is at most k^2 times the number of hops. Since k does not depend on n, we conclude that the delay, $D(n) = \Theta\left(1/\sqrt{a(n)}\right)$.

Now we can compute the energy-per-bit $\mathcal{E}(n)$ for this scheme. Since the throughput is

$T(n)$ and there are $n/2$ S-D pairs, $nT(n)L/2$ bits are communicated from the sources to their destinations over a long enough period of L time-slots. The total transmission energy spent in the network during this time is that due to $1/k^2a(n)$ cells in each time slot, which is equal to $Pb(n)^\delta L/k^2a(n)$. The total circuit energy spent is proportional to the number of hops in the network, which is $L/k^2a(n)$. Hence

$$\begin{aligned}\mathcal{E}(n) &= O\left(PA(n)^\delta a(n)^{\delta-1}/nT(n) + 1/na(n)T(n)\right) \\ &= O\left(A(n)^\delta a(n)^{\delta-1/2} + 1/\sqrt{a(n)}\right).\end{aligned}$$

This concludes the proof of Theorem 4.2. □

4.4 Proof of optimality

In this section, we present a converse to Theorem 4.2 to show that the trade-off provided by our trade-off scheme is indeed the optimal trade-off as far as scaling is concerned. That is, we show that no scheme can provide a better scaling trade-off than the one achieved by the scheme presented in Section 4.3. This is the content of Theorem 4.3 and this along with Theorem 4.2 proves Theorem 4.1 thus establishing the optimal scaling trade-off between throughput, delay and energy.

To establish a converse, we need to show that if any scheme has throughput $T(n)$ then its delay scaling is $\Omega(nT(n))$, i.e., its delay scaling can be no better than that achieved by Scheme Π in Section 4.3. By the definitions of the performance metrics, this means, we need to show that if any scheme has throughput $T(n)$ *whp* over all realizations then its expected delay over all realizations is $\Omega(nT(n))$. We also need to show a similar relationship between delay and energy scaling. Before doing this, we consider a fixed realization of a network and determine how the throughput, delay and energy-per-bit of any scheme depend on the average transmission range. The analysis of the trade-off scheme in Section 4.3 showed that the transmission range of scheme determines the amount of hops used by S-D pairs and this in turn determines the throughput, delay and energy-per-bit. The following lemma shows that the transmission range of a scheme puts a bound on its performance and that our trade-off scheme uses multi-hopping in the best possible way, as far was scaling is concerned.

Lemma 4.1. *Consider any realization of the random network with $2n$ nodes in area $A(n)$. Let d_i be the distance between S-D pair $i, 1 \leq i \leq n$ and let $\bar{L} = \frac{1}{n}\sum_{i=1}^{n} d_i$. With any scheme for this realization, let the throughput be λ, the average transmission range be $\bar{r}$, the average number of hops per bit be $\bar{h}$ and the average energy-per-bit be $\bar{\mathcal{E}}$. Then the following*

hold:

(a) $\bar{h} \geq \bar{L}/\bar{r}$,

(b) $\lambda \leq c_3 A(n)\bar{h}/\left(n\bar{L}^2\right)$,

(c) $\bar{\mathcal{E}} \geq \beta \bar{L}^{2\delta}\bar{h}^{1-2\delta} + c_0\bar{h}$.

Proof. (a) Consider any realization of the random network with $2n$ nodes. Suppose that a scheme provides throughput λ for this realization. Then given a sufficiently long time interval T, each source communicates λT bits to its destination. The total number of bits is $B = \lambda nT$ since there are n S-D pairs. The most general scheme can transmit bits via multiple hops and paths in the network. Suppose that bit $b, 1 \leq b \leq B$, is communicated to its destination by H_b hops and let $r(b,h), 1 \leq b \leq B, 1 \leq h \leq H_b$ be the length of hop h of bit b. Thus $r(b,h)$ is the transmission range at hop h of bit b.

First note that the average number of hops,

$$\bar{h} = \frac{1}{B}\sum_{b=1}^{B} H_b, \tag{4.4}$$

and the average transmission range for the scheme is

$$\bar{r} = \frac{1}{\sum_{b=1}^{B} H_b}\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h). \tag{4.5}$$

Recall that the distance between the source and destination of S-D pair $i, 1 \leq i \leq n$, is d_i. Since each bit belonging to S-D pair i has to travel at least distance d_i, it follows that

$$\lambda T \sum_{i=1}^{n} d_i \leq \sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h). \tag{4.6}$$

Starting from (4.4), we obtain the following.

$$\begin{aligned} \bar{h} &= \frac{1}{\lambda nT}\sum_{b=1}^{B} H_b \\ &= \frac{1}{\lambda nT\sum_{i=1}^{n} d_i}\frac{1}{n}\sum_{i=1}^{n} d_i \sum_{b=1}^{B} H_b \end{aligned}$$

$$\begin{aligned} &\overset{(a)}{\geq} \frac{1}{n}\sum_{i=1}^{n} d_i \frac{\sum_{b=1}^{B} H_b}{\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h)} \\ &\overset{(b)}{=} \frac{\bar{L}}{\bar{r}}, \end{aligned}$$

where inequality (a) is due to (4.6) and (b) is by the definitions of $\bar{r}$ and $\bar{L}$.

(b) The proof of this part of the lemma is essentially the same as that of Theorem 2.3 in Chapter 2 which uses the Protocol model and follows from the equivalence between the Physical model and the Protocol model as shown in [23].

Let the position of node i be denoted by X_i and let $|X_i - X_j|$ denote the distance between nodes i and j. Then as shown in the proof of Theorem 2.1 of [23], the Physical model implies that if node i is transmitting to node j, and k is any other simultaneously transmitting node then

$$|X_k - X_j| \geq (1+\Delta)|X_i - X_j|,$$

where $\Delta = \left(\frac{\beta P_{min}}{P_{max}}\right)^{\frac{1}{2\delta}} - 1$.

As a result of this equivalence with the Protocol model, as shown in [23], in every time-slot t

$$\sum_{(b,h)\in\Gamma_t} r(b,h)^2 \leq c_3 A(n),$$

where Γ_t is the set of (b,h) pairs such that hop h of bit b occurs in time-slot t and c_3 is a constant that depends only on W. This is based on the idea that each transmission consumes an area proportional to the square of the range of transmission and the total area is $A(n)$. Summing over all T time-slots, we obtain

$$\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h)^2 \leq c_3 A(n) T. \tag{4.7}$$

By convexity,

$$\bar{r}^2 \leq \frac{1}{\sum_{b=1}^{B} H_b} \sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h)^2.$$

Combining the above two equations and rearranging, we obtain

$$\sum_{b=1}^{B} H_b \leq \frac{c_3 A(n) T}{\bar{r}^2}. \tag{4.8}$$

Substituting from (4.5) into (4.6) and using (4.8), we obtain

$$\lambda T \sum_{i=1}^{n} d_i \leq \frac{c_3 A(n) T}{\bar{r}^2} \bar{r}.$$

This can be rewritten as

$$\lambda n \left(\frac{1}{n} \sum_{i=1}^{n} d_i \right) \leq \frac{c_3 A(n)}{\bar{r}}.$$

Now using part (a) of the lemma, we obtain

$$\lambda n \bar{L} \leq \frac{c_3 A(n) \bar{h}}{\bar{L}},$$

which proves part (b) of the lemma.

(c) Consider hop h of bit b and let $P(b,h)$ be the power used for this transmission. Suppose this transmission occurs in time-slot t. Then as per the Protocol model

$$\frac{P(b,h) r(b,h)^{-2\delta}}{N + \sum_{(i,j)\in\Gamma_t} P(i,j) r(i,j)^{-2\delta}} \geq \beta.$$

Thus by ignoring the interference, we obtain

$$P(b,h) \geq \beta N r(b,h)^{2\delta}.$$

Therefore the average transmission energy-per-bit over time T,

$$\begin{aligned}
\bar{\mathcal{E}}_T &= \frac{1}{B} \sum_{b=1}^{B} \sum_{h=1}^{H_b} P(b,h) \\
&\geq \frac{\sum_{b=1}^{B} H_b}{B} \frac{1}{\sum_{b=1}^{B} H_b} \sum_{b=1}^{B} \sum_{h=1}^{H_b} P(b,h) \\
&\overset{(a)}{\geq} \bar{h} \beta \bar{r}^{2\delta} \\
&\overset{(b)}{\geq} \beta \bar{L}^{2\delta} \bar{h}^{1-2\delta}, \qquad (4.9)
\end{aligned}$$

where inequality (a) is due to convexity and inequality (b) is due to part (a) of the lemma.

As per our energy model, each hop consumes a constant amount of energy, c_0 and hence

the average circuit energy, $\bar{\mathcal{E}}_C = c_0\bar{h}$. As a result, the average energy-per-bit

$$\bar{\mathcal{E}} = \bar{\mathcal{E}}_T + \bar{\mathcal{E}}_C \geq \beta\bar{L}^{2\delta}\bar{h}^{1-2\delta} + c_0\bar{h},$$

which proves part (c) of the lemma. □

We would like to note that (4.9) in the proof of part (c) of the above lemma is a natural extension of the minimum transmission energy-per-bit for an AWGN channel to the case of a network with multiple hops. For the AWGN channel with noise power N, the rate when using power P to communicate over a distance r is $\frac{1}{2}\log\left(1 + Pr^{-2\delta}/N\right)$, which implies that the minimum transmission energy-per-bit is $Nr^{2\delta}/2$. In our case, if we ignore the interference due to other simultaneous transmissions, the minimum energy-per-bit for a single transmission over distance r would be $\beta Nr^{2\delta}/W$, which is the same as that for the AWGN channel except for the constant β/W. Thus the above lemma lower bounds the minimum energy-per-bit in the case of a network by taking into account multiple hops and ignoring the effect of interference due to other simultaneous transmissions. Ignoring interference does not hurt in determining the correct scaling, as long as it does not dominate the noise N and this is what our trade-off scheme does.

Using the above lemma, we prove the converse to Theorem 4.2 for the cases of the T-D-E and D-T-E trade-offs. This converse establishes that no scheme can provide a better T-D-E or D-T-E scaling trade-off than that provided by our trade-off scheme.

Theorem 4.3. *In a random network with area $A(n)$, if a communication scheme has has throughput, $T(n)$, delay, $D(n)$ and energy-per-bit, $\mathcal{E}(n)$ then*

$$D(n) = \Omega\left(nT(n)\right) \quad \textit{and}$$

and if the scheme has $D(n) = \Theta(nT(n))$ then

$$\mathcal{E}(n) = \Omega\left(D(n) + A(n)^{\delta}D(n)^{1-2\delta}\right).$$

Proof. From part (b) of Lemma 4.1, we have

$$\lambda \leq \frac{c_3 A(n)\bar{h}}{n\bar{L}^2}. \tag{4.10}$$

Combining this with the obvious fact that $\lambda \leq W$, we obtain

$$\lambda n \leq \min\left\{\frac{c_3 A(n)\bar{h}}{\bar{L}^2},\ nW\right\}. \tag{4.11}$$

Now we can use the same steps as in the proof of Theorem 2.3 following (2.10) to conclude that $D(n) \geq E[\bar{h}] = H(n)$. Thus we have shown that for any scheme,

$$D(n) = \Omega(H(n)) = \Omega(nT(n)). \tag{4.12}$$

This establishes the desired relationship between throughput and delay. Next we deal with energy-per-bit. Taking expectation on both sides of part (c) of Lemma 4.1, we obtain

$$\begin{aligned}
\mathcal{E}(n) &= E\left[\bar{\mathcal{E}}\right] \\
&\geq \beta E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}\right] + c_0 E\left[\bar{h}\right] \\
&= E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}\right] + c_0 H(n).
\end{aligned} \tag{4.13}$$

Now $\bar{L} = \Theta\left(\sqrt{A(n)}\right)$ *whp* and moreover the rate of convergence in the law of large numbers is exponential in n. Let A be the event that $\bar{L} = \Theta\left(\sqrt{A(n)}\right)$ then

$$E[\bar{h}^{1-2\delta}] = E[\bar{h}^{1-2\delta}|A]P(A) + E[[\bar{h}^{1-2\delta}|A^c](1-P(A)). \tag{4.14}$$

Now $\bar{h}^{1-2\delta}$ is a polynomial in n and since $1-P(A)$ decays exponentially fast to 0, it follows that

$$E\left[\bar{h}^{1-2\delta}\right] = E\left[\bar{h}^{1-2\delta}|A\right]P(A)(1-o(1)).$$

Using the above, we obtain

$$\begin{aligned}
E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}\right] &\geq E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}|A\right]P(A) \\
&\geq c_4 A(n)^{\delta} E\left[\bar{h}^{1-2\delta}|A\right]P(A) \\
&= c_4 A(n)^{\delta} E\left[\bar{h}^{1-2\delta}\right](1-o(1)) \\
&\overset{(a)}{\geq} c_4 A(n)^{\delta} E\left[\bar{h}\right]^{1-2\delta}(1-o(1)) \\
&= c_4 A(n)^{\delta} H(n)^{1-2\delta}(1-o(1)),
\end{aligned}$$

where inequality (a) is due to Jensen's inequality.

Using the above equation, (4.13) can be rewritten as

$$\mathcal{E}(n) = \Omega\left(A(n)^{\delta} H(n)^{1-2\delta} + H(n)\right).$$

Now assume that the optimal trade-off between throughput and delay is achieved by a scheme, i.e., $D(n) = \Theta(nT(n))$. Then it follows from (4.12) that $D(n) = \Theta(H(n))$. As a result, we have

$$\mathcal{E}(n) = \Omega\left(A(n)^{\delta} D(n)^{1-2\delta} + D(n)\right).$$

□

The above converse shows that the trade-off obtained by Scheme II in Section 4.3 is optimal in terms of scaling and this establishes the optimal T-D-E and D-T-E trade-offs for the random network. This also proves part (i) of Theorem 4.1. Converses for the remaining two parts can be proved similarly. The additional variable $B(n)$ arises in Theorem 4.1 because, to ensure connectivity in the network, the number of hops must be $O\left(\sqrt{n/\log n}\right)$.

4.5 Discussion

Using a random network model to study large, ad hoc wireless networks, previous work established the optimal throughput-delay trade-off. The optimal number of hops to minimize energy consumption for a given placement of nodes has been studied in a separate body of work. In this chapter, we used a random network model to unify these results by establishing the optimal trade-offs between throughput, delay and energy-per-bit. In so doing, we also determined the amount of hopping needed to operate at an optimal point of the trade-off between these performance metrics. We also showed that at any optimal trade-off point, delay scales as the number of hops and the amount of hopping determines the trade-off point at which the network operates depending on the node density. This is a consequence of the interference-limited nature of communication, due to which the amount of hopping determines the optimal power to use for transmissions.

Maximum hopping is required for the highest throughput but results in the worst delay. Whether it is energy efficient or not depends on the network node density. Hopping is expensive in terms of energy in a constant area network with node density n and the minimum energy-per-bit is obtained using $\Theta(1)$ hops. On the other hand, in a constant density network, hopping is necessary for energy efficiency. But even with maximum hopping that provides the highest energy efficiency, the energy-per-bit increases as the network grows in size.

Our results suggest some general guidelines for building ad hoc wireless networks.

1. In high density networks consisting of low data-rate nodes, where the main concerns are energy and delay, our results suggest the use of minimal hopping. This also saves the energy and delay overheads of implementing complex multi-hopping protocols.

2. In high density networks consisting of high data-rate nodes with limited energy, maximum hopping is needed to accommodate the high throughput requirement. As our results show, this requires the nodes to operate at far from minimum energy consumption in addition to the overhead of high protocol complexity. In this case it may be necessary to have an infrastructure of relay nodes with unconstrained access to energy.

3. In applications where the network extends over a large geographical area, in spite of maximal hopping being optimal, energy efficiency can be low. Again this problem can be mitigated by adding wired or wireless infrastructure.

4. We showed that a cellular, TDM network architecture with equal node transmission powers determined by the cell size achieves the optimal scaling. Such an architecture is attractive due to its simplicity.

In Scheme Π presented in Section 4.3, the size of packets scaled down with n. This fluid model was used to avoid the problem of scheduling packets in the network. However, it is easy to see that by using the scheduling scheme in Chapter 3, the same trade-off can also be achieved using packets of constant size. Hence the results in this chapter also hold with packets of constant size.

Chapter 5

Throughput-Delay Trade-off in Mobile Networks

Grossglauser and Tse [21] introduced a mobile random network model and showed that constant throughput scaling can be achieved if the nodes move independently with an ergodic and uniform stationary distribution. The corresponding delay was not quantified, however, it was expected to be high, since mobility was utilized to communicate packets. In this chapter, we begin by showing that the corresponding delay is of order $n \log n$. This sets the stage for studying the throughput-delay trade-off, which provides some key insights into the role of mobility in wireless networks.

5.1 Model and main results

Definition 12 (Mobile random network model). The random network consists of n nodes distributed uniformly at random in a unit torus. The nodes are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for packetized transmission. For simplicity, we assume that the time-slots are of unit length. In a *static network* nodes do not move. In the case of a *mobile network*, the mobility model, which we denote as the *random walk (RW) model*, is as follows. The unit torus is divided into n square cells of area $1/n$ each, resulting in a two-dimensional $\sqrt{n} \times \sqrt{n}$ discrete torus. The initial position of each node is equally likely to be any of the n possible cells independent of others. Each node independently performs a simple random walk on the two-dimensional $\sqrt{n} \times \sqrt{n}$ discrete torus. By a simple random walk, we mean the following: let a node be in cell $(i, j) \in \{0, \ldots, \sqrt{n} - 1\}^2$ at time t, then, at time $t + 1$ the node is equally likely to be in any of the

four adjacent cells $\{(i-1,j),(i+1,j),(i,j-1),(i,j+1)\}$, where addition and subtraction are modulo $\sqrt{n}$.

Note that, implicitly this models a situation where each node moves $1/\sqrt{n}$ distance in unit time, that is, velocity scales as $1/\sqrt{n}$. Further, note that in the random walk model, nodes move independently according to a uniform stationary and ergodic distribution as in previous work [21]. The additional assumption of the random walk model is not required for throughput results and is used only in the analysis of delay.

At this point, we would like to argue that the random walk model is a good model for capturing real motion in the physical world due to its Markovian nature, so that the present position determines the distribution of the future position. It is sufficiently simple and well studied by probabilists, so as to allow analysis of complicated quantities such as queuing delay, which depends heavily on the motion model.

The definitions of throughput, delay, and the trade-off between them is as in Chapter 2. Further, we use the fluid model, that is, the packets are allowed to be arbitrarily small.

The main results of this chapter are as follows.

Mobile network at $\mathbf{T(n) = \Theta(1)}$:

In [21], a two-hop scheme that achieves constant throughput scaling in mobile wireless networks was presented. It was expected that the delay would be high since mobility was utilized to communicate packets. Delay scaling, however, was not quantified. In Chapter 5, we introduce Scheme 2, which is essentially the same as the two-hop scheme in [21]. We show that this scheme achieves constant throughput scaling for the mobile network and determine its associated constant. Using results and methods from random walks and queuing theory, we determine the exact order of delay for the random walk model of node mobility.

Theorem 5.1. *Scheme 2 for the mobile random network has throughput $T(n) = \Theta(1)$ and its delay scales as*

$$D(n) = \Theta\left(n \log n\right).$$

Point R in Figure 5.1 corresponds to the throughput-delay scaling provided by Scheme 2. Packet size remains constant in Scheme 2 and hence it does not require a fluid model. Note that any trade-off that can be achieved using constant-size packets can obviously be achieved using the fluid model since the constraint of requiring packets to have a constant size is removed.

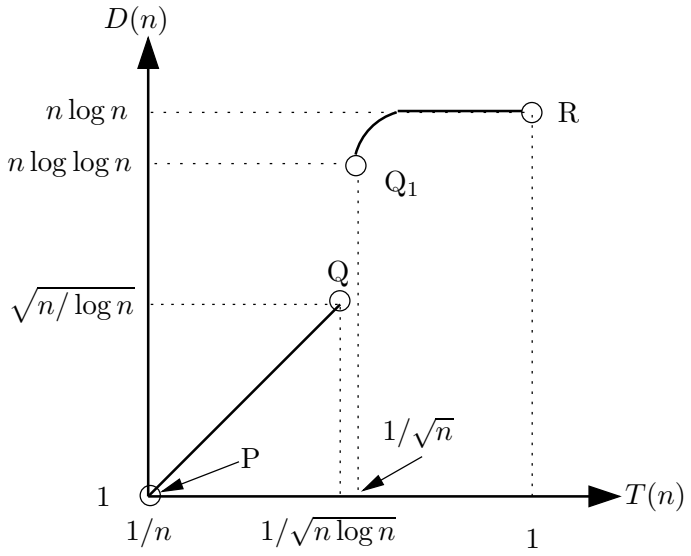


Figure 5.1: Throughput-delay trade-off for the mobile random network. Again, the scales of the axes are in terms of the orders in n.

Trade-off in the mobile random network:

In Chapter 5 we introduce Schemes 3(a) and 3(b) that achieve the optimal throughput-delay trade-off in mobile networks. To provide lower delay, Scheme 3(a) does not use mobility to relay packets. In fact, mobility makes this scheme significantly more complex than Scheme 1, even though the throughput-delay trade-off achieved is the same for both schemes. This is because packets need to chase the nodes to achieve low delay. However, mobility is essential for higher throughput and this is harnessed by Scheme 3(b) at the cost of higher delay. The throughput-delay results for mobile networks are as follows.

Theorem 5.2. *The optimal throughput-delay trade-off for the mobile random network is as follows.*

(a) For the range of $T(n) = O\left(1/\sqrt{n \log n}\right)$*, similar to the case of static network,*

$$T(n) = \Theta\left(\frac{D(n)}{n}\right).$$

(b) For $T(n) = \omega\left(1/\sqrt{n \log n}\right)$ *and* $T(n) = O(1/\log n)$ *the optimal throughput-delay*

trade-off is characterized as,

$$T(n) = \Theta\left(\frac{1}{\sqrt{n\ a(n)\ \log n}}\right) \text{ and}$$

$$D(n) = \Theta\left(n \log\left(\frac{1}{a(n)}\right)\right),$$

where $a(n)$ *is a parameter such that* $a(n) = \Omega(\log n/n)$ *and* $a(n) \leq 1$.

Segment PQ in Figure 5.1 corresponds to Theorem 5.2(a) and segment Q_1R corresponds to Theorem 5.2(b). Note that the effect of mobility is to significantly increase the range of achievable throughput albeit at the expense of a very large delay.

Independently and around the same time, it has been shown in [33], using a Brownian motion model for node mobility, that any throughput higher than $\Theta(1/\sqrt{n})$ comes at the expense of a very high delay. The same conclusion follows from the optimal throughput-delay trade-off for mobile networks obtained using the random walk mobility model in this chapter.

The results for mobile networks provide several insights into the role of mobility in wireless networks.

- For throughput of $O\left(1/\sqrt{n \log n}\right)$, the trade-off in mobile networks is identical to that in static networks. This suggests that, although mobility can enhance the throughput of wireless networks, it does not alter the trade-off between throughput and delay for the range of throughputs achievable in static wireless networks. Further, the scheme achieving the above trade-off does not use the mobility of the nodes to communicate packets. This suggests that for low delay applications, mobility is in fact a hindrance and makes communication schemes significantly more complex.

- As soon as mobility is used to boost the throughput beyond $\Theta\left(1/\sqrt{n \log n}\right)$, the delay jumps up to $\Theta(n \log \log n)$. Thereafter, even though the throughput increases to $\Theta(1)$, the delay only increases to $\Theta(n \log n)$. In this sense there is almost no trade-off between throughput and delay for this range of high throughputs. This also means that if mobility is used to boost the throughput even slightly beyond that in static wireless networks then the delay shoots up to its highest value.

We would like to note that the trade-off with random walk mobility is dramatically different from the one with the less realistic i.i.d. mobility [38]. With i.i.d. mobility the optimal trade-off is given by $D(n) = \Theta(nT(n)$ for $T(n) = O(1)$.

5.2 Scheme with constant throughput scaling

In this section, we present a scheme (which is similar to the one in [21]) and show that it achieves constant throughput and then analyze its delay in Subsection 5.2.2. The analysis of delay for this scheme will also help in characterizing the throughput-delay trade-off in mobile wireless networks in Section 5.3.

Scheme 2: $T(n) = \Theta(1)$ in mobile networks

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1. Divide the unit torus into n square cells, each of area $1/n$.
2. Each cell becomes active once in every $1 + c_1$ time-slots (Lemma 2.2 then ensures that all transmissions are successful).
3. In an active cell, the transmission is always between two nodes within the same cell.
4. Each time-slot is divided into two sub-slots A and B. The following is done in each active cell.
   (a) In sub-slot A, if one or more source nodes are present and the cell contains two or more nodes, pick one source node at random. With probability $0 < p_1 < 1$, the randomly chosen node transmits its packet to another randomly chosen node in the same cell, which acts as a relay. This node could also happen to be the destination. And with probability $1 - p_1$ it does nothing.
   (b) In sub-slot B, if the cell contains one or more destination nodes and two or more nodes in all, pick one destination node at random. Another randomly chosen node in the same cell acts as a relay and transmits to this destination a packet that is destined for it if it has one. This node could also happen to be the source. Otherwise nothing happens.

5.2.1 Achievability of constant throughput scaling

We now show that this scheme achieves a constant throughput scaling using a simpler proof than the one in [21]. Our proof is based on showing that the total network throughput is $\Theta(n)$. The symmetry of the scheme and the use of at most one relay ensure that this total network throughput is equally divided among the $n/2$ S-D pairs resulting in $T(n) = \Theta(1)$.

Theorem 5.3. *The throughput in a mobile random network using Scheme 2 is* $T(n) = \Theta(1)$.

Proof. Consider the transmission of packets from sources in sub-slot A over a period of T time-slots. Due to division into sub-slots the packet size is $W/2$ bits. Let $A(i,t)$ be the number of packets transmitted from source i in time-slot t and let $A(t) = \sum_{i=1}^{n/2} A(i,t)$ be the total number of packets transmitted in time-slot t. The number of bits transmitted by source i in time-slot t is just $WA(i,t)/2$.

Next we determine $E[A(t)]$. Let $C(t)$ be the number of cells which contain at least one of the $n/2$ source nodes and two or more nodes in all. Then from the description of Scheme 2 it follows that $E[A(t)] = p_1 E[C(t)]/(1+c_1)$. Now let I_i be the indicator for the event that cell i contains at least one source node and two or more nodes in all. Let E_1 be the event that cell i contains exactly one source node and E_2 be the event that cell i contains two or more source nodes. Similarly let F_1 be the event that cell i contains one or more destination nodes. Also let

$$p_k = \binom{n/2}{k}\left(\frac{1}{n}\right)^k\left(1-\frac{1}{n}\right)^{n/2-k} \rightarrow \frac{e^{-1/2}}{2^k k!}.$$

That is, p_k is equal to the probability that k nodes are in cell i. Then for $1 \le i \le n$,

$$\begin{aligned} E[I_i] &= P\left((E_1 \cap F_1) \cup E_2\right) \\ &= P(E_1)P(F_1) + P(E_2) \\ &= p_1(1-p_0) + (1-p_0-p_1) \\ &\rightarrow \frac{1}{2}e^{-1/2}\left(1-e^{-1/2}\right) + 1 - e^{-1/2} - \frac{1}{2}e^{-1/2} \\ &= c_3 > 0, \end{aligned} \tag{5.1}$$

as $n \rightarrow \infty$. Therefore,

$$\begin{aligned} E[A(t)] &= \frac{p_1}{1+c_1}E[C(t)] \\ &= \frac{p_1}{1+c_1}E\left[\sum_{i=1}^{n} I_i\right] \\ &= \frac{np_1}{1+c_1}E[I_1] \\ &\rightarrow c_4 n, \end{aligned} \tag{5.2}$$

where $c_4 = c_3 p_1/(1+c_1) > 0$.

Now the mobile random network is an irreducible finite-state Markov chain and $A(t)$ is a bounded non-negative function of the state of this Markov chain at time t. Therefore by

the ergodicity of such a Markov chain,

$$\lim_{T\to\infty} \frac{1}{t} \sum_{t=1}^{T} A(t) = E[A(t)] \to c_4 n.$$

Thus the total rate at which packets are transmitted from sources is $\Theta(n)$. From the symmetry of the nodes and the randomness of the scheme it follows that each of the $n/2$ sources transmits at rate of $\Theta(1)$. These packets either reach the destination or are queued at the relay nodes, in which case they are transmitted to their final destinations in some of the B sub-slots. By choosing $0 < p_1 < 1$ in Scheme 2, we have ensured that the arrival rate to each queue is less than the rate at which the queue can be serviced. This ensures the stability of the queues as a result of which the throughput per S-D pair is just the rate at which each source transmits data. And we have shown that this is $\Theta(1)$ thus proving that Scheme 2 yields $T(n) = \Theta(1)$. □

The above proof also shows that the constant throughput per S-D pair that can be achieved is close to $c_4 W/2$ bits per second for large enough n. Now $c_3 = 0.2095$ and reasonably small values of Δ result in $1 + c_1 = 16$, which results in $c_4 = 0.13 p_1$. Thus under Scheme 2, for large enough n, the throughput between each source-destination pair is about 0.65% of the maximum possible value of W bits per second.

5.2.2 Analysis of delay

The nodes perform independent random walks. Hence only $\Theta(1/n)$ of the packets belonging to any S-D pair reach their destination in a single hop (which happens when both S and D are in the same cell in sub-slot A). Thus, most of the packets reach their destination via a relay node, where the delay has two components: (i) *hop-delay*, which is constant and independent of n, and (ii) *mobile-delay*, which is the time a packet spends at the relay while it is moving until it is delivered to its destination. Next, we analyze the mobile-delay.

Relay-queue: From the description of Scheme 2, for each S-D pair, each of the remaining $n - 2$ nodes acts as a relay. Each node maintains a separate queue for each of the S-D pairs as illustrated in Figure 5.2. Thus the mobile-delay is the expected delay at such a relay-queue. By symmetry, all such queues at all relay nodes are identical. Consider one such relay-queue, i.e., fix an S-D pair and a relay node R. To compute the expected delay of this relay-queue, we need to study the characteristics of its arrival and departure processes.

A packet arrives at the relay-queue when (i) R is in the same cell as S, (ii) the cell becomes active, (iii) S and R are chosen as a transmit-receive pair, and (iv) S transmits

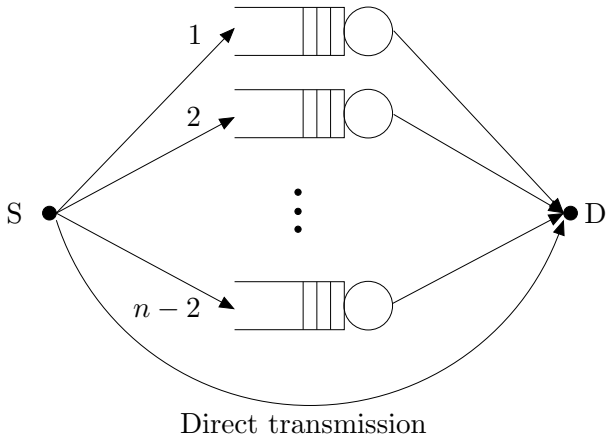

Figure 5.2: For any S-D pair, the remaining $n-2$ nodes act as relays. Each node maintains a separate queue for each of the $n-2$ S-D pairs.

a packet (which happens with probability p_1). Similarly, a departure can occur from the queue when (i) R is in the same active cell as D, (ii) the cell becomes active, and (iii) R and D are chosen as a transmit-receive pair. Such a time-slot is called a *potential departure instant* and the sequence of inter-potential-departure times is called the *potential-departure process*. A packet actually departs, only if, in addition to the above, R also has a packet for D, i.e., the relay-queue is not empty. In the analysis below, we ignore the effect of the the cell becoming active once in $1+c_1$ time-slots since the actual delay is $1+c_1$ times the delay computed in this manner by ignoring it.

We say that two nodes *meet* if they are in the same cell. The joint position of two nodes due to independent random walks can also be viewed as a difference random walk relative to the position of one node. Then the inter-meeting times are just the inter-visit times of state $(0,0)$ for the difference random walk on a $\sqrt{n}\times\sqrt{n}$ discrete torus. Hence the inter-meeting times of nodes S and R, $\tau_0, \tau_1, \ldots$, form an i.i.d. sequence. Let τ be a random variable with their common distribution. The moments of τ that will be required later are given in the following lemma, which is proved in Subsection 5.4.

Lemma 5.1. *The first and second moments of τ are given by*

$$E[\tau] = n, \quad E[\tau^2] = \Theta(n^2 \log n).$$

In what follows, we obtain upper and lower bounds of the same order on the delay of the relay-queue, thus pinning down the exact order of delay scaling. To obtain an upper

bound, we progressively define queues that are simpler to analyze. We first upper bound the delay of the relay-queue by that of another queue, $\mathcal{Q}_1$, which has i.i.d. inter-arrival times. The delay of $\mathcal{Q}_1$ is then upper bounded by that of $\mathcal{Q}_2$, which, in addition, has i.i.d. inter-potential-departure times. The delay of $\mathcal{Q}_2$ is then upper bounded by the sum of delays through two queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, in tandem. Queue, $\mathcal{Q}_3$, is a GI/GI/1 queue, so that Kingman's upper bound can be used. Queue, $\mathcal{Q}_4$, is not a GI/GI/1 queue, but we are still able to bound the delay using the moments of the inter-arrival and inter-potential-departure times. For both these queues, the required moments can be expressed in terms of the moments of the inter-meeting time of two nodes, which we obtain using random walk analysis. We now proceed to the details.

Upper bound: As mentioned above, the inter-meeting times of nodes S and R form an i.i.d. sequence. The inter-arrival times, however, are not independent because an arrival occurs with probability p_1 only if S and R are chosen as a transmit-receive pair. The probability that S and R are chosen as a transmit-receive pair depends on the number of other nodes in the same cell. If S and R are not chosen, in spite of being in the same cell, the likelihood of there being many more nodes in the same cell increases. Due to the random walk model of the node mobility, if there is a crowding of nodes in some part of the network, it remains crowded for some time in the future. Hence due to the Markovian nature of node mobility, inter-arrival times in the relay-queue are not independent.

Consider a queue, $\mathcal{Q}_1$, in which there is an arrival with probability p_1 whenever S and R meet, irrespective of whether S and R are chosen as a transmit-receive pair. The inter-arrival times of this queue are then stochastically dominated by the inter-arrival times of the relay-queue. Let the potential-departure process of $\mathcal{Q}_1$ be the same as that of the relay-queue. Then the delay of $\mathcal{Q}_1$ provides an upper bound on the delay of the relay queue. It is easy to see that the sequence of inter-arrival times of $\mathcal{Q}_1$ is i.i.d. and that the common distribution is that of the sum of G independent copies of τ, where $G \sim \text{Geom}(p_1)$. Recall that τ is the inter-meeting time of S and R.

Let the potential-departure process of the relay-queue (and also $\mathcal{Q}_1$) be denoted by $\{S_i\}$. Next we will study this process in order to replace it with another coupled process with less frequent potential departures. Recall that the potential departure time-slots are the ones in which a packet can be emptied from the queue if the queue is not empty. These are the times when R and D are in the same cell and are chosen as a transmit-receive pair. Let $\alpha_0, \alpha_1, \ldots$ be the time-slots in which R and D are in the same cell and let $E_i, i = 0, 1, \ldots$, be the indicator of the event that α_i is also a potential service instant. That is, E_i is an indicator for the event that R and D are chosen as a transmit-receive pair at time α_i.

Let α^i denote $\{\alpha_0, \ldots, \alpha_i\}$ and E^i denote $\{E_0, \ldots, E_i\}$. Then $P(E_i = 1|\alpha^i, E^{i-1}), i = 0, 1, \ldots$ is the probability that $E_i = 1$ given $\alpha_0, \ldots, \alpha_i$ and $E_0, \ldots, E_{i-1}$. This is the probability that R and D are chosen as a transmit-receive pair given that they are in the same cell in time-slot α_i and given the entire past consisting of $\alpha_0, \ldots, \alpha_i$ and $E_0, \ldots, E_{i-1}$. From the description of Scheme 2 it is clear that the probability of the event $\{E_i = 1\}$ depends on the number of other nodes in the cell containing R and D. This, in turn, depends on the entire past of the processes $\{E_i\}$ and $\{\alpha_i\}$ due to the Markovian nature of node mobility.

Thus the potential-departure process is generated by the processes $\{\alpha_i\}$ and $\{E_i\}$. This is because time-slot α_i, when R and D are in the same cell for the ith time, is chosen as a potential departure instant with probability $P(E_i = 1|\alpha^i, E^{i-1})$.

Due to the dependence on the past of $\{E_i\}$, the inter-potential-departure times are also dependent. We will next show that this dependence is not too much, in the sense that irrespective of the past, the probability of the event $\{E_i = 1\}$ is greater than a positive constant that does not depend on n. The following lemma is proved in Subsection5.4.

Lemma 5.2. *There exists a constant, c_6, (independent of n) such that for all large enough n,*

$$P\left(E_i = 1|\alpha^i, E^{i-1}\right) \geq c_6 > 0.$$

Now let $\mathcal{Q}_2$ be a queue such that each time-slot in which R and D meet is chosen to be a potential departure instant with probability c_6. Then by Lemma 5.2, the inter-potential-departure times for this queue would be stochastically dominated by those for $\mathcal{Q}_1$. If $\mathcal{Q}_2$ has the same arrival process as $\mathcal{Q}_1$ then the delay of $\mathcal{Q}_2$ is an upper bound on that of $\mathcal{Q}_1$.

As before, the sequence of inter-potential-departure times of $\mathcal{Q}_2$ is i.i.d. and the common distribution is that of the sum of G independent copies of τ, where $G \sim \text{Geom}(c_6)$. As a result, we have upper bounded the delay of the relay-queue by the delay of $\mathcal{Q}_2$, which has i.i.d. inter-arrival times and i.i.d. inter-potential-departure times. To obtain an upper bound on the delay, we only use the first two moments of the inter-arrival time and the inter-potential-departure time. Since both of these are sums of a Geometric number of independent inter-meeting times, it is easy to check that their moments are of the same order as that of the inter-meeting time. As a result, the constants p_1 and c_6 do not affect the delay scaling. Further, $\mathcal{Q}_2$ is stable as long as the arrival rate is less than the service rate, i.e., $p_1 < c_6$. Since we are interested in determining the delay scaling, for simplicity, we assume that in $\mathcal{Q}_2$, an arrival occurs whenever S and R meet with probability 0.5 and a potential departure occurs whenever R and D meet.

Now we bound the delay of $\mathcal{Q}_2$ by the sum of the delays through two queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, in tandem. Both $\mathcal{Q}_3$ and $\mathcal{Q}_4$ will be shown to have delay of $O(n \log n)$, which implies that the delay of $\mathcal{Q}_2$ is $O(n \log n)$. Queues $\mathcal{Q}_3$ and $\mathcal{Q}_4$ are constructed as follows. The arrival process of $\mathcal{Q}_3$ is the same as that of $\mathcal{Q}_2$. The potential-departure process of $\mathcal{Q}_3$ is an i.i.d. Bernoulli process with parameter $2/3n$ (or potential departure rate $\frac{2}{3n}$). An arrival occurs at $\mathcal{Q}_4$ whenever there is a potential-departure at $\mathcal{Q}_3$. If $\mathcal{Q}_3$ is non-empty, then the arrival to $\mathcal{Q}_4$ is the head-of-line packet transferred from $\mathcal{Q}_3$ to $\mathcal{Q}_4$ or else a dummy packet is fed to $\mathcal{Q}_4$. Thus the arrival process at $\mathcal{Q}_4$ is the same as the potential-service process at $\mathcal{Q}_3$. By construction, the delay of a packet through this tandem of queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, upper bounds the delay experienced by a packet through $\mathcal{Q}_2$. Now from Lemmas 5.3 and 5.4 stated below and proved in Subsection 5.4, both $\mathcal{Q}_3$ and $\mathcal{Q}_4$ have an expected delay of $O(n \log n)$.

Lemma 5.3. *The expected delay of a packet through $\mathcal{Q}_3$ is $O(n \log n)$.*

Lemma 5.4. *The expected delay of a packet through $\mathcal{Q}_4$ is $O(n \log n)$.*

Hence the expected delay of the packets of each S-D pair relayed through each relay R is $O(n \log n)$. The delay of a scheme is the expectation of the packet delay averaged over all S-D pairs and all relay nodes. Hence it follows that the delay of Scheme 2 is $O(n \log n)$.
Lower bound: We now establish a lower bound on the delay of Scheme 2. Consider a packet arrival at the relay node when it is in cell (i, j). Let the destination be in cell (k, l), which is equally likely to be any one of the n cells since the destination performs an independent random walk. Using the difference random walk, the delay is at least equal to the time required for the random walk to reach state (k, l) starting from state (i, j). Hence the expected value of the delay can be lower bounded as

$$\begin{aligned} E[D] &\geq \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \pi(i,j)\pi(k,l) E_{(i,j)} T_{(k,l)} \\ &= \Theta(n \log n), \end{aligned} \tag{5.3}$$

where (5.3) is from p. 11 in Chapter 5 of [1].

Combining this lower bound with the earlier upper bound leads to the following theorem.

Theorem 5.4. *The delay of Scheme 2 is*

$$D(n) = \Theta\left(n \log n\right).$$

5.3 Throughput-delay trade-off in mobile networks

In this section we establish the optimal throughput-delay trade-off for mobile random networks under the random walk (RW) model for node mobility. To achieve this trade-off, we introduce Scheme 3. The scheme is divided into two parts based on the range of throughput handled by it. Scheme 3(a) is for $T(n) = O(1/\sqrt{n \log n})$, while Scheme 3(b) is for $T(n) = \omega(1/\sqrt{n \log n})$. These ranges are dealt with separately since the schemes achieving the optimal trade-off in these ranges are fundamentally different. In the low throughput range, the optimal scheme cannot use the mobility of nodes to communicate packets. In fact, Scheme 3(a) is significantly more complex than Scheme 1. Even though it provides the same trade-off, it needs to overcome difficulties created by node mobility. On the other hand, in the high throughput range, it is essential to use the mobility of nodes to communicate packets.

Both these schemes divide the network into square cells of area $a(n)$, which is a parameter that determines the point of trade-off. We would like to note that these cells are a part of the scheme and are unrelated to the n cells used in the definition of the random walk mobility model.

5.3.1 Trade-off for low throughput

Scheme 3(a) described below requires the packet size to scale down as $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, and similar to Scheme 1, it is a cellular TDM scheme. Due to the mobility of the nodes, the packets need to chase their destination nodes, which makes the scheme and its analysis significantly more complex. For the sake of our proof technique, the scheme drops a packet that is unsuccessful in chasing down its destination for long. A more precise description of the scheme follows.

Scheme 3(a): Mobile networks at low throughput

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1. Divide the unit torus into square cells, each of area $a(n)$ (see Figure 2.2).

2. A cellular TDMA transmission scheme is used, in which, each cell becomes active at regularly scheduled cell time-slots (see Figure 2.3). From Lemma 2.2, each cell gets a chance to be active once every $1 + c_1$ cell time-slots.

3. A packet is sent from its source S to its destination D by chasing the destination for at most $k(n) = \Theta(\log \log n)$ stages as follows:

(a) Consider a packet that is generated at S, when S is in cell C^0. Let D be in cell C^1 at that time.

(b) Set $k = 1$.

(c) In stage k, the packet is sent from cell C^{k-1} to cell C^k via hops along adjacent cells on the line joining centers of cells C^{k-1} and C^k.

(d) When the packet reaches cell C^k, assume that the destination D is in the cell C^{k+1}. If $C^{k+1} = C^k$ then the packet is delivered to D. Otherwise, set $k = k + 1$.

(e) If $k < k(n)$, repeat (c)-(d), else drop the packet.

4. In every active time-slot of a cell, each of the source nodes residing in the cell at that time generates a new packet for its respective destination. These new packets are transmitted in the same time-slot.

5. Packet size scales as $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ and hence in an active time-slot a cell can transmit $\Theta\left(n\sqrt{a(n)}\right)$ packets. If at any time instant a cell has more packets than it can transmit then the excess packets are dropped.

6. Packets transmitted to a cell not containing any node are dropped.

In the above scheme packets are dropped if one of the following three scenarios occurs.

(i) Empty cells: Due to mobility of nodes, it is possible that a cell may be empty in some time-slot. In part b) of step 3) above, a packet may be lost if it is transmitted to any empty cell.

(ii) Overloading of cells: In the case of Scheme 1 we could provide a guarantee *whp* on the maximum number of S-D lines passing through each cell. In this case, due to mobility, the number of S-D lines passing through a cell may exceed its capacity of $\Theta\left(n\sqrt{a(n)}\right)$ packets. If this occurs, the excess packets are dropped as mentioned in step 5) above.

(iii) Unsuccessful chasing: A packet that does not reach its destination after $k(n)$ stages of chasing is also dropped.

The following theorem shows that the fraction of packets dropped is negligible, i.e., goes to 0 as $n \to \infty$. We assume that error correction is employed to combat this packet loss, however this requires only a constant fraction of the total throughput and hence does not affect the throughput scaling. Thus, in spite of node mobility, Scheme 3(a) achieves the same throughput-delay trade-off as Scheme 1 for static networks.

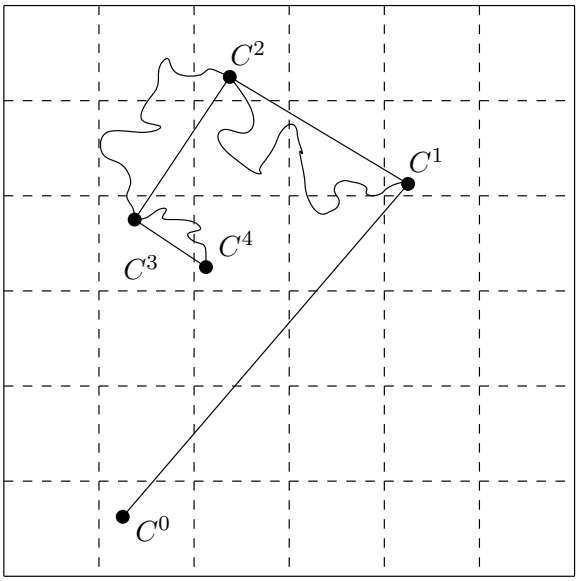

Figure 5.3: In scheme 3(a), the unit torus is divided into square cells of area $a(n)$. In stage 1 of chasing, a packet starts from cell C^0 containing its source, and moves by hops along adjacent cells towards cell C^1 which contains its destination node at that time. By the time it reaches cell C^1, its destination has moved to cell C^2. So in stage 2 of chasing, the packet hops from cell C^2 to cell C^3. And this continues for at most $k(n)$ stages or until the packet reaches the cell containing its destination.

Theorem 5.5. *Scheme 3(a) achieves the throughput-delay trade-off given by:*

$$T(n) = \Theta\left(\frac{D(n)}{n}\right), \quad \textit{for} \;\; T(n) = O\left(\frac{1}{\sqrt{n \log n}}\right).$$

We first analyze the throughput of Scheme 3(a). The delay of a packet, as per the description of the scheme, is $(1 + c_1)$ times the number of hops taken by a packet. In the process of determining the throughput, we shall determine the average number of hops per packet during the $k(n)$ stages. This will allow us to determine the average packet delay.

As explained in the description of Scheme 3(a) above, each source generates a packet of size $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ each time the cell it belongs to become active. Each cell becomes active once every $1 + c_1$ time-slots, according to Lemma 2.2. Hence, independent random walk of nodes under the RW model implies that each node is in an active cell for a constant fraction $1/(1+c_1)$ of the time *w.p. 1* due to ergodicity. That is, each source node generates traffic at rate $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ under Scheme 3(a). Thus, to show that each S-D pair achieves throughput $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, it is sufficient to show that the fraction of the packets dropped under Scheme 3(a) goes to 0 as $n \to \infty$.

To show that the fraction of dropped packets goes to 0 as $n \to \infty$, we need to bound the total traffic generated by all $k(n)$ stages. The total traffic due to all $k(n)$ stages is the number of packets that a cell is required to transmit in a time-slot. We analyze this in the following three lemmas by utilizing arguments similar to that in the proof of Lemma 2.3. See Subsection 5.4 for proofs of Lemmas 5.5 and 5.6. For simplicity of analysis, we assume that each cell becomes active every time-slot instead of $1+c_1$ time-slots. This simplification does not change the results in the order notation.

Lemma 5.5. *The number of packets of stage* 1 *passing through each cell in a time-slot is* $O\left(n\sqrt{a(n)}\right)$ *with probability at least* $1 - 1/n^3$.

Lemma 5.6. *For* $k \geq 2$, *the number of packets of stage* k *passing through each cell in a time-slot is* $O\left(\left(\frac{n^2}{m}\right)^{2^{-k}}\right)$ *with probability at least* $1 - 1/n^3$.

Lemma 5.7. *The number of packets passing through each cell in a time-slot is* $O\left(n\sqrt{a(n)}\right)$ *with probability at least* $1 - 1/n^{2.9}$.

Proof. There are $k(n) = O(\log \log n)$ stages. Using the union bound over $k(n)$ stages and the bounds given by Lemma 5.5 and Lemma 5.6, we can show that the total number of

packets (due to all $k(n)$ stages), $P(n)$, passing through each cell is

$$P(n) \;=\; O(n\sqrt{a(n)}) + \sum_{k=2}^{k(n)} O\left(\left(\frac{n^2}{m}\right)^{2^{-k}}\right), \tag{5.4}$$

with probability at least $1 - k(n)/n^3 \geq 1 - 1/n^{2.9}$.

Next, we evaluate the summation on the right hand side of (5.4). Consider the largest index k such that

$$\left(\frac{n^2}{m}\right)^{2^{-k+1}} \geq 2\left(\frac{n^2}{m}\right)^{2^{-k}}. \tag{5.5}$$

This gives

$$\left(\frac{n^2}{m}\right)^{2^{-k}} \geq 2. \tag{5.6}$$

That is,

$$k \leq \frac{\log\log n^2/m - \log\log 2}{\log 2}.$$

Note that the parameter $k(n)$ in Scheme 3(a) is chosen to satisfy this condition. Moreover for $k < k(n)$, the ratio of consecutive terms,

$$\frac{\left(\frac{n^2}{m}\right)^{2^{-k+1}}}{\left(\frac{n^2}{m}\right)^{2^{-k}}} = \left(\frac{n^2}{m}\right)^{2^{-k}} \geq 2,$$

due to (5.6). As a consequence,

$$\begin{aligned}
\sum_{k=2}^{k(n)} \left(\frac{n^2}{m}\right)^{2^{-k}} &\leq \sum_{k=2}^{k(n)} \left(\frac{n^2}{m}\right)^{2^{-2}} 2^{-k+2} \\
&= \frac{\sqrt{n}}{m^{1/4}} \sum_{k=0}^{k(n)-2} 2^{-k} \\
&\leq \frac{2\sqrt{n}}{m^{1/4}}.
\end{aligned} \tag{5.7}$$

Replacing (5.7) in (5.4), we obtain that, for large enough n, with probability at least $1 - 1/n^{2.9}$,

$$P(n) \;=\; O(n\sqrt{a(n)}) + O\left(\frac{\sqrt{n}}{m^{1/4}}\right). \tag{5.8}$$

Now, $a(n) = \Omega(\log n/n)$. Hence, $n\sqrt{a(n)} = \Omega\left(\sqrt{n \log n}\right)$. Since $m = \Omega(1)$, this will imply that the right hand side of (5.8) is $O(n\sqrt{a(n)})$. □

Remark: The packet size is of order $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ and the associated constant in this Θ notation is chosen such that the total data due to all the packets can be supported in one time-slot.

Proof of Theorem 5.5: We show that the fraction of packets dropped due to (i) overloading of cells, (ii) unsuccessful chasing, and (iii) empty cells, goes to 0 as $n \to \infty$. This will immediately imply that the throughput of Scheme 3(a) is as claimed in Theorem 5.5.

Packets dropped due to overloading of cells: Since the number of hops in each stage is $O\left(1/\sqrt{a(n)}\right)$ and there are at most $\Theta(\log \log n)$ stages, the total number of packets in the network is $O\left(n \log \log n/\sqrt{a(n)}\right)$. Thus the process describing the number of packets in each of the $1/a(n)$ cells is a finite-state Markov chain that is induced by the underlying Markov chain due to the random walk of nodes on the $\sqrt{n} \times \sqrt{n}$ discrete torus as specified by the RW model. This is aperiodic and hence an ergodic Markov chain. Hence by choosing packets of size $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, the fraction of time when the the number of packets through any cell exceeds its capacity is $p_n \leq 1/n^{2.9}$. Now consider a long time duration t in which $nT(n)t$ bits corresponding to nt packets are sent from the sources to their destinations. Of these, at most $O\left(n \log \log n p_n t/\sqrt{a(n)}\right)$ is dropped due to overloading in cells since the total number of packets in the network is $O\left(n \log \log n/\sqrt{a(n)}\right)$. Hence if $p_n = o\left(\log \log n\sqrt{a(n)}\right)$, the fraction of packets dropped approaches 0 as $n \to \infty$, which is indeed the case here.

Packets dropped due to unsuccessful chasing: We need to determine the fraction of packets dropped due to the destination moving away from its initial position in the last stage, i.e., stage $k(n)$. Recall that the choice of $k(n)$ is such that it is the largest index with property (5.5). Hence

$$\left(\frac{n^2}{m}\right)^{2^{-k(n)}} \geq 2\left(\frac{n^2}{m}\right)^{2^{-k(n)-1}},$$

which in turn yields

$$m^{2^{-k(n)}} \leq 4. \tag{5.9}$$

Using (5.9) in (5.55), we obtain

$$l_{k(n)} = O\left(\frac{m}{n}\right).$$

Now using the fact that $m = O\left(n/\log n\right)$, it follows that $l_{k(n)} \to 0$ as $n \to \infty$. On the other hand, for our motion model where the nodes move according to a two-dimensional random

walk on the $\sqrt{n} \times \sqrt{n}$ discrete torus and cell size of $\Omega(\log n/n)$, the average time taken by a node to move out of a cell is $\Omega(\log n)$. Hence the probability that a packet is dropped due to unsuccessful chasing tends to 0 as $n \to \infty$.

Packets dropped due to empty cells: Finally in any time-slot, the packets that need to be relayed through cells that do not contain any nodes are lost. Now consider a fixed time-slot. Each node is equally likely to be in any of the cells, thus by Lemma 2.1, the probability that any cell is empty is $\leq 1/n^2$. Again, using ergodicity and the bound on the number of packets transmitted per time-slot, which is $O\left(n\sqrt{a(n)}\right)$, the fraction of packets dropped due to cells being empty goes to 0 as $n \to \infty$.

Thus, the net fraction of packets dropped due to (i) overloading in cells, (ii) empty cells and (iii) unsuccessful chasing goes to 0 as $n \to \infty$. In other words, almost all the packets that are generated in Scheme 3(a) reach their destination successfully. As noted before, the number of packets generated by each source per unit time is $\Theta(1)$, and since each packet is of size $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, the net throughput per S-D pair is $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$. This completes the proof of the achievability of throughput as claimed in Theorem 5.5.

Average Delay: Next we compute the average delay of packets. Under Scheme 3(a), the average delay of a packet in stage k is l_k as it makes l_k hops on average in the k^{th} stage. Hence from (5.55), we obtain that the average delay, $D(n)$, is

$$\begin{aligned} D(n) &= \sum_{k=1}^{k(n)} l_k \\ &\leq \sqrt{c_5 m} + \sum_{k=2}^{k(n)} \frac{c_5 m}{n} \left(\frac{n^2}{m}\right)^{2^{-k}}. \end{aligned} \tag{5.10}$$

Using (5.7) in (5.10), for large enough n, we obtain

$$\begin{aligned} D(n) &\leq \sqrt{c_5 m} + \frac{2c_5 m\sqrt{n}}{nm^{1/4}} \\ &\leq c_5\sqrt{m} + 2c_5\sqrt{m}\sqrt{\frac{\sqrt{m}}{n}} \\ &= O\left(\sqrt{m}\right), \end{aligned} \tag{5.11}$$

where (5.11) holds because $m = O(n/\log n) = o(n)$. Thus as claimed the average packet delay $D(n) = O\left(\sqrt{m}\right)$. This completes the proof of Theorem 5.5.

■

5.3.2 Trade-off for high throughput

To obtain a throughput higher than $\Theta\left(1/\sqrt{n \log n}\right)$, we need to use mobility, and to keep the delay as low as possible, we need to use multiple hops cleverly. This naturally leads to a scheme that combines Scheme 3(a) (which provides low throughput with low delay) and Scheme 2 (which provides high throughput with high delay). A first approach would be to *time-share* between Scheme 3(a) and Scheme 2. It is easy to show that such a scheme can achieve any throughput in the range from $\Theta\left(1/\sqrt{n \log n}\right)$ to $\Theta(1)$, but the average delay remains fixed at $\Theta(n \log n)$. By using Scheme 3(b), which is a careful combination of Scheme 2 and Scheme 3(a), the throughput-delay trade-off can be slightly improved. And it turns out that the performance of Scheme 3(b) is optimal.

Scheme 3(b) uses the chasing technique of Scheme 3(a) by using hopping along adjacent cells of size $\Theta(\log n/n)$ as the underlying packet transport mechanism. However this chasing is not done all the way from each source to its destination. Instead, the mobility of an intermediate mobile relay node is employed as in Scheme 2 to increase the throughput. The chasing technique of Scheme 3(a) is used only to send packets from a source to a mobile relay node or from a mobile relay node to a destination when they are sufficiently close. Nodes get sufficiently close to each other due to their mobility and the amount of closeness is captured by the parameter $b(n)$ of the scheme, which determines the trade-off point.

In the description of the scheme and the proof we refer to two ways of measuring distance between two nodes.

1. The *step-distance* or distance in terms of steps between two nodes with positions (i_1, j_1) and (i_2, j_2) on the $\sqrt{n} \times \sqrt{n}$ discrete torus is $|i_2 - i_1| + |j_2 - j_1|$, where the subtraction is modulo $\sqrt{n}$. Thus this distance is simply the Manhattan distance on the underlying discrete torus.

2. The *hop-distance* or distance in terms of hops, which is the number of hops a packet would take along adjacent cells of the straight line joining the nodes to reach from one node to the other. The cells in this case are determined by the scheme and are always of area of $\Omega(\log n/n)$ in Scheme 3(b).

Scheme 3(b): Mobile networks at high throughput

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1. Divide the unit torus into square cells each of area $a(n) = \Theta(\log n/n)$ as in Figure 5.4. The scheme uses a parameter $b(n)$ that determines the point of trade-off. Let $l(n) = c_6\sqrt{nb(n)}$ and $c_0(n) = c_7\sqrt{nb(n)}$.

2. A cellular TDMA transmission scheme is used, in which, each cell becomes active at regularly scheduled cell time-slots. By Lemma 2.2, these active time-slots are at most $1 + c_1$ time-slots apart.

3. Each node in the network maintains a separate FIFO queue for each of the $n/2$ S-D pairs in the network.

4. Each time-slot is divided into two equal subslots, A and B. The network operates in two phases – SR (source to relay) phase in the A subslots and RD (relay to destination) phase in the B subslots.

5. SR phase: (Source-to-Relay in the A subslots)

   (a) For the SR phase, each source node maintains a counter and a state variable for every other node. Let S_{ij}^{SR} be the value of the state variable and C_{ij}^{SR} be the value of the counter at source node i for some mobile node j. The state variable S_{ij}^{SR} is binary valued and is used to determine whether node i should send packets to node j or not. Each counter is initially at count 0 and is operated as follows:

      i. If $C_{ij}^{\mathrm{SR}} = 0$ and the step-distance between nodes i and j is greater than $l(n)$ then set $C_{ij}^{\mathrm{SR}} = -1$.

      ii. If $C_{ij}^{\mathrm{SR}} = -1$ and the step-distance between nodes i and j is no greater than $l(n)$ then set $C_{ij}^{\mathrm{SR}} = c_0(n)$ and with probability $p_0, 0 < p_0 < 1$ set $S_{ij}^{\mathrm{SR}} = 1$ otherwise reset to $S_{ij}^{\mathrm{SR}} = 0$.

      iii. In each A subslot of the SR phase, the counter decrements by one until it reaches 0.

   (b) In the SR phase when a cell becomes active, every source node i in the cell sends a packet intended for its destination to every other node j in the network for which $C_{ij}^{\mathrm{SR}} > 0$ and $S_{ij}^{\mathrm{SR}} = 1$. These nodes act as relay nodes for this source node. These packets reach these relay nodes using the transport mechanism of Scheme 3(a) during the A subslots of the SR phase.

6. RD phase: (Relay-to-Destination in the B subslots)

   (a) For the RD phase, each mobile node i maintains a counter for every destination node j denoted by C_{ij}^{RD}. They are initially set to 0 and operated in the same way as the counters for the SR phase as follows:

That is, each C_{ij}^{RD} is set to 0,-1 or $c_0(n)$ based on the step-distance between nodes i and j and the previous value of the counter. Each counter is initially at count 0 and is operated as follows:

i. If $C_{ij}^{\mathrm{RD}} = 0$ and the step-distance between nodes i and j is greater than $l(n)$, set $C_{ij}^{\mathrm{RD}} = -1$.

ii. If $C_{ij}^{\mathrm{RD}} = -1$ and the step-distance between nodes i and j is no greater than $l(n)$, set $C_{ij}^{\mathrm{RD}} = c_0(n)$.

iii. In each B subslot of the SR phase, the counter decrements by one until it reaches 0.

(b) In the RD phase when a cell becomes active, every node i in the cell sends a packet to every other destination node j in the network, for which $C_{ij}^{\mathrm{RD}} > 0$, if it has a packet intended for that destination node. That is, if the FIFO queue corresponding to a destination is not empty, then a packet for that destination is emptied out of the queue. These packets reach their respective destinations using the transport mechanism of Scheme 3(a) during the B subslots constituting the RD phase.

7. Packet size scales as $\Theta\left(1/\sqrt{n^3 b(n)^3 \log n}\right)$ and hence in an active time-slot, a cell can transmit $\Theta\left(\sqrt{n^3 b(n)^3 \log n}\right)$ packets. If in any time-slot, a cell has more packets than it can transmit then the excess packets are dropped.

8. Packets transmitted to a cell not containing any node are dropped.

As shown in Figure 5.4, in Scheme 3(b), a source node S sends a packet intended for its destination to a mobile relay node R, which is no farther than $l(n)$ hops initially. It continues to do so for $c_0(n)$ time slots during which it is improbable that S and R get too far (i.e. farther than $\Theta(l(n))$ step-distance) from each other due to the random walk mobility model. These packets are sent using the chasing strategy of Scheme 3(a) by hops along adjacent cells of size $\Theta(\log n/n)$. This mobile relay R, in turn, sends the packet to the destination D when R and D get sufficiently close. Sending with probability $p_0 < 1$ ensures that arrival rate to each relay node is less than the service rate so that the queues at the mobile relay nodes are stable.

Theorem 5.6. *Scheme 3(b) provides the following throughput and delay.*

(i)

$$T(n) = \Theta\left(\frac{1}{\sqrt{nb(n)\log n}}\right),$$

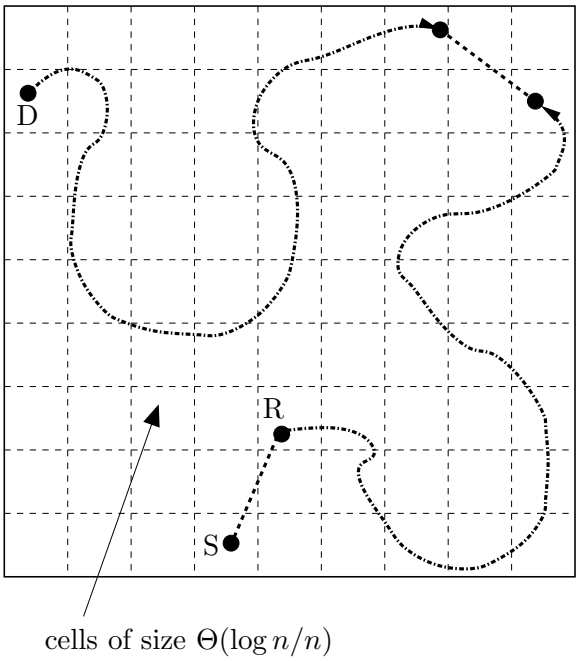


Figure 5.4: Scheme 3(b) is a cellular TDM scheme like Scheme 3(a). All other mobile nodes act as relays for each S-D pair as in Scheme 2. Sources send packets to relays and relays send packets to destinations only when they are sufficiently close using the chasing technique of Scheme 3(a).

(ii)

$$D(n) = O\left(n \log\left(\frac{1}{b(n)}\right)\right),$$

where the parameter satisfies $b(n) = \Omega(\log n/n)$ *and* $b(n) = O(1)$*. That is, the achievable throughput-delay trade-off for Scheme 3(b) is*

$$T(n) = \Theta\left(\frac{D(n)}{n}\right).$$

We first analyze the throughput of this scheme and then its delay. The throughput analysis requires extensions of the techniques used in the proof of Theorem 5.5.

Proof of Theorem 5.6(i): The throughput analysis involves showing that (i) if no packets are dropped then the throughput is as claimed, and that (ii) the fraction of packets dropped by the underlying packet transport mechanism of Scheme 3(a) is negligible.

First note that the traffic in the SR and RD phases are similar and hence it is sufficient to analyze just the SR phase. Let $C_{ij}(t)$ denote the value of the counter at source node i for some mobile node j at time t. Let $X_{ij}(t)$ be the indicator of the event that $C_{ij}(t)$ is positive, i.e., $X_{ij}(t) = 1$ if $C_{ij}(t) > 0$ and zero otherwise. For simplicity, assume that each cell becomes active every time-slot (instead of once every $1 + c_1$ time slots) and whenever $X_{ij}(t) = 1$, source i transmits a new packet to a relay j (instead of also considering the state variable $S_{ij}(t)$ which is positive with probability p_0). It is easy to see that these assumptions do not affect the results in the order notation.

Now let $\{\tilde{T}_k\}$ be the intervals between consecutive transitions of $X_{ij}(t)$ from 0 to 1 (see Figure 5.5). Then $\{\tilde{T}_k\}$ is an i.i.d. sequence due to the independent random walks of the nodes. Let $\tilde{T}$ be a random variable with the common distribution of these random variables. As shown in Lemma 5.8

$$E[\tilde{T}] = \Theta\left(\sqrt{n/b(n)}\right).$$

The random walk of nodes i and j on the discrete torus is a stationary and ergodic process and hence the process $X_{ij}(t)$ derived from it is also stationary and ergodic. Hence *w.p. 1*,

$$\lim_{s\to\infty} \sum_{t=1}^{s} X_{ij}(t)/s = E[X_{ij}(t)].$$

The quantity above is the rate at which packets are sent from source node i to relay node j in t time-slots.

Now each time $X_{ij}(t)$ makes a transition from 0 to 1, it remains at 1 for $c_0(n)$ time-slots. Hence using the random variable version of the elementary renewal theorem (see [44]), it is

easy to see that *w.p. 1*,

$$\begin{aligned}\lim_{s\to\infty}\sum_{t=1}^{s} X_{ij}(t)/s &= c_0(n)/E[\tilde{T}] \\ &= \Theta\left(\sqrt{nb(n)}\frac{1}{\sqrt{n/b(n)}}\right) \\ &= \Theta(b(n)). \end{aligned} \tag{5.12}$$

Hence we have

$$P\{X_{ij}(t)=1\} = E[X_{ij}(t)] = \Theta(b(n)). \tag{5.13}$$

Each source node uses the other $n-2$ nodes as relays. By considering all these $n-2$ relays and the direct path, it follows from (5.12) that if no packets are dropped then the throughput for each S-D pair in terms of packets is

$$\lim_{s\to\infty}\sum_{t=1}^{s}\sum_{j\neq i} X_{ij}(t)/s = \Theta(nb(n)).$$

Now since the packet size is $\Theta\left(1/\left(n^{3/2}b(n)^{3/2}\sqrt{\log n}\right)\right)$ it is immediate that if no packets are dropped then the throughput per S-D pair is $\Theta\left(1/\sqrt{nb(n)\log n}\right)$ as claimed. The rest of the proof shows that the fraction of packets dropped goes to zero. In Scheme 3(b), as in Scheme 3(a), packets are dropped due to (i) overloading in cells, (ii) transmission to empty cells, and (iii) unsuccessful chasing. Scheme 3(b) uses cells of size $\Theta(\log n/n)$ and hence as shown in the proof of Scheme 3(a), the fraction of packets being dropped due to (ii) and (iii) goes to 0. To establish (i), it is sufficient to show that the number of packets passing through each cell is $O\left(\sqrt{n^3b(n)^3\log n}\right)$ *whp*, which we do next.

Next let $D_{ij}(t)$ be the distance in steps from node i to node j. If $X_{ij}(t)$ makes a transition from 0 to 1 at time τ then we know that $D_{ij}(\tau) = l(n)$. After this $X_{ij}(t)$ stays at 1 for $c_0(n)$ time-slots. Since nodes i and j are moving according to independent random walks, the distance between them increases by at most two steps in each time-slot. Therefore, $D_{ij}(t) \leq l(n) + c_0(n) = O(l(n))$ for $\tau \leq t \leq \tau + c_0(n)$. Thus we have

$$D_{ij}(t) = O(l(n)) \quad \text{given} \quad X_{ij}(t) = 1. \tag{5.14}$$

Now let $L_{ij}(t)$ be the distance in hops between nodes i and j if $X_{ij}(t) = 1$ and 0 otherwise.

Then $L_{ij}(t) = O(D_{ij}/\sqrt{\log n}))$ since the cells are of size $\log n/n$ and so

$$\begin{aligned} & E[L_{ij}(t)] \\ &= E[L_{ij}(t)|X_{ij}(t)=1]P\{X_{ij}(t)=1\} \\ &= E[L_{ij}(t)|X_{ij}(t)=1]\Theta(b(n)) && (5.15) \\ &= O(E[D_{ij}(t)|X_{ij}(t)=1]b(n)/\sqrt{\log n}) && (5.16) \\ &= O(l(n)b(n)/\sqrt{\log n}) \\ &= O(b(n)^{3/2}\sqrt{n/\log n}), && (5.17) \end{aligned}$$

where (5.15) is due to (5.13) and (5.16) follows from (5.14).

For exactly the same reasons as those pointed out while analyzing the traffic of Scheme 3(a), the traffic in stage 1 of Scheme 3(b) dominates the traffic of all other stages. Hence we will only analyze traffic of stage 1 here. Now let $Y_{ij}^c(t)$ be the number of packets in stage 1 of source i to node j passing through cell c at time t. As shown during the analysis of traffic in stage 1 of Scheme 3(a), we have $Y_{ij}^c(t) \in \{0,1,2\}$.

Now a packet sent out from source i at time $t-k$ passes through some cell in the network at time t if and only if $L_{ij}(t-k) > k$. There are $m = n/\log n$ cells in the network and summing over all these cells we obtain

$$\sum_{c=1}^{m} Y_{ij}^c(t) = \sum_{k=0}^{\infty} I\{L_{ij}(t-k) > k\},$$

where $I(A)$ is the indicator function of the event A. Taking expectations on both sides, using the symmetry of the cells and 5.17, this gives

$$\begin{aligned} mE[Y_{ij}^c(t)] &= \sum_{k=0}^{\infty} P\{L_{ij}(t-k) > k\} \\ &= \sum_{k=0}^{\infty} P\{L_{ij}(t) \geq k\} \\ &= E[L_{ij}(t)] \\ &= O(b(n)^{3/2}\sqrt{n/\log n}). \end{aligned}$$

Hence we have

$$E[Y_{ij}^c(t)] = O\left(b(n)^{3/2}\sqrt{\log n/n}\right). \quad (5.18)$$

The total number of packets passing through cell c in time-slot t is given by $Y =$

$\sum_i \sum_{j \neq i} Y_{ij}^c(t)$. The total number of distinct source-relay pairs is $\Theta(n^2)$. For each such pair, by symmetry and (5.18), $E[Y_{ij}^c(t)] = O\left(b(n)^{3/2}\sqrt{\log n/n}\right)$. Hence the expected number of packets passing through cell c in time-slot t is

$$E[Y] = \Theta\left(n^2 E[Y_{SR}^C(t)]\right) = O\left(\sqrt{n^3 b(n)^3 \log n}\right). \tag{5.19}$$

Thus it is sufficient to show that $Y = \Theta\left(E[Y^C(t)]\right)$ *whp* in order to establish that the number of packets through each cell is $O\left(\sqrt{n^3 b(n)^3 \log n}\right)$ *whp*. Next, we establish this.

Note that terms of the form $Y_{ii}^c(t)$ are always 0 and terms of the form $Y_{ij}^c(t)$ and $Y_{ik}^c(t)$ for $j \neq k$ are independent. So are terms of the form $Y_{ij}^c(t)$ and $Y_{kj}^c(t)$ and terms where all indices are different. Another important property which we use crucially is that if nodes j and k are not in cell c and $Y_{ij}^c = Y_{ik}^c = 1$ then $Y_{jk}^c(t)$ is necessarily 0. This happens because $Y_{ij}^c(t) = Y_{ik}^c(t) = 1$ implies that nodes j and k lie on the same side of cell c under consideration and hence the line connecting j and k does not pass through cell c. The same holds when $Y_{ji}^c = Y_{ki}^c = 1$. This is not true however if either of the nodes j and k lies in cell c. However this can be handled by dealing with the first and last hops separately which is easy since it depends only on the distribution of nodes themselves. Hence we ignore this aspect for the sake of simplicity.

Now fix a cell c and a time-slot t and define $Z_{ij} = 1$ if $Y_{ij}^c(t) > 0$ and 0 otherwise. Then $Y_{ij}(t) \leq 2Z_{ij}$ and hence

$$\begin{aligned} Y &\leq 2\sum_{i=1}^{n/2}\sum_{j=1}^{n} Z_{ij} \\ &= \sum_{i=1}^{n/2}\sum_{j=i+1}^{n} Z_{ij} + \sum_{i=2}^{n/2}\sum_{j=1}^{i-1} Z_{ij}. \end{aligned}$$

In what follows, we will show that the first sum above is of the same order as its expected value *whp*. The same technique would show that the same is true for the second sum also *whp* and hence that the total traffic Y is of the same order as its expected value *whp*.

So consider the $M = \Theta(n^2)$ terms in the first sum, i.e., $\{Z_{ij}, 1 \leq i \leq n/2, i < j \leq n\}$. These are identically distributed Bernoulli random variables with $p_n = P\{Z_{ij} = 1\} = O\left(b(n)^{3/2}\sqrt{\log n/n}\right)$. The dependence properties of $\{Y_{ij}(t)\}$ carry over to $\{Z_{ij}\}$ so that, for example, $Z_{ij} = Z_{ik} = 1$ implies that $Z_{jk} = Z_{kj} = 0$. For simplicity we rewrite $\{Z_{ij}, 1 \leq i \leq n/2, i < j \leq n\}$ as $\{Z_i, 1 \leq i \leq M\}$. Now let $\{\tilde{Z}_i, 1 \leq i \leq M\}$ be i.i.d. Ber(p_n) random

variables and let $Z = \sum_{i=1}^{M} Z_i$ and $\tilde{Z} = \sum_{i=1}^{M} \tilde{Z}_i$. Now

$$\begin{aligned} E[Z^k] &= \sum_{(i_1,\ldots,i_k)\in\{1,\ldots,M\}^k} E\left[Z_{i_1},\ldots,Z_{i_k}\right] \\ &= \sum_{(i_1,\ldots,i_k)\in\{1,\ldots,M\}^k} P\{Z_{i_1} = \ldots = Z_{i_k} = 1\}. \end{aligned}$$

Similarly, we can write $E[\tilde{Z}^k]$ as the sum of terms of the form $P\{\tilde{Z}_{i_1} = \ldots = \tilde{Z}_{i_k} = 1\}$. Consider a term of the form $P\{Z_{i_1} = \ldots = Z_{i_k} = 1\}$. If all the Z_is contained in this term are independent then this is the same as the corresponding term of $E[\tilde{Z}^k]$. When there is a dependence, the term becomes zero due to the particular nature of dependence as mentioned above, due to which for some distinct indices i, j, k, $Z_i = Z_j = 1$ implies $Z_k = 0$. However, for $E[\tilde{Z}^k]$ the corresponding term is still non-zero. Hence $E[Z^k] \leq E[\tilde{Z}^k]$ for all $k \geq 0$. As a result, for any $t > 0$,

$$E[\exp(tZ)] \leq E[\exp(t\tilde{Z})]. \tag{5.20}$$

So we can write for $t > 0$,

$$\begin{aligned} P\{Z > (1+\delta)E[Z]\} &\leq \frac{E[\exp(tZ)]}{\exp((1+\delta)tZ)} \\ &\leq \frac{E[\exp(t\tilde{Z})]}{\exp((1+\delta)t\tilde{Z})} \\ &\leq \exp(-E[\tilde{Z}]\delta^2/2), \end{aligned}$$

using the Chernoff bounding technique for the sum of i.i.d Bernoulli random variables. Now $E[\tilde{Z}] = O(Mp_n) = O\left(n^{3/2}b(n)^{3/2}\sqrt{\log n}\right)$. Choosing an appropriate δ we have $Z = O(E[Z])$ *whp*. Hence the total number of packets through a cell is $O\left(n^{3/2}b(n)^{3/2}\sqrt{\log n}\right)$ *whp*. Using the union bound over the $n/\log n$ cells establishes the claim about the throughput of Scheme 3(b). ■

Next we analyze the delay for Scheme 3(b). The delay of a packet is determined by the queuing delay at a relay node. We first upper bound this queuing delay by that of a GI/GI/1 queue and then use Kingman's upper bound for the GI/GI/1 queue. To use this upper bound, we require to compute the first two moments of the inter-arrival times and the service times. For Scheme 3(b), these moments are related to the corresponding moments of the hitting times of subsets of the torus.

Proof of Theorem 5.6(ii): Two types of relaying are used in Scheme 3(b). First, there

is relaying by hops along adjacent cells of size of $\Theta(\log n/n)$ when a packet is sent from a source to a mobile relay node which we call relaying by hops. Second, the mobile relay node carries the packet until it gets near the destination for further hop-relaying. Thus the total delay experienced by a packet in moving from its source S to its destination D via a mobile relay R involves two types of delay - (i) the hop-delay $D_h(n)$, which is the delay in relaying by hops from S to R and then from R to D, and (ii) the queuing delay $D_q(n)$ in the queue at R for that S-D pair. Only packets that reach from S to R directly (i.e., without an intermediary mobile relay node) are not subject to queuing delay, which form a negligible fraction.

First we determine $D_h(n)$, the delay due to relaying by hops. Consider a packet that is relayed by hops from its source S to a mobile relay node R or its destination. The counters in Scheme 3(b) ensure that this process starts only when S and R are initially within a step-distance of $l(n)$ hops. Thereafter due to the random walk model for mobility of nodes, the average distance between S and R monotonically increases with time and at time $c_0(n)$ the average distance in terms of number of steps is less than $l(n) + \Theta(\sqrt{c_0(n)}) = \Theta(l(n))$ since $l(n) = \Theta(c_0(n))$. Hence from the analysis of Scheme 3(a) it can be seen that the delay due to relaying by hops along cells of size $\Theta(\log n/n)$ is

$$D_h(n) = \Theta\left(l(n)/\sqrt{\log n}\right) = \Theta\left(\sqrt{nb(n)/\log n}\right). \tag{5.21}$$

Now we proceed to determine $D_q(n)$, the queuing delay. Since the underlying packet transport mechanism is that of Scheme 3(a), packets are dropped in the network. However packets are dropped during the relaying by hops and hence does not affect the queuing delay at the mobile relay nodes.

For any S-D pair the delay at each mobile relay node is the same and by symmetry each S-D pair has the same delay. Hence we only need to compute the queuing delay for any one such queue. So fix an S-D pair and a mobile relay node R and let the queue at this relay node be called $\mathcal{Q}_1$. First consider the arrival process to this queue which is depicted in Figure 5.5. The solid line in in the figure is non-zero when S and R when S and R are at a distance no more than $l(n)$ hops. The counter is set to $c_0(n)$ the first time this happens and the dashed line is non-zero when the counter is positive. Packets are sent from S to R during this period when the counter is positive if the corresponding state variable is 1. Note that some of the arriving packets may be dropped due to the way Scheme 3(a) operates. However by considering a queue $\mathcal{Q}_2$ in which these packet drops are ignored we obtain an upper bound on the queuing delay in $\mathcal{Q}_1$.

Packets arrive at R with a delay of $D_h(n)$ due to hopping. However these packets arrive

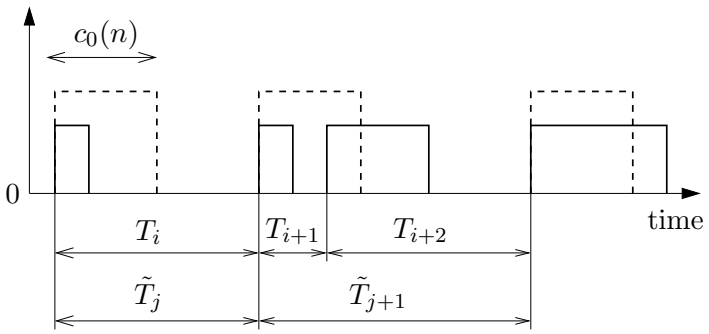


Figure 5.5: The solid line is non-zero when S and R are within a distance of $l(n)$ hops of each other. The dashed line is non-zero when the counter at S for R is positive. Packets are sent from S to R when the dashed line is non-zero if the state variable is 1 as explained in the description of Scheme 3(b).

in order (although some of the packets may arrive together) because of the way Scheme 3(a) works. Now the step-distance between S and R when any packet departs is at most $l(n)+c_0(n)$ and so $D_h(n) \leq (c_6+c_7)\sqrt{nb(n)/\log n} = D_h^{max}$ (say). So if we consider a queue in which each packet arrives exactly D_h^{max} time-slots after its departure from S, then the sum of the average delay of this queue and D_h^{max} is an upper bound to the delay in $\mathcal{Q}_2$. Now as will be shown later (and as claimed in the Theorem), it turns out the the queuing delay is of an order strictly greater than that of D_h^{max} hence it does not matter in the order of the average delay of $\mathcal{Q}_2$. Moreover the arrival and service processes are jointly stationary since they are based on the motion of nodes S, R and D. Hence in order to determine an upper bound of the same order as the actual delay we can instead consider a queue, say $\mathcal{Q}_3$, in which arrivals occur when the counter is positive and the state variable is 1.

Let $\tilde{T}$ be a random variable denoting the time interval between two instants when the counter is set to $c_0(n)$ consecutively. Then $\{\tilde{T}_j\}$ in Figure 5.5 is a sequence of i.i.d. random variables each with the same distribution as $\tilde{T}$. Also let T be a random variable denoting the time interval between two slots when the distance between S and R decreases to $l(n)$ from greater than $l(n)$ consecutively. Then $\{T_i\}$ in Figure 5.5 is a sequence of i.i.d. random variables each with the same distribution as T.

The departure process is similar and departures occur when the corresponding counter is positive if the queue is not already empty. The delay for this queue is the same as that for $\mathcal{Q}_4$, a queue in which a single arrival occurs at the start of each period of length $c_0(n)$ of arrivals in $\mathcal{Q}_3$ and a departure occurs at the beginning of each period of $c_0(n)$ departures in $\mathcal{Q}_3$. The inter-arrival times in $\mathcal{Q}_4$ form an i.i.d. process and the distribution of the

inter-arrival time is the same as the sum of K independent copies of $\tilde{T}$ where K is an independent Geometric random variable with parameter p_0. Hence the first two moments of the inter-arrival time in $\mathcal{Q}_4$ are the same as those of $\tilde{T}$ which is the time interval between two instants when the counter is set to $c_0(n)$ consecutively. Similarly the inter-service times are also i.i.d. and hence as done in the delay analysis of Scheme 2, we can upper bound the delay of $\mathcal{Q}_4$ by that of a GI/GI/1 queue $\mathcal{Q}_5$ whose service time distribution is the same as the inter-service time distribution of $\mathcal{Q}_4$. Further using Kingman's upper bound for the GI/GI/1 queue as in the delay analysis of Scheme 2 and the fact that the moments of the inter-arrival time and service time in $\mathcal{Q}_5$ are of the same order, we can write

$$D_q(n) = O\left(E[\tilde{T}^2]/E[\tilde{T}]\right). \tag{5.22}$$

Substituting the moments of $\tilde{T}$ from Lemma 5.8 which is presented after this proof, we obtain

$$D_q(n) = O(n \log(1/b(n))). \tag{5.23}$$

As a result the delay for Scheme 3(b) scales as

$$D(n) = D_h(n) + D_q(n) = O(n \log(1/b(n))).$$

■

Remark: At the choice of $b(n) = \Theta(1)$ the performance of the scheme is vastly different for $b(n) = 1$ and $b(n) \neq 1$. This discontinuity of trade-off is the consequence of using mobility since $b(n) = 1$ means that the mode of operation is that of Scheme 3(a) where mobility is not used to move the packets toward their destinations. The delay jumps up immediately as mobility is used with $b(n) \neq 1$.

Lemma 5.8. *For $\tilde{T}$, as defined earlier,*

$$E[\tilde{T}] = \Theta\left(\sqrt{\frac{n}{b(n)}}\right), \quad \textit{and}$$

$$E[\tilde{T}^2] = O\left(\log\left(\frac{1}{b(n)}\right)\sqrt{\frac{n^3}{b(n)}}\right).$$

Proof. Consider the counter at a source node for some other node. Recall that $\tilde{T}$ is a random variable denoting the time interval between two consecutive transitions of the counter from -1 to $c_0(n)$. For simplicity, assume that each cell becomes active every time-slot (instead of once in $1 + c_1$ time slots). Then by definition, $\tilde{T} \geq c_0(n)$. Let T be a random variable

denoting the time interval between two slots when the distance between S and R decreases to $l(n)$ from greater than $l(n)$ consecutively. Then $\{T_i\}$ in Figure 5.5 is a sequence of i.i.d. random variables each with the same distribution as T.

Consider the random variable $K = \inf\{k : \ T_1 + \cdots + T_k > c_0(n)\}$, where T_i are i.i.d. with the distribution of T. Then, by definition

$$\tilde{T} \ = \ \sum_{k=1}^{K} T_k. \tag{5.24}$$

From definition of K, $\sum_{k=1}^{K-1} T_k \leq c_0(n)$. Hence, from (5.24)

$$\tilde{T} > T_K \ \text{ and } \ \tilde{T} \leq c_0(n) + T_K. \tag{5.25}$$

This implies that,

$$E[T] \ \leq \ E[\tilde{T}] \ \leq \ c_0(n) + E[T], \tag{5.26}$$

and

$$E[\tilde{T}^2] \ \leq \ c_0^2(n) + c_0(n)E[T] + E[T^2]. \tag{5.27}$$

Next we compute $E[T], E[T^2]$ to determine $E[\tilde{T}], E[\tilde{T}^2]$. In the random walk motion model, each node moves according to a simple random walk on the discrete $\sqrt{n} \times \sqrt{n}$ torus. Let $X(t) \in \{(i,j) : 0 \leq i,j \leq \sqrt{n} - 1\}$ be such a random walk on the $\sqrt{n} \times \sqrt{n}$ torus. Since T is determined by the independent random walks of S and R on the torus, equivalently we can study it using a difference random walk of a single node as was done in the analysis of delay for Scheme 2.

Now let A be the set of cells of the torus which are at a distance no greater than $l(n)$ from $(0,0)$, i.e.,

$$A = \{(i,j) : d((i,j),(0,0)) \leq l(n)\}.$$

And let ∂A be the set of cells of the torus which are exactly at distance $l(n)$ from $(0,0)$, i.e.,

$$\partial A = \{(i,j) : d((i,j),(0,0)) = l(n)\}.$$

Then T as defined above is the time taken by a node performing a difference random walk to perform another transition from A^c to A starting from such a transition. Since we are interested only in the exact order of the moments, we can consider the simple random walk instead of the difference random walk on the discrete torus. Hence to determine $E[T]$ and $E[T^2]$ we can redefine T as follows.

Let π denote the stationary distribution for the random walk on the torus which we know is uniform and let for a set B let $\pi(B) = \sum_{v \in B} \pi(v)$ be the probability of the set B under π. By the probability distribution π_B, we mean $\pi_B(v) = \pi(v)/\pi(B)$ if $v \in B$ and zero otherwise.

Now instead of the random walk $\{X(t)\}$, consider a Markov chain given by $Z(t) = (X(t-1), X(t))$. The state space of $\{Z(t)\}$ is clearly the set of directed edges of the torus. Let ∂A^+ be the set of edges of the torus directed from A to A^c and let ∂A^- be the set of edges of the torus directed form A^c to A. For this new Markov chain $\{Z_t\}$, T is the first return time to the set ∂A^-, i.e.,

$$T = \inf\{t \geq 1 : Z(t) \in \partial A^-\},$$

starting from $Z(0) \sim \pi_{\partial A^-}$.

Before proceeding to compute the first two moments of T, note that there are $4n$ states in the state space corresponding to the 4 directed edges emanating from each vertex of the torus. Moreover the stationary distribution of $\{Z(t)\}$ is also uniform. This can be easily verified (e.g. see the proof of Lemma 6.5 in [37] or the proof of Lemma 7 in Chapter 3 of [1]). Also note that the number of states in A is $4l(n) = 4c_6\sqrt{nb(n)}$. Now using Kac's formula, we obtain

$$\begin{aligned} E[T] = &= E_{\pi_{\partial A^+}} T \\ &= 1/\pi(\partial A^+) \qquad (5.28) \\ &= 4n/c_8\sqrt{nb(n)} \\ &= c_8\sqrt{n/b(n)}. \qquad (5.29) \end{aligned}$$

Now using equation (21) in Chapter 2 of [1], we obtain

$$\begin{aligned} E[T^2] &= E_{\pi_{\partial A^+}}[T^2] \\ &= \left(2E_\pi T + 1\right)/\pi(\partial A^+) \\ &= \left(2E_\pi T + 1\right) E[T], \qquad (5.30) \end{aligned}$$

where the last equality follows from (5.28).

Let $\mathcal{E}$ be the set of all directed edges of the torus and let $\mathcal{E}_A$ be the set of directed edges that are between vertices in A. Consider the following two possible cases, based on the starting position of $Z(0)$, to compute $E_\pi[T]$.

Case 1. Suppose $Z(0) = e \in \mathcal{E} - (\mathcal{E}_A \cup \partial A^-) \stackrel{\triangle}{=} \mathcal{E}_1$. Under this situation for $Z(\cdot)$ to visit any edge in ∂A^-, the original random walk needs to enter A starting from a node in A^c chosen with the uniform distribution restricted to A^c. Thus it is the same as T_A, the hitting time of set A of vertices, starting with initial distribution $\pi(A^c)$. Using Lemma 5.9 following this proof, it follows that

$$E_{\pi_{A^c}} T_A = O(n \log(1/b(n))).$$

Thus,

$$E_{\pi_{\mathcal{E}_1}}[T] = O(n \log(1/b(n))). \tag{5.31}$$

Case 2. Suppose $Z(0) = e \in \mathcal{E}_A \cup \partial A^- \stackrel{\triangle}{=} \mathcal{E}_2$. Under this starting condition, the original random walk starts inside the set A. Hence visiting edge of ∂A^- requires that the original random walk first get out of the set A, and then visit ∂A^- given that $Z(\cdot)$ is in $\mathcal{E} - (\mathcal{E}_A \cup \partial A^-)(= \mathcal{E}_1)$. From basic first passage time results for one-dimensional random walks, it is easy to see that the expected time to get out of set A starting from any position inside A is $O\left(l(n)^2\right)$. Using this and *Case 1*, we obtain

$$\begin{aligned} E_{\pi_{\mathcal{E}_2}}[T] &\leq E_{\pi_{\mathcal{E}_1}}[T] + O(l(n)^2) \\ &= O\left(n \log(1/b(n)) + O\left(l(n)^2\right)\right. \\ &= O\left(n \log(1/b(n))\right). \end{aligned} \tag{5.32}$$

From Case 1 and Case 2, using (5.31) and (5.32), we obtain

$$E_\pi T = O(n \log(1/b(n))). \tag{5.33}$$

Finally combining (5.30) and (5.33),

$$E[T^2] = O\left(\log(1/b(n)) \sqrt{\frac{n^3}{b(n)}}\right). \tag{5.34}$$

□

The following lemma used in the above proof is proved in Subsection 5.4.

Lemma 5.9. *Consider the subset $A = \{(i,j) : 0 \leq i,j < \sqrt{m}\}$ on a two-dimensional $\sqrt{n} \times \sqrt{n}$ discrete torus. That is, A is a square set of $\sqrt{m} \times \sqrt{m}$ cells of the discrete torus.*

Let T_A be the hitting time of A then

$$E_{\pi_{A^c}} T_A = O\left(n \log\left(\frac{n}{m}\right)\right).$$

5.3.3 Optimality of the trade-offs

In this section, we establish the optimality of Scheme 3 under the random walk (RW) model. Consider any communication scheme operating under the RW model. Then the distance traveled by a packet between its source and destination is the sum of the distances traveled by hops and the total distance traveled by the mobile relays that are used by the packet under this scheme. Let $\bar{l}(n)$ be the sample mean of the distance traveled by hops (i.e., by wireless transmission) and let $\bar{r}(n)$ be the sample mean of the distance traveled by a packet per hop. In the following lemma, proved in Subsection 5.4, we obtain a bound on throughput scaling as a function of $\bar{l}(n)$ and $\bar{r}(n)$ using a technique similar to the one used in Theorem 2.3. We then show that to achieve this optimal throughput, the minimum delay incurred is of the same order as the delay of Scheme 3, which will establish the optimality of Scheme 3.

Lemma 5.10. *Consider any scheme such that the sample mean of the distance traveled by hops is $\bar{l}(n)$ and the sample mean of the distance traveled per hop is $\bar{r}(n)$. Then its achievable throughput,*

$$T(n) = O\left(\frac{1}{n\bar{l}(n)\bar{r}(n)}\right). \tag{5.35}$$

The above lemma is proved in Subsection 5.4. Next we state a result regarding the mean hitting time of a subset of cells for a random walk on the $\sqrt{n} \times \sqrt{n}$ discrete torus. It is a consequence of Lemma 2.1 in [9] and the strong approximation (of random walk by Brownian motion) results as used in the proof of Theorem 1.1 in [9].

Lemma 5.11. *Let T_r be the time to hit a set of cells of the discrete torus contained in a disk of radius $r < R/2$ around a point x starting from the boundary of a disk of radius R around x. Then for a symmetric random walk on $\sqrt{n} \times \sqrt{n}$ discrete torus,*

$$E[T_r] = \Theta(n \log r^{-1}).$$

Now we are ready to prove the optimality of Schemes 3(a) and 3(b) using Lemma 5.10 and Lemma 5.11.

Optimality of Scheme 3(a): Consider any communication scheme that uses cellular transmission and possibly utilizes node mobility to achieve the optimal throughput-delay trade-off for $T(n) = O(1/\sqrt{n \log n})$. Let $\bar{l}(n)$ be the sample mean of the distance traveled by hops under this scheme. Let $\bar{r}(n)$ be the sample mean of the hop distance, then by Lemma 5.10, $T(n) = O\left(1/\left(n\bar{l}(n)\bar{r}(n)\right)\right)$. For this scheme, the average delay due to hops alone, i.e., the time taken to travel by hops is $\Theta\left(\bar{l}(n)/\bar{r}(n)\right)$.

Now, if $\bar{l}(n) = \Theta(1)$, $T(n) = O\left(1/\left(n\bar{r}(n)\right)\right)$ and $D(n) = \Omega(1/\bar{r}(n))$. That is, $T(n) = O\left(D(n)/n\right)$. Hence from Theorem 5.5, among all schemes that have $\bar{l}(n) = \Theta(1)$, Scheme 3(a) is optimal. As a consequence, if Scheme 3(a) is not optimal then it must be that $\bar{l}(n) = o(1)$ for the optimal scheme.

If $\bar{l}(n) = o(1)$, then at least a constant fraction of the packets must travel a distance of $\Theta(1)$ using the node mobility. From Lemma 5.11, it follows that for such packets, the delay is $\Omega(n)$. But then, $D(n) = \Omega(n) = \omega(nT(n))$ since $T(n) = o(1)$. Hence, Scheme 3(a) is optimal among all schemes with $\bar{l}(n) = o(1)$.

This shows that Scheme 3(a) provides the optimal throughput-delay trade-off as far as the scaling is concerned.

Optimality of Scheme 3(b): Consider an optimal communication scheme that uses cellular transmission, possibly along with the mobility of nodes, to achieve the optimal throughput-delay trade-off for $T(n) = \omega(1/\sqrt{n \log n})$. Let $\bar{l}(n)$ and $\bar{r}(n)$ be as defined above. By Lemma 5.10, $T(n) = \omega(1/\sqrt{n \log n})$ requires that $\bar{l}(n) = o(1)$. But from the preceding discussion, when $\bar{l} = o(1)$, the mobile-delay (the time spent at a mobile relay node) dominates the hop-delay (the time spent in performing hops). Thus when throughput is $\omega(1/\sqrt{n \log n})$, to maximize the throughput for a given delay any optimal scheme must have the smallest possible $\bar{r}(n)$, which is $\Theta(\sqrt{\log n/n})$. Therefore, any optimal scheme for this range of high throughput has $T(n) = \Theta\left(1/\left(\bar{l}(n)\sqrt{n \log n}\right)\right)$.

Consider a throughput-delay optimal scheme. For any such scheme, fixing a throughput $T(n)$, fixes $\bar{l}(n)$, which is the average distance traveled by hops. The goal of an optimal scheme then, is to travel this distance by hops in a manner so as to minimize the average time for a packet to reach its destination.

Consider the transmission of a packet p starting from its source S and moving toward its destination D, initially at a distance d from S. Recall that a packet travels a distance $\bar{l} = \bar{l}(n)$ through hops and the rest through the motion of the nodes relaying it. Define t_p to be the time it takes the packet p, after leaving its source S, to reach its destination D. We ignore the time required for hops as the mobile delay dominates the total delay. Let $E[t_p]$ be the expectation of t_p for a given $\bar{l}$ and d. Note that the expectation is over the

distribution induced by the random walks of the nodes.

We claim the following.

Lemma 5.12. *For any $\bar{l}$ and d, a scheme that minimizes $E[t_p]$ must perform all the hops the first time the packet is at a distance less than or equal to $\bar{l}$ from its destination D.*

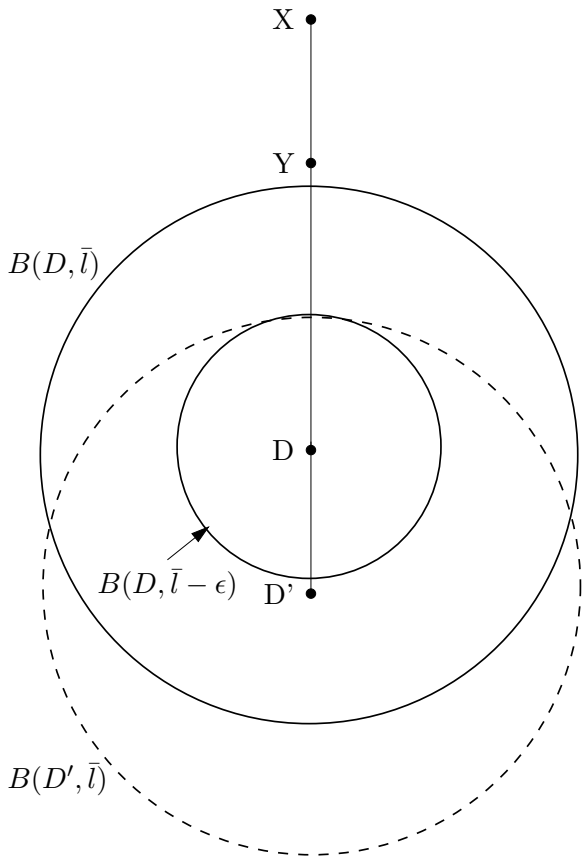
Proof. For $\bar{l} \geq d$, the Lemma clearly holds. Next consider the case where $\bar{l} < d$. To prove the Lemma for this case, consider the following two schemes. Scheme A uses the entire hop distance $\bar{l}$ when the packet reaches within a distance $\bar{l}$ of D for the first time, which is consistent with the claim of the lemma. Scheme B uses a hop of length ϵ when the packet is at a distance $\tilde{d}$ ($d > \tilde{d} > \bar{l}$) from D, and uses the remaining hop distance $\bar{l} - \epsilon$ at the end, as in Scheme A.

We want to show that, on average, a packet takes longer to reach D in Scheme B than in Scheme A. For simplicity, we assume that D is fixed. This does not affect generality as all nodes perform independent symmetric random walks.

Consider the path of a packet originating at distance d from D. Until the packet reaches within a distance $\tilde{d}$ of D, its path is the same in both schemes. As illustrated in Figure 5.3.3, under Scheme B, at point X, which is at a distance $\tilde{d}$ from D, the packet travels a distance ϵ by hops toward D to reach Y. Under Scheme A, the packet remains at point X. At this instant, the remaining time for the packet to reach D under Scheme A, t_A, is the time taken to reach a ball $B(D, \bar{l})$ starting from X, and under Scheme B, it is the time t_B taken to reach $B(D, \bar{l} - \epsilon)$, starting from Y. We now show that on average $t_{\mathrm{A}} < t_{\mathrm{B}}$. Consider a point D' on the line X–D at distance ϵ from D (as depicted in Figure 5.3.3). Since all nodes perform independent symmetric random walks, the probability that a path starting from X reaches $B(D, \bar{l})$ is the same as the probability that any path starting from Y reaches $B(D', \bar{l})$. Note that, by construction, $B(D, \bar{l} - \epsilon) \subset B(D', \bar{l})$. Hence the time for a packet at Y to reach $B(D', \bar{l})$ is stochastically dominated by the time needed to reach $B(D, \bar{l} - \epsilon)$. This proves that the time taken by Scheme A is strictly smaller than the time taken by Scheme B on average.

Using the above argument inductively for all hops establishes the Lemma. □

The above lemma shows that a throughput-delay optimal scheme must utilize all the hops at the end. Since in Scheme 3(b), half the hops are performed at the end, it follows that its throughput-delay trade-off is of the same order as that of an optimal scheme. The argument used above is for the case when the scheme allows only one copy of each packet in the network at any time. Clearly the scaling is unaffected by the use of $\Theta(1)$ copies of each packet.


X
Y
$B(D, \bar{l})$
D
$B(D, \bar{l} - \epsilon)$
D'
$B(D', \bar{l})$

The optimality of Schemes 3(a) and 3(b) establishes the following theorem.

Theorem 5.7. *Among all schemes such that the number of copies of any packet in the network at any time is* $\Theta(1)$*, Scheme 3 obtains the optimal throughput-delay trade-off for mobile networks.*

5.4 Remaining proofs

Proof of Lemma 5.2: Let the cells on the torus be numbered 1 to n. Let $X^i(t)$ be the cell in which node i resides at time t for $i = 1, \ldots, n$. Without loss of generality let node 1 be the relay node and node 2 be the destination node. Now $X(t) = (X^3(t), \ldots, X^n(t))$ is a Markov chain formed by the independent random walks of the $n-2$ nodes other than the relay and the destination on the two-dimensional torus. The corresponding equilibrium distribution is independent and uniform, i.e., each node is equally likely to be in any of the n cells. This implies that at time α_0, $X^1(\alpha_0) = X^2(\alpha_0) = U_0$ where U_0 is uniformly distributed on $\{1, \ldots, n\}$. And nodes 3 to n are also distributed independently and uniformly over the n cells. Due to symmetry on the torus, by arguing inductively, it can be seen that for $k = 0, 1, \ldots, X^1(\alpha_k) = X^2(\alpha_k) = U_k \sim \text{Uniform}\{1, \ldots, n\}$.

For typographical ease, let $F = \{E_i = 1\}$. Then for a network of n nodes, we have

$$\begin{aligned}
&P\left(E_i = 1 | \alpha^i, E^{i-1}\right) \\
&= \sum_{X(\alpha_i)} P\left(F | X\left(\alpha_i\right), \alpha^i, E^{i-1}\right) P\left(X\left(\alpha_i\right) | \alpha^i, E^{i-1}\right) \\
&= \sum_{X(\alpha_i)} P\left(F | X\left(\alpha_i\right)\right) P\left(X\left(\alpha_i\right) | \alpha^i, E^{i-1}\right) \qquad (5.36) \\
&\geq \min_{X(\alpha_i)} P(F | X(\alpha_i)) \sum_{X(\alpha_i)} P(X(\alpha_i) | \alpha^i, E^{i-1}) \\
&= \min_{X(\alpha_i)} P\left(F | X\left(\alpha_i\right)\right), \qquad (5.37)
\end{aligned}$$

where (5.36) is true since E_i is independent of everything else given $X\left(\alpha_i\right)$ since it determines the configuration of occupancies of all the n cells by the other $n-2$ nodes. Next we show that the configuration that minimizes $P(F|X(\alpha_i))$ is the one in which $n-2$ cells contain one node each.

In time-slot α_i, let F_k be the event that R and D meet in cell k and let s_k, d_k denote the cell occupancies, i.e., number of source and destination nodes respectively in cell k (not including R and D) for $1 \leq k \leq n$. Obviously, s_k, d_k are determined by $X(\alpha_i)$. Recall that

R and D are equally likely to meet in any of the n cells and hence

$$\begin{aligned} P(F|X(\alpha_i)) &= \sum_{k=1}^{n} P(E_i = 1|X(\alpha_i), F_k)P(F_k) \\ &= \frac{1}{n}\sum_{k=1}^{n} \frac{1}{(1+d_k)(1+s_k+d_k)}. \end{aligned} \tag{5.38}$$

Now clearly for $l \geq 0$ and $k \geq 1$

$$\frac{1}{(k+1)(k+2)(k+l+1)} < \frac{1}{2},$$

and for $l \geq 1$ and $k \geq 0$

$$\frac{1}{(k+1)(k+l+1)(k+l+2)} < \frac{1}{2}.$$

With simple algebraic manipulations, these can be rewritten as

$$\frac{1}{(k+l+1)(k+1)} + \frac{1}{2} < \frac{1}{(k+2)(k+l+1)} + 1, \tag{5.39}$$

$$\frac{1}{(k+l+1)(k+1)} + \frac{1}{6} < \frac{1}{(k+1)(k+l+2)} + 1. \tag{5.40}$$

Consider two cells with occupancies $s_1 = l, d_1 = k+1$ and $s_2 = 0, d_2 = 0$. Then their contribution to the sum in (5.38) is given by the right hand side of (5.39). Now by moving one of the nodes from cell 1 into cell 2 we obtain $s_1 = l, d_1 = k$ and $s_2 = 0, d_2 = 1$. For this case, the left hand side of (5.39) gives the contribution to the sum in (5.38). Thus the last inequality says that if a cell contains more than 1 destination nodes and there is another empty cell then $P(F|X(\alpha_i))$ can be reduced by moving one of the destination nodes to an empty cell. Similarly (5.40) says that if a cell contains more than 1 source nodes and there is another empty cell then $P(F|X(\alpha_i))$ can be reduced by moving one of the source nodes to an empty cell. But in our case with n cells and $n-2$ nodes, empty cells always exist. Hence starting from any initial cell occupancies, we progressively obtain that $P(F|X(\alpha_i))$ is minimized when each of the $n-2$ nodes occupies a different cell.

Next we compute the term in (5.37), which is the probability of R and D being chosen as a transmit-receive pair for the worst case of cell occupancies mentioned above. Using

(5.38), we obtain

$$\begin{aligned}\min_{X(\alpha_i)} P\left(F|X\left(\alpha_i\right)\right) &= \frac{1}{n}\left(\frac{(n-2)}{2}\left[\frac{1}{4}+\frac{1}{2}\right]+2\right) \\ &= \frac{3}{8}-\frac{5}{4n} \\ &\rightarrow \frac{3}{8}. \end{aligned} \tag{5.41}$$

This proves the lemma for any choice of $c_6 < 3/8$. ■

Remark: It is interesting to obtain an upper bound on $P(E_i = 1|\alpha^i, E^{i-1})$ in the same manner. It is easy to see that the configuration that maximizes this probability is the one in which all $n-2$ nodes are in the same cell. Hence using (5.38) and recalling that $F = \{E_i = 1\}$, we obtain

$$P(E_i = 1|\alpha^i, E^{i-1}) \leq \frac{1}{n}\left((n-1)+\frac{4}{n^2}\right) = 1 - \frac{1}{n} + \frac{4}{n^3} \rightarrow 1.$$

This shows that $3/8 - 5/4n \leq P(E_i = 1|\alpha^i, E^{i-1}) \leq 1 - 1/n + 4/n^3$, indicating the extent of dependence in the process.

Proof of Lemma 5.1: We need to compute the first and second moments of τ, which is the inter-meeting time of two nodes, i and j, moving according to independent random walks on a $\sqrt{n} \times \sqrt{n}$ discrete torus. Let the position of node i at time t be $X^i(t) = (X_1^i(t), X_2^i(t))$, where $X_k^i(t) \in \{0, \ldots, \sqrt{n}-1\}$ for $k \in \{1, 2\}$.

Now consider the difference random walk between nodes i and j, defined by $X^{ij}(t) = (X_1^{ij}(t), X_2^{ij}(t))$, where $X_k^{ij}(t) = X_k^i(t) - X_k^j(t) \mod \sqrt{n}$, for $k = 1, 2$. The meeting time of two nodes $i,\ j$ is identified by the event $\{X^{ij}(t) = (0,0)\}$. Thus the inter-meeting time is the stopping time

$$T^{ij} = \inf\{t \geq 1 : X^{ij}(t) = (0,0), X^{ij}(0) = (0,0)\}.$$

This is in fact the first return time to state $(0,0)$. Since we are only interested in the scaling orders of the first two moments, we instead consider the first return time to state $(0,0)$ for a simple random walk $X(t)$ on a $\sqrt{n} \times \sqrt{n}$ discrete torus. The first return time to state $(0,0)$ is given by

$$T = \inf\{t \geq 1 : X(t) = (0,0),\ X(0) = (0,0)\}.$$

First note that, $X(t)$ is a Markov chain with a uniform equilibrium distribution π, i.e. $\pi(i,j) = 1/n$ for $0 \leq i, j \leq \sqrt{n} - 1$. For any finite-state ergodic Markov chain,

the expectation of the first return time to any state is the reciprocal of the equilibrium probability of the Markov chain being in that state. In particular, the average first return time to state $(0,0)$ is n, i.e. $E[T] = n$. By the same argument, $E[\tau] = n$.

Next, we wish to compute $E[T^2]$, which is of the same order as $E[\tau^2]$. First consider the quantity

$$T_0 = \inf\{t \geq 0 : X(t) = (0,0)\},$$

which is the hitting time of state $(0,0)$. Let $E_{(i,j)}T_{(k,l)}$ denote the expected time to first hit state (k,l) starting from state (i,j). Then

$$\begin{aligned}
E_\pi[T_0] &= \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)} T_{(0,0)} \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)} T_{(k,l)} \qquad (5.42) \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \frac{1}{n} \pi(i,j) E_{(i,j)} T_{(k,l)} \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \pi(i,j)\pi(k,l) E_{(i,j)} T_{(k,l)} \\
&= \Theta(n \log n), \qquad (5.43)
\end{aligned}$$

where (5.42) holds because $\sum_{ij} E_{(i,j)}T_{(0,0)} = \sum_{ij} E_{(i,j)}T_{(k,l)}$ for any $0 \leq k, l \leq \sqrt{n} - 1$ due to symmetry of states corresponding to cells on the torus. The validity of (5.43) is from page 11 of Chapter 5 in [1].

Using Kac's formula (see Corollary 24 in Chapter 2 of [1]) and (5.43), we obtain

$$\begin{aligned}
E[T^2] &= \frac{2E_\pi[T_0] + 1}{\pi(0,0)} \\
&= 2\Theta\left(n^2 \log n\right) + n.
\end{aligned}$$

Therefore, we obtain $E[\tau] = n$ and $E[\tau^2] = \Theta(n^2 \log n)$. ■

Equipped with the above lemma, we are now ready to prove Lemma 5.3.

Proof of Lemma 5.3: An arrival occurs to $\mathcal{Q}_3$ when S and R meet with probability 0.5. Let $\{X_i\}$ be the sequence of inter-arrival times to this queue. Then, X_i are i.i.d. with $E[X_1] = 2E[\tau] = 2n$ and $E[X_1^2] = \Theta(E[\tau^2]) = \Theta(n^2 \log n)$ from Lemma 5.1. The potential-departure process is an i.i.d. Bernoulli process with parameter $1/1.5n$. Let $\{Y_i\}$

be the sequence of service times then Y_i is a Geometric random variable with mean $1.5n$. Hence $E[Y_1] = 1.5n$ and $E[Y_1^2] = \Theta(n^2)$. Let D_3 denote the delay of a packet through $\mathcal{Q}_3$. The service process is independent of the arrival process and hence $\mathcal{Q}_3$ is a GI/GI/1 FCFS queue. Then, by Kingman's upper bound [44] on the expected delay for a GI/GI/1 – FCFS queue, the expected delay of $\mathcal{Q}_3$ is upper bounded as

$$\begin{aligned} E[D_3] &= O\left(\frac{E[X_1^2] + E[Y_1^2]}{E[X_1]}\right) = O\left(\frac{n^2 \log n + n^2}{n}\right) \\ &= O\left(n \log n\right). \end{aligned}$$

■

A GI/GI/1 queue has i.i.d. inter-arrival times and i.i.d. service times. But unlike $\mathcal{Q}_3$, $\mathcal{Q}_4$ is not a GI/GI/1 queue because the inter-potential departure times are i.i.d., and not the service times. As a result, Kingman's upper bound cannot be used. Instead we obtain an upper bound by considering a queue sampled at potential departure instants and by exploiting the memorylessness of the inter-arrival times.

Proof of Lemma 5.4: Consider the service process of $\mathcal{Q}_4$, which is 1 at a potential departure instant and 0 otherwise. This is a stationary, ergodic process since the inter-potential-departure times are i.i.d. with mean n. The Bernoulli arrival process to $\mathcal{Q}_4$ is independent of the service process with mean inter-arrival time $1.5n$. Since the arrival and service processes form a jointly stationary and ergodic process with mean service time strictly less than mean inter-arrival time, the queue has a stationary, ergodic distribution with finite expectation as shown by [34]. Thus $\mathcal{Q}_4$ is stable.

Let $\tilde{Q}_t$ be the number of packets in the queue in time-slot t and let Q_i be the number of packets in the queue at potential departure instant i. Thus the process $\{Q_i\}$ is obtained by sampling $\{\tilde{Q}_t\}$ at potential departure instants. Let A_{i+1} be the number of arrivals between potential departure instants i and $i+1$. Then the evolution of Q_i is given by

$$Q_{i+1} = Q_i - \mathbf{1}_{\{Q_i > 0\}} + A_{i+1}. \tag{5.44}$$

Comparing the evolution of the process $\{Q_i\}$ with that of $\{\tilde{Q}_t\}$ shows that $\{Q_i\}$ also has a stationary, ergodic distribution. Recall that τ is the inter-meeting time of R and D. Then since the arrival process is Bernoulli and the inter-potential departure times are i.i.d. with common distribution that of τ, it is clear the $\{A_i\}$ is a stationary process. Let $\tilde{Q}$, Q and A be random variables with the common stationary marginals of $\{\tilde{Q}_t\}$, $\{Q_i\}$ and $\{A_i\}$

respectively. Then taking expectation in (5.44) under the stationary distribution, we obtain

$$P(Q > 0) = E[A]. \tag{5.45}$$

The arrival process is i.i.d. Bernoulli and hence conditioned on τ, the distribution of A is Binomial $(\tau, 2/3n)$. Since $E[\tau] = n$ from Lemma 5.1, we obtain

$$E[A] \;=\; E[E[A|\tau]] \;=\; E\left[\frac{\tau}{1.5n}\right] \;=\; \frac{2}{3}. \tag{5.46}$$

Squaring (5.44), taking expectation, using the independence of Q_i and A_{i+1} and then rearranging terms, we obtain

$$2(1 - E[A])E[Q] = P(Q > 0) + E[A^2] - 2E[A]P(Q > 0).$$

Using (5.45) and (5.46) in the above, we obtain

$$\begin{aligned} E[Q] &= \frac{E[A] + E[A^2] - 2E[A]^2}{1(1 - E[A])} \\ &= \frac{3}{2}\left(E[A^2] - \frac{2}{9}\right). \end{aligned} \tag{5.47}$$

Recall that conditioned on τ the distribution of A is Binomial $(\tau, 2/3n)$ and hence

$$\begin{aligned} E[A^2] &= E[E[A^2|\tau]] \\ &= \frac{2E[\tau]}{3n} + \frac{4}{9n^2}\left(E[\tau^2] - E[\tau]\right) \\ &= \left(\frac{2}{3} - \frac{4}{9n}\right) + \frac{4}{9n^2}\Theta(n^2 \log n) \\ &= \Theta(\log n), \end{aligned} \tag{5.48}$$

where we used Lemma 5.1. As a result it follows from (5.47) that

$$E[Q] = \Theta(\log n). \tag{5.49}$$

Next, we will bound $E[\tilde{Q}]$ using $E[Q]$. To this end, consider a time-slot t and let the number of potential departures before time-slot t be $I(t)$. Thus time-slot t is flanked by potential departures $I(t)$ and $I(t) + 1$. Then $\tilde{Q}_t \leq Q_{I(t)} + A_{I(t)+1}$. Also using the fact that

$\{\tilde{Q}_t\}$ is ergodic, with probability 1, we have

$$\begin{aligned}
E[\tilde{Q}] &= \lim_{T\to\infty} \frac{1}{T} \sum_{k=1}^{T} \tilde{Q}_k \\
&\leq \lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{I(T)+1} (Q_j \tau_{j+1} + A_{j+1}\tau_{j+1}) \\
&= \lim_{T\to\infty} \frac{I(T)+1}{T} \frac{1}{I(T)+1} \\
&\qquad \sum_{j=1}^{I(T)+1} (Q_j \tau_{j+1} + A_{j+1}\tau_{j+1}) \\
&= \frac{1}{E[\tau]} \left(E[Q_1\tau_2] + E[A_1\tau_1]\right) \qquad (5.50) \\
&= \frac{1}{n} \left(E[Q]E[\tau] + \frac{2}{3n} E[\tau^2] \right) \qquad (5.51) \\
&= O(\log n). \qquad (5.52)
\end{aligned}$$

We used the fact that $I(T)/T \to 1/E[\tau]$ by the elementary renewal theorem [44] in (5.50) and the independence of Q_j and τ_{j+1} in (5.51). Let D_4 denote the delay of a packet through $\mathcal{Q}_4$. Using Little's formula, since the arrival rate is $2/3n$, we conclude that

$$E[D_4] = \frac{3n}{2} E[\tilde{Q}] = \frac{3n}{2} O(\log n) = O(n \log n).$$

■

Proof of Lemma 5.5: Consider a fixed cell, say cell 1, out of $m = 1/a(n)$ cells. First we determine the traffic due to stage 1 in time-slot 0 which is the number of packets that cell 1 is required to transmit in time-slot 0.

Let Y_i be the number of packets of the S-D pair i at cell 1 in time-slot 0. We claim that no more than two packets of S-D pair i can be passing through cell 1 in time-slot 0. This is because k packets of S-D pair i can pass through cell 1 at the same time if and only if for k consecutive time-slots, the source node S moves closer to cell 1 by 1 hop along the line joining S and D. Now, cells are of size at least $\Theta(\log n/n)$, whereas according to the RW model, the nodes can only move $\Theta\left(1/\sqrt{n}\right)$ distance in unit time. Hence this can hold for at most $k = 2$. Thus, $Y_i \in \{0, 1, 2\}$. Next, we compute $E[Y_i]$.

Let $L_i(s)$ be the length (in terms of number of hops) of the straight line joining the center of cells that contain S and D at time s of S-D pair i. Now, a packet generated at

time $-s, s \geq 0$ can be part of stage 1 at time 0, only if $L_i(-s) > s$. This is true because otherwise packet generated at time $-s$ is already transmitted for s times till time 0 and hence it is either in stage 2 or has reached the destination. Using this, we obtain

$$\sum_{j=1}^{m} Y_i(j) \leq \sum_{s=0}^{\infty} I(\{L_i(-s) > s\}),$$

where $I(A)$ is the indicator function of the event A and $Y_i(j)$ is the number of packets of stage 1 of the i^{th} S-D pair at cell j at time 0. By symmetry, $E[Y_i(j)] = E[Y_i(1)] = E[Y_i]$. Then,

$$\begin{aligned} mE[Y_i] &\leq \sum_{s=0}^{\infty} P\{L_i(-s) > s\} \\ &\stackrel{(a)}{=} \sum_{s=0}^{\infty} P\{L_i(0) > s\} \\ &= E[L_i(0)] \\ &\stackrel{(b)}{=} \Theta\left(1/\sqrt{a(n)}\right). \end{aligned}$$

Above, (a) holds because of the stationarity of $L_i(\cdot)$ under the RW model. And (b) is true because under the RW model the average distance between an S-D pair is $\Theta(1/\sqrt{a(n)})$ hops since the actual physical distance is $\Theta(1)$.

From above, $E[Y_i] = \Theta\left(\sqrt{a(n)}\right)$. Now let $Z_i = I(\{Y_i \neq 0\})$. Then by Markov's inequality,

$$P(\{Z_i = 1\}) = P(\{Y_i \geq 1\}) \leq E[Y_i].$$

The total number of packets passing through cell 1 at time 0 is $\sum_{i=1}^{n/2} Y_i \leq 2\sum_{i=1}^{n/2} Z_i$. Since under the RW model, all $n/2$ S-D pairs move independently, $\{Z_i\}$ are independent and due to symmetry they are distributed identically. Thus we can define $\{\tilde{Z}_i\}$ to be i.i.d. Bernoulli random variables with parameter $p_n = \Theta\left(\sqrt{a(n)}\right)$ so that Z_i is stochastically dominated by $\tilde{Z}_i$. As a result, the total number of packets is stochastically dominated by $2\sum_{i=1}^{n/2} \tilde{Z}_i$. By an application of the Chernoff bound, $\sum_{i=1}^{n/2} \tilde{Z}_i \leq np_n$ with probability at least $1 - 1/n^4$ for large enough n. Thus, cell 1 has $O(np_n) = O\left(n\sqrt{a(n)}\right)$ packets passing through it in time-slot 0 with probability at least $1 - 1/n^4$. Due to symmetry, the same is true for all m cells. Hence, by the union bound, each of the m cells has $O\left(n\sqrt{a(n)}\right)$ packets of stage 1 in time-slot 0 with probability at least $1 - m/n^4 \geq 1 - 1/n^3$. ■

Proof of Lemma 5.6: Let $H(C^{k-1}, C^k)$ denote the number of hops made by a packet

in stage k for $k = 1, \ldots, k(n) - 1$. For a given packet this is the hop distance between C^{k-1} and C^k (as explained in the description of Scheme 3(a)). Let $l_k = E[H(C^{k-1}, C^k)]$. Since, each source node S is at a physical distance of $\Theta(1)$ from its destination on average, $l_1 = \Theta(1/\sqrt{a(n)})$. It is clear that the traffic of stage k depends on l_k, which in turn depends on l_{k-1}. Hence, we first determine the relation between l_k and l_{k-1} for $k \geq 2$ in order to evaluate l_k.

By definition, $H(C^{k-1}, C^k)$ is the displacement of the destination node of the packet while the packet was being transported in stage k from cell C^{k-2} to cell C^{k-1}. Under Scheme 3(a), if a packet is not dropped, it reaches C^{k-1} from C^{k-2} in time $H(C^{k-2}, C^{k-1})$ since each hop takes a constant amount of time. The average distance moved by destination node in this time is $\Theta\left(\sqrt{H(C^{k-2}, C^{k-1})/n}\right)$ under the RW model. That is, the destination node is displaced from C^{k-1} by $\Theta\left(\sqrt{H(C^{k-2}, C^{k-1})/na(n)}\right)$ hops on average. Putting this together, we obtain the following.

$$\begin{aligned}
l_k &= E[H(C^{k-1}, C^k)] \\
&= E\left[E\left[H(C^{k-1}, C^k)|H(C^{k-2}, C^{k-1})\right]\right] \\
&= E\left[\Theta\left(\sqrt{\frac{H(C^{k-2}, C^{k-1})}{na(n)}}\right)\right] \\
&= \Theta\left(\frac{E\left[\sqrt{H(C^{k-2}, C^{k-1})}\right]}{\sqrt{na(n)}}\right) \\
&= O\left(\frac{\sqrt{E[H(C^{k-2}, C^{k-1})]}}{\sqrt{na(n)}}\right), \qquad (5.53)
\end{aligned}$$

where (5.53) follows from Jensen's inequality. Equation (5.53) gives us the following recursion. For $k \geq 2$,

$$l_k = O\left(\sqrt{\frac{ml_{k-1}}{n}}\right). \qquad (5.54)$$

Now, as noted earlier, $l_1 = O\left(1/\sqrt{a(n)}\right) = O\left(\sqrt{m}\right)$. For ease of presentation, let $c_5 \geq 1$ be constant such that for large enough n, $l_1 \leq \sqrt{c_5 m}$ and $l_k \leq \sqrt{c_5 m l_{k-1}/n}$. Putting all

this together, we obtain

$$\begin{aligned} l_k &\leq \sqrt{\frac{c_5 m l_{k-1}}{n}} \\ &\leq \left(\frac{c_5 m}{n}\right)^{\sum_{j=1}^{k-1} 2^{-j}} \quad l_1^{2^{-k+1}} \\ &\leq \left(\frac{c_5 m}{n}\right)^{1-2^{-k+1}} \quad (c_5 m)^{2^{-k}} \\ &\leq \frac{c_5 m}{n} \left(\frac{n^2}{m}\right)^{2^{-k}}. \end{aligned} \tag{5.55}$$

Now we are ready to analyze the traffic load on cells due to stage k, $k \geq 2$. The analysis of stage 1 showed that at most 2 packets of the same S-D pair can pass through a cell in the same time-slot. This happens only if the source moves in a particular way so that in adjacent time-slots the distance to the cell being considered reduces by one hop. Similarly by considering the motion of both the source and the destination it can be shown that in stage 2 the number of packets of the same S-D pair that can pass through a cell is at most 3. For stage $k \geq 3$, since a packet originates from a fixed cell, rather than from a mobile node like S, it follows that the number of packets of the same S-D pair passing through a cell in the same time-slot is no more than 3.

Consider stage k. As in stage 1, let Y_i be the number of packets for S-D pair i at cell 1 at time 0. From the above discussion, $Y_i \in \{0, 1, 2, 3\}$. Let $L_i(s)$ denote the length in hops, $H(C^{k-1}, C^k)$, for a packet of the S-D pair i entering stage k in time-slot s. Due to the symmetry of the cells under the RW model (using arguments similar to that used in analysis of stage 1) and (5.55), we obtain

$$\begin{aligned} mE[Y_i] &\leq \sum_{s=0}^{\infty} P(\{L_i(-s) > s\}) \\ &= \sum_{s=0}^{\infty} P(\{L_i(0) > s\} \\ &= E[L_i(0)] \\ &= O\left(\frac{m}{n}\left(\frac{n^2}{m}\right)^{2^{-i}}\right). \end{aligned}$$

Thus,

$$E[Y_i] \;=\; O\left(\frac{1}{n}\left(\frac{n^2}{m}\right)^{2^{-i}}\right). \tag{5.56}$$

Again using the same method as in the analysis of stage 1, it can be shown that the total number of stage k packets passing through each cell is $O\left(\left(\frac{n^2}{m}\right)^{2^{-k}}\right)$ with probability at least $1 - 1/n^3$. ■

Proof of Lemma 5.9: Let $X(t) = (X_1(t), X_2(t))$ be a two-dimensional simple RW on the $\sqrt{n} \times \sqrt{n}$ discrete torus. Let $B = \{1, \sqrt{m}, \ldots, (\sqrt{n/m} - 1)\sqrt{m}\}$, i.e., B is the set of elements of $\{0, \ldots, \sqrt{n} - 1\}$ which are multiples of $\sqrt{m}$. Let $Y_1(t) \in B$ be the discrete-time process such that $Y_1(t)$ is the last (including the current) state visited by $X_1(t)$ in B. Thus the process $Y_1(t)$ is a coarser version of $X_1(t)$ that changes state only when $X_1(t)$ moves $\sqrt{m}$ steps away on the one-dimensional discrete torus. Similarly, define $Y_2(t)$ to be the last state visited by $X_2(t)$ in B and let $Y(t) = (Y_1(t), Y_2(t))$.

Now let $Z(t)$ be the process obtained by sampling $Y(t)/\sqrt{m}$ whenever its value changes. Thus $Z(t) \in \{0, \ldots, \sqrt{n/m}\}$ proceeds in steps and ignores the random amount of time that $Y(t)$ spends in each step. Now $Z(t)$ is a random walk on the discrete $\sqrt{n/m} \times \sqrt{n/m}$ torus, such that the next state is one of the eight possible neighbors. Let T_0^Z be the hitting time of state $(0, 0)$ for the random walk $Z(t)$. This is the number of steps taken by $Z(t)$ to hit $(0, 0)$. Since we are only interested in the order, we can use the corresponding moment for the simple random walk instead. Thus we obtain

$$E_\pi T_0^Z = O\left(\frac{n}{m} \log\left(\frac{n}{m}\right)\right).$$

Now $Z(T_0^Z) = (0, 0)$ implies that for some time T, $X(T) \in A$. In fact, T is the random time required for T_0^Z steps of $Z(t)$. Hence $E[T]$ is an upper bound on $E[T_A]$. Let $\tilde{T}(i)$ be the time required for step i of $Z(t)$, i.e., the time for $Y(t)$ to change state for the i^{th} time. The $\tilde{T}(i)$ are clearly i.i.d. Moreover, T_0^Z is independent of $\tilde{T}(i)$ and

$$T = \sum_{i=1}^{T_0^Z} \tilde{T}(i).$$

Let $\tilde{T}_1$ and $\tilde{T}_2$ be the random times required for $Y_1(t)$ and $Y_2(t)$ respectively, to change their states. Then $E[\tilde{T}_1] = E[\tilde{T}_2] = m$, since this is the time required for a simple random walk on the integers to exit from $\{-\sqrt{m} + 1, \ldots, \sqrt{m} - 1\}$. Moreover, $\tilde{T}(i)$ is dominated by

$\tilde{T}_1$ since $Y(t)$ changes state if either of $Y_1(t)$ or $Y_2(t)$ change. Hence $E[\tilde{T}(i)] \leq E[\tilde{T}_1] = m$.

Now in using T to upper bound T_A, we need to take care of the possibility that $X(t)$ may not start from a state that is an element of $B \times B$ and hence $Y(t)$ is undefined until both $X_1(t)$ and $X_2(t)$ hit some element of B. But this time is at most equal to $\max T_1, T_2 \leq T_1 + T_2$. Hence allowing for the initial expected time for $X(t)$ to reach one of the elements of B, which is at most $2m$, we obtain

$$\begin{aligned}
\frac{n-|A|}{n} E_{\pi_{A^c}} T_A &= E_\pi T_A \\
&\leq E_\pi T + 2m \\
&\leq E_\pi \sum_{i=1}^{T_0^Z} \tilde{T}(i) + 2m \\
&= E\left[T_0^Z\right] E\left[\tilde{T}(1)\right] + 2m \\
&= O\left(n \log\left(\frac{n}{m}\right)\right).
\end{aligned}$$

■

Proof of Lemma 5.10: If the throughput of the network is $T(n)$, the number of bits transmitted in a large enough time t is at least $ntT(n)/2$. We will ignore this factor of $1/2$ as it does not affect the scaling. Now consider a bit b, where $1 \leq b \leq ntT(n)$. Let $h(b)$ denote the number of hops taken by bit b and let $r(b,h)$ denote the length of hop h of bit b. Then

$$\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} r(b,h) \geq ntT(n)\bar{l}(n). \tag{5.57}$$

Using the same reasoning as the one which led to (2.7), we obtain

$$\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{\pi}{4} \left(\frac{\Delta}{2} r(b,h)\right)^2 \leq Wt. \tag{5.58}$$

Let the total number of hops taken by all bits be

$$H = \sum_{b=1}^{ntT(n)} h(b).$$

Then by convexity, we obtain

$$\left(\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h) \right)^2 \leq \sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)^2. \tag{5.59}$$

Combining (5.58) and (5.59) we obtain,

$$\left(\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h) \right)^2 \leq \frac{16Wt}{\pi \Delta^2 H} = c_3 \frac{t}{H}, \tag{5.60}$$

where c_3 is a constant that does not depend on n. Substituting from (5.57) into (5.60) and rearranging we obtain

$$\frac{ntT(n)\bar{l}(n)}{H} \bar{r}(n) \leq c_3 \frac{t}{H}, \tag{5.61}$$

where

$$\bar{r}(n) = \sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)$$

is the sample mean of hop-lengths over H hops as defined earlier. Rearranging we obtain

$$T(n) \leq \frac{c_3}{n\bar{l}(n)\bar{r}(n)}. \tag{5.62}$$

This completes the proof of Lemma 5.10. ■

5.5 Discussion

This chapter established the optimal trade-off between throughput and packet delay for mobile wireless networks. We presented a scheme similar to the one in [21] and showed that the delay corresponding to $\Theta(1)$ throughput scales as $\Theta(n \log n)$, when the nodes move according to independent random walks. Further, we described and analyzed a scheme that achieves the optimal throughput-delay trade-off for mobile networks by varying the number of hops, the transmission range, and the degree to which node mobility is used. For the low throughput range achieved in static networks, we found that the trade-off for mobile networks is the same as that for static networks. For higher throughputs, there is almost no trade-off between throughput and delay – the same maximum delay is incurred regardless of the throughput.

Several questions remain to be tackled for a better theoretical understanding of the

data communication aspect of random wireless networks. With the exception of Scheme 2, the chapter assumed a fluid model in which the packet size is allowed to be arbitrarily small. A priori, it is not clear what the trade-off will be with the additional constraint that packet size remains constant. In Chapter 3, we showed that for static networks, the trade-off remains unchanged even with packets of constant size. We believe that the same should hold for mobile networks, but such a result remains to be established.

In this chapter we assumed a random walk model for node mobility. One expects that the scaling results would be the same for a larger class of Markovian motion models. Determining the class of such motion models would be another future challenge.

Chapter 6

Throughput and Delay With Restricted Mobility

Grossglauser and Tse [21] showed that by allowing the nodes to move, the throughput scaling changes dramatically. Indeed, if node motion is independent across nodes and has a uniform stationary distribution, a constant throughput scaling ($\Theta(1)$) per S-D pair is feasible. This raised the question: what kind of mobility is necessary for achieving constant throughput scaling? Diggavi, Grossglauser and Tse [10] considered a restricted mobility model where each node is allowed to move along a randomly chosen great circle on the unit sphere with a uniform stationary distribution along the great circle. They showed that a constant throughput per S-D pair is feasible even with this restricted mobility model. Thus they established that node motion with a stationary distribution on the entire network area is not necessary for achieving constant throughput scaling.

The constant throughput scaling result of [10] for a network with restricted mobility raises the question whether the high throughput in spite of restricted mobility is at the expense of increased delay. Motivated by this question, we study the delay scaling for constant throughput scaling in a network with restricted mobility. Somewhat surprisingly, we find that delay scaling is not affected by this mobility restriction either. That is, delay scales as $\Theta(n \log n)$, which is the same as the delay scaling when mobility is not restricted.

This seemingly surprising result can be explained as follows. Since there are n nodes in a network of constant area, the neighborhood of each node is $\Theta(1/n)$. Based on this, let us say that two nodes *meet* or are *neighbors* when they are within a distance of $\Theta\left(1/\sqrt{n}\right)$. The following condition ensures constant throughput scaling in the mobile network models presented in [21], [10] and this chapter: *for $\Theta(1/n)$ fraction of the time, each node is a*

neighbor of every other node with only $\Theta(1)$ *other nodes in its neighborhood.* This ensures that the total network throughput is $\Theta(n)$ and that it is distributed evenly among the $n/2$ S-D pairs, so that the throughput is $\Theta(1)$. Delay is determined by the first and second moments of the inter-meeting time of the nodes. In the case of unrestricted mobility, the inter-meeting time of any two nodes is equivalent to the inter-visit time to state $(0,0)$ for a 2-D random walk on a $\sqrt{n} \times \sqrt{n}$ grid. In the restricted mobility case also the inter-meeting time turns out to be equivalent to the inter-visit time to state $(0,0)$ for a slightly different random walk. However the first two moments are still of the same order and hence the queuing delay is the same, leading to the same delay scaling. As a result, even with this particular mobility restriction, the maximal throughput scaling and the corresponding delay scaling remain unchanged.

The rest of the chapter is organized as follows. In Section 6.1, we introduce the random mobile network model, some definitions and notation. In Section 6.2, we present a scheme using random relaying and show that it achieves constant throughput scaling. In Section 6.3, we show that the delay for this scheme is $\Theta(n \log n)$.

6.1 Model and definitions

In this section, we present the network model, and the definitions of the performance metrics – throughput and delay.

We begin with the meaning of uniform distribution of great circles on a sphere. Let S^2 denote the surface of a sphere in $\mathbb{R}^3$ with unit area. For $x \in S^2$, let $x' \in S^2$ be the diametrically opposite point of x. Let $G(x)$ denote the great circle obtained by the intersection of S^2 with the plane passing through the center of S^2 and perpendicular to the line xx'. Let x be called the pole of $G(x)$. If the pole of a great circle is chosen according to a uniform distribution on S^2 then the great circle is said to have a uniform distribution.

Definition 13 (Natural random walk). A natural random walk on a discrete torus of size m is the process $S(t) \in \{0, \ldots, m-1\}$, $t = 0, 1, \ldots$, such that $S(0)$ is uniformly distributed over $\{0, \ldots, m-1\}$ and $S(t+1)$ is equally likely to be any element of $\{S(t), S(t) - 1 \mod m, S(t) + 1 \mod m\}$.

This differs from a simple random walk, where $S(t+1)$ is equally likely to be any element of $\{S(t) - 1 \mod m, S(t) + 1 \mod m\}$. In this chapter, we are interested only in scaling results, which, as we show later, depend on the first two moments of various hitting times for two-dimensional random walks. Hence we use the terms simple and natural random walks interchangeably.

Definition 14 (Random network). The random network consists of n nodes that are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for transmission. Associated with each node is a great circle of S^2 chosen independently according to a uniform distribution.

The great circle of each node has $\sqrt{n}$ equidistant lattice points numbered from 0 to $\sqrt{n}-1$ placed on it arbitrarily resulting in a one-dimensional discrete torus of size $\sqrt{n}$. Each node moves according to a natural random walk on these lattice points on its great circle. Figure 6.1 shows a realization of the random network model. Note that since the sphere has unit area, its radius is $1/2\sqrt{\pi}$. Hence each great circle has perimeter $\sqrt{\pi}$ because of which the distance between two adjacent lattice points is $\sqrt{\pi/n}$.

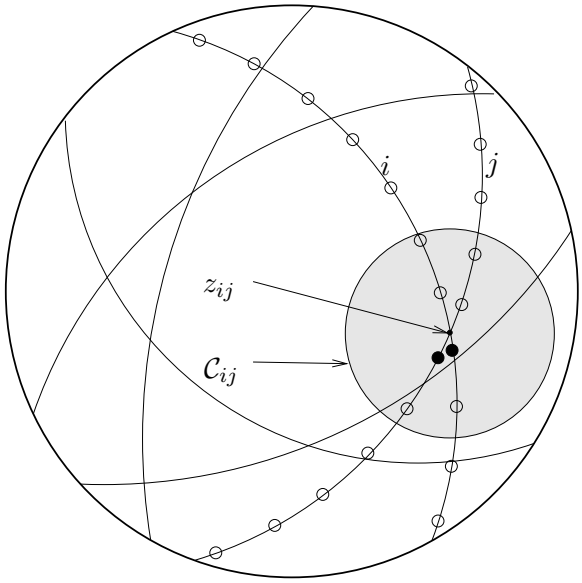


Figure 6.1: A realization of the random network model. Only the lattice points on the great circles of nodes i and i are shown. The intersection of their great circles is z_{ij}. The shaded circle is $\mathcal{C}_{ij}$ and i and j become neighbors when they are at the two dark lattice points.

We use the Relaxed Protocol model for successful transmission, where the distance between nodes is the distance on the sphere. The definitions of throughput and delay are as in Chapter 2.

The differences between this model and the model in [10] are: (i) the Relaxed Protocol model is used instead of the Physical model, and (ii) each node is assumed to move according to a natural random walk instead of just a stationary, ergodic motion with uniform stationary distribution on the great circle. However, this model has the same 1-D mobility

restriction. Further, the proofs clearly show that the assumption of mobility according to a natural random walk is not necessary for achieving constant throughput scaling and is used only for computing delay.

Now observe that some realizations of the random network may result in the configuration of nodes being such that it is not possible to achieve constant throughput scaling. Hence we first define a typical configuration which captures the fact that the distribution of great circles is sufficiently uniform everywhere on the sphere. We need some notation to introduce this definition.

Let G_i denote the great circle of node $i \in \{1, \ldots, n\}$. For any two nodes $i \neq j$, G_i and G_j are not identical with probability 1 under the random network model. Two distinct great circles must intersect in exactly two points. For each pair $i \neq j$, select one of the two distinct intersection points of G_i and G_j uniformly at random and call it z_{ij}. Let $\mathcal{C}_{ij}$ denote the disk on the sphere centered at z_{ij} with radius $(2+\Delta)\sqrt{\pi/n}$. See Figure 6.1 for an illustration.

Definition 15 (Typical configuration). A configuration (i.e., realization of the random network) is said to be *typical* if the number of great circles passing through each $\mathcal{C}_{ij}$ is $\Theta(\sqrt{n})$.

Definition 16 (Neighbor). We say that nodes i and j are *neighbors* at time t if both nodes i and j are at the lattice points of their respective great circles that are closest to z_{ij}.

In Figure 6.1, the lattice points for nodes i and j that are closest to z_{ij} have been darkened. Under the random walk model, it is possible that in some time-slot, a node may not have any neighbors.

6.2 Scheme with constant throughput scaling

In this section we present Scheme Π and show that it achieves constant throughput scaling. In the next section its delay scaling will be analyzed. Before presenting the scheme, we prove a property of the random network model which makes the scheme feasible.

Lemma 6.1. *Configurations are typical* whp.

Proof. Consider any two nodes i and j. First note that the probability that G_i and G_j coincide is zero. Also any two distinct great circles necessarily intersect at exactly two points. By definition, $\mathcal{C}_{ij}$ has area c_1/n (for some positive constant c_1) since it has radius $(2+\Delta)\sqrt{\pi/n}$.

Let $I_k,\ k = 1, \ldots, n,\ k \neq i, j$, be an indicator random variable for the event that the great circle of node k, G_k, passes through $\mathcal{C}_{ij}$. By definition, I_k are i.i.d. Bernoulli random variables with parameter p, where $p = c_2/\sqrt{n}$ where c_2 is a positive constant. This is because a great circle passes through a disk of radius R if and only if its pole lies in an equatorial band of width $2R$. The probability of this event is $\Theta(R)$ as the position of pole is uniformly distributed over the sphere.

Thus, the total number of great circles passing through $\mathcal{C}_{ij}$ is given by a random variable $X = \sum_k I_k$ with $E[X] = (n-1)c_2/\sqrt{n} = \Theta\left(\sqrt{n}\right)$. An application of the well-known Chernoff bound for the sum of i.i.d. Bernoulli random variable (e.g., see [37]), yields

$$\begin{aligned} P\{|X - E[X]| \geq \delta E[X]\} &\leq 2\exp\left(-\delta^2 E[X]/2\right) \\ &= \frac{1}{n^3}, \quad \text{for } \delta = \sqrt{\tfrac{2(\log 2 + 3\log n)}{E[X]}}. \end{aligned} \tag{6.1}$$

The choice of δ in (6.1) shows that $X = \Theta(E[X]) = \Theta\left(\sqrt{n}\right)$ with probability at least $1 - 1/n^3$. Hence by the union bound over all $n(n-1)/2$ possible $\mathcal{C}_{ij}$ for $i, j = 1, \ldots, n$, we obtain that with probability at least $1 - 1/n$, the number of great circles passing through each $\mathcal{C}_{ij}$ is $\Theta\left(\sqrt{n}\right)$. □

The operation of Scheme Π depends on whether the configuration is typical or not. If the configuration is not typical, direct transmission is used between the S-D pairs along with time-division multiplexing. That is, the sources transmit to their destinations once in $2/n$ time-slots in a round-robin fashion. If the configuration is typical then Policy Σ_n as described below is used. Policy Σ_n is a variant of the policies presented in [21], [10].

Policy Σ_n:

1. Each time-slot is divided into two sub-slots – A and B.
2. Sub-slot A
   (a) Each source node independently becomes *active* with probability $p_\Delta > 0$.
   (b) If an active node has one or more neighbors then with probability $0 < \alpha < 1$, it chooses one at random and a packet intended for its destination is transmitted to this randomly chosen neighbor, which acts as a relay node.
3. Sub-slot B
   (a) Each node independently becomes *active* with probability $p_\Delta > 0$.
   (b) If an active node has one or more neighbors that are destination nodes, it chooses one at random. The active node, which acts as a relay, transmits a packet intended for this destination node, if it has any, in FIFO order.

In policy Σ_n, each node acts as a relay for all the other $n/2-1$ S-D pairs. A packet reaches from its source to its destination as shown in Figure 6.2. A source node, S, transmits its packet to a random relay node, R, which may also happen to be the destination itself. The random relay node then moves around carrying the packet. Finally, when it becomes a neighbor of the destination, D, the packet is transmitted to D. A relay node may receive several packets from a source before it gets a chance to transmit to the destination. To handle this, each relay node maintains a separate queue for each of the other $n/2-1$ S-D pairs. The actual mechanism is slightly more complicated. Since each node decides to transmit at random, it is possible that two nearby nodes transmit simultaneously so that some transmissions are not successful under the Protocol model.

6.2.1 Achievability of constant throughput scaling

In order to analyze the throughput of Scheme Π, we first state a result about the probability of successful transmission between two nodes when they are neighbors under policy Σ_n.

Lemma 6.2. *Under policy Σ_n, the following hold in a typical configuration.*

(a) In sub-slot A, if nodes S and R are neighbors of each other, S transmits a packet to R successfully with a strictly positive probability, independent of n.

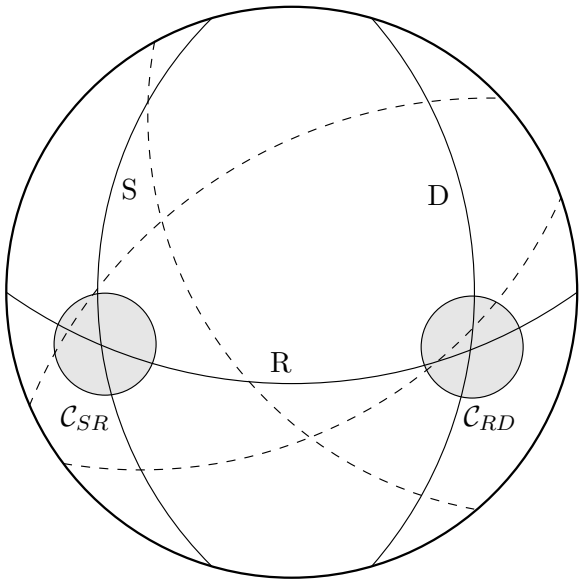


Figure 6.2: Source node, S, transmits its packet to a random relay node, R. The packet is carried by R, until its transmission to the destination node, D, when R and D become neighbors. The dotted great circles correspond to other nodes which can act as relays.

(b) In sub-slot B, if nodes R and D are neighbors of each other, R transmits a packet to D successfully with a strictly positive probability, independent of n.

Proof. We shall only prove for the case of sub-slot A since the proof for the other part is similar. Consider a sub-slot A in which S and R are neighbors. Let E_1 be the event that S becomes active and E_2 be the event that S chooses R as a random relay and no other source node in $\mathcal{C}_{SR}$ becomes active. If both events E_1 and E_2 occur, S transmits to R and the transmission is successful under the Relaxed Protocol model. Thus,

$$\begin{aligned} P(\text{ S transmits to R successfully}) &= P(E_1 \cap E_2) \\ &= P(E_1)P(E_2|E_1). \end{aligned} \tag{6.2}$$

From the description of Policy Σ_n it is clear that $P(E_1) = \alpha p_\Delta$, which is a strictly positive constant. Next we compute $P(E_2|E_1)$ and show that it is lower bounded by a strictly positive constant, independent of n, which will imply the statement of the lemma.

Given that S is active, the probability of successful transmission to R depends on how many other nodes are present in $\mathcal{C}_{SR}$ since these nodes could interfere, i.e., transmit simultaneously so that the transmission from S to R is not successful under the Relaxed Protocol

model.

Since we have a typical configuration, the number of distinct great circles of source nodes that intersect $\mathcal{C}_{SR}$ is $\Theta(\sqrt{n})$, that is, between $c_3\sqrt{n}$ and $c_4\sqrt{n}$ for some constants c_3, c_4. Moreover each great circle has $\Theta(1)$ lattice points that are in $\mathcal{C}_{SR}$. For a natural random walk on a discrete torus of size $\sqrt{n}$, the probability of being at any particular position is $1/\sqrt{n}$. Hence the probability that any of the $\Theta(\sqrt{n})$ source nodes whose great circles intersect $\mathcal{C}_{SR}$ is present in $\mathcal{C}_{SR}$ is $\Theta(1/\sqrt{n})$, that is, between $c_5\sqrt{n}$ and $c_6/\sqrt{n}$ for some constants c_5, c_6. Due to the independent movement of all nodes, we obtain that for a typical configuration, the probability of k nodes being present in $\mathcal{C}_{SR}$ is at least

$$\binom{c_3\sqrt{n}}{k}\left(\frac{c_5}{\sqrt{n}}\right)^k\left(1-\frac{c_6}{\sqrt{n}}\right)^{c_4\sqrt{n}-k} \approx \frac{(c_3c_5)^k\exp(-c_4c_6)}{k!},$$

for large enough n. If $\mathcal{C}_{SR}$ has k nodes not including S and R then S certainly has no more than $k+1$ neighbors. In this situation, R is chosen by S with probability at least $1/(k+1)$. Further there are at most k other source nodes and the probability that no other node in $\mathcal{C}_{SR}$ becomes active is at least $(1-p_\Delta)^k$. Thus,

$$\begin{aligned} P(E_2|E_1) &\geq \sum_{k=0}^{n-2}\frac{(c_3c_5)^k\exp(-c_4c_6)}{k!}\frac{1}{k+1}(1-p_\Delta)^k \\ &\geq \exp(-c_4c_5)\sum_{k=0}^{n-2}\frac{(c_3c_5(1-p_\Delta))^k}{(k+1)!}. \end{aligned}$$

It is easy to see that for $0 < p_\Delta < 1$, the term on the right hand side is lower bounded by a strictly positive constant. Hence, $P(E_2|E_1)$ is strictly positive. This completes the proof of the lemma. □

Theorem 6.1. *Scheme* Π *achieves* $T(n) = \Theta(1)$.

Proof. Consider a typical configuration so that policy Σ_n is used. Fix a source node S and a relay node R. Let $A(t)$ be the number of bits transmitted from S to R in sub-slot A of time-slot t. If S transmits to R successfully in sub-slot A of time-slot t, $A(t) = W/2$ otherwise $A(t) = 0$.

First we determine $E[A(t)]$. Let F_1 be the event that S and R are neighbors and F_2 be the event that S transmits to R successfully. Then

$$E[A(t)] = \frac{W}{2}P\{F_1 \cap F_2\} = \frac{W}{2}P\{F_1\}P\{F_2|F_1\}. \tag{6.3}$$

From Lemma 6.2(a), $P\{F_2|F_1\} \geq c_5 > 0$. Due to the independent motion of nodes S and R according to natural random walks, the joint description of their positions is a two-dimensional random walk on a discrete torus of size $\sqrt{n} \times \sqrt{n}$. It is easy to see that the stationary distribution for this process is the uniform distribution on n joint positions. Since S and R become neighbors when they are in one particular joint position out of these n joint positions, it follows that the probability of S and R being neighbors is $1/n$, i.e., $P(F_1) = 1/n$. Hence from (6.3) it follows that $E[A(t)] = \Theta(1/n)$.

Now the positions of nodes S and R form an irreducible, finite state Markov chain and $A(t)$ is a bounded, non-negative function of the state of this Markov chain at time t. Therefore by the ergodicity of such a Markov chain, the long-term throughput between S and R is

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A(t) = E[A(t)] = \Theta(1/n).$$

Thus the throughput between a source node S and any other node in sub-slot A is $\Theta(1/n)$. Similarly, it can be shown that the throughput between any node and a destination node D in sub-slot B is also $\Theta(1/n)$. The value of $0 < \alpha < 1$ guarantees that the arrival rate of packets belonging to every S-D pair at any relay node is strictly less than the service rate. This ensures the stability of the queues formed at the relay nodes, which in turn implies that the throughput between each S-D pair is simply the sum of the throughputs between S and the other $n - 1$ nodes in sub-slot A. Hence the throughput of each S-D pair is $\Theta(1)$.

We have shown that in a typical configuration, Scheme Π provides $\Theta(1)$ throughput between all S-D pairs. From Lemma 6.1, configurations are typical *whp*. Hence it follows that Scheme Π has throughput $T(n) = \Theta(1)$. □

Note that for the unrestricted mobility models in Chapter 1 and the one in [21], it is possible to prove a stronger result that each S-D pair has $\Theta(1)$ throughput for any n with probability 1, instead of probability approaching 1 as n tends to infinity, as in the present case.

6.3 Delay of the scheme

Under Scheme Π, if the configuration is not typical, direct transmission is used, in which case the delay for each packet is 1. Since the delay of a scheme is defined to be the expectation over all configurations of the average delay, the delay for Scheme Π is determined by the expected delay over typical configurations. So we shall assume that the configuration is typical.

Consider a particular S-D pair. Packets from S reach D either directly by a single hop in sub-slot A or through any of the other $n-2$ nodes, which act as relays. Since the nodes perform independent random walks, only $\Theta(1/n)$ of the packets belonging to any S-D pair reach their destination in a single hop. Thus, most of the packets reach their destination via a relay node, in which case the delay is two time-slots for two hops plus the mobile-delay, which is the time spent by the packet at the relay node.

Each relay node maintains a separate queue for each of the S-D pairs. Fix a relay node, R, and consider the queue for the S-D pair under consideration. The mobile-delay mentioned above is the delay at this relay-queue.To compute the average delay for this relay-queue, we need to study the characteristics of its arrival and potential departure processes.

6.3.1 Lower bound

First we obtain a lower bound on the delay at the relay-queue. Each node performs a random walk on a 1-D torus of size $\sqrt{n}$ on its great circle. We say that an S-D pair intersects node R's great circle k vertices apart if the lattice points where R can become neighbors of S and D are k lattice points (vertices) apart on the 1-D discrete torus of R.

Fix an S-D pair and consider a particular relay node R and suppose that this S-D pair intersects the great circle of R i vertices apart. Suppose that when a packet is transmitted successfully from S to R, D is j lattice points away from the position where it can become a neighbor of S. Let T_{ij} be the random time it takes for a random walk on a $\sqrt{n} \times \sqrt{n}$ torus to hit $(0,0)$ starting from (i,j). The first time R and D meet after a packet is transmitted from S is distributed as T_{ij}. Hence the delay for this packet in reaching D is at least T_{ij}. Of course, the delay can be much more since the packet may not be successfully transmitted or even transmitted at all when R and D meet for the first time after the arrival of the packet. Now, when a packet is transmitted successfully from S to R, D is equally likely to be in any of its $\sqrt{n}$ lattice points since it performs an independent random walk. Hence if the S-D pair intersects the great circle of R i vertices apart then the expected delay for packets of this S-D pair relayed through R is lower bounded by $\frac{1}{\sqrt{n}} \sum_{j=0}^{\sqrt{n}-1} T_{ij}$.

Using the Chernoff bound for the sum of i.i.d. Bernoulli random variable (e.g., see [37]), it can be shown that $\Theta(\sqrt{n})$ S-D pairs intersect the great circle of each node i points apart for $0 \leq i \leq \sqrt{n}-1$ *whp*. Hence the delay of Scheme Π, which is the expected delay over all packets is

$$D(n) = \Omega\left(E\left[\frac{1}{n}\sum_{i,j=1}^{\sqrt{n}-1} T_{ij}\right]\right).$$

As shown in [1], $E\left[\frac{1}{n}\sum_{i,j=1}^{\sqrt{n}-1} T_{ij}\right] = \Theta(n \log n)$. Therefore,

$$D(n) = \Omega(n \log n). \tag{6.4}$$

6.3.2 Upper bound

The rest of this section derives an upper bound which is of the same order as the lower bound. It is hard to obtain an upper bound on the delay in the relay-queue since the arrival and service processes are complicated and dependent. We progressively obtain queues that are simpler to analyze and upper bound the delay of the previous queue as follows. We first upper bound the delay in the relay-queue by that in another queue, $\mathcal{Q}_1$, in which the arrival process is simpler. The delay of $\mathcal{Q}_1$ is upper bounded by that in $\mathcal{Q}_2$, which has a relatively simpler service process. However, the arrival and service process are not independent. The final part consists of introducing a virtual server with i.i.d. Geometric service times to break this dependence. With this overview, we proceed to the details.

Recall that a packet arrives at the relay-queue when (i) S and R are neighbors, (ii) S becomes active (which happens with probability αp_Δ), (iii) S chooses R as a random relay, and (iv) the transmission from S to R is successful. Similarly, a packet can depart from the queue when (i') R and D are neighbors, (ii') R becomes active (which happens with probability p_Δ), (iii') R chooses D as the destination node, and (iv') the transmission is successful. We call such a time-slot a potential departure instant and the sequence of inter-potential-departure times is called the potential-departure process. Let the potential-departure process of the relay-queue be called $\{S_i\}$. The qualifier potential is used since a departure can occur only if R has a packet for D.

Upper bound on delay of relay-queue by that of $\mathcal{Q}_1$

Consider a queue $\mathcal{Q}_1$ in which arrivals happen whenever (i), (ii) and (iii) above are satisfied, irrespective of whether (iv) is satisfied or not. The potential departure process for $\mathcal{Q}_1$ is the same as that for the relay-queue. Then it is clear that the expected delay in $\mathcal{Q}_1$ provides an upper bound on that in the relay-queue.

Recall that the motion of each node is an independent 1-D random walk on a discrete torus of size $\sqrt{n}$. We will say that two nodes *meet* when they become neighbors. Since nodes move independently the joint position of nodes R and D is a random walk on a $\sqrt{n} \times \sqrt{n}$ discrete torus and R and D become neighbors when the 2-D random walk is in state $(0, 0)$, without loss of generality. Therefore, the inter-meeting time of R and D is distributed like the inter-visit time of state $(0, 0)$ of a 2-D random walk. Since this is a Markov chain with n

states having a uniform stationary distribution, we know that the sequence of inter-meeting times of nodes R and D, denoted by $\{\tau_i, i \geq 0\}$, is an i.i.d. process. Further, if τ is a random variable with the common distribution then

$$E[\tau] = n. \tag{6.5}$$

However a potential departure instant does not occur each time R and D meet. A potential departure instant occurs only if R also becomes active, chooses D as the random destination and the transmission is successful. If R and D are not chosen in spite of being in the same cell, it increases the likelihood of there being many more nodes in the same cell. Due to the random walk model of the node mobility, if there is a crowding of nodes in some part of the network then it remains crowded for some time in the future. Hence due to the Markovian nature of node mobility, the inter-potential-departure times are not independent.

Upper bound on delay of $\mathcal{Q}_1$ by that of $\mathcal{Q}_2$

We want to obtain an upper bound on the delay of $\mathcal{Q}_1$ which has potential-departure process $\{S_i\}$. To do this we will consider a queue, $\mathcal{Q}_2$, which has the same arrival process as $\mathcal{Q}_1$ but a different departure process $\{\tilde{S}_i\}$ such that $\tilde{S}_i|\tilde{S}_{i-1}, \ldots$ stochastically dominates $S_i|S_{i-1}, \ldots$. Then the expected delay in $\mathcal{Q}_2$ would provide an upper bound on the the expected delay in the relay-queue.

Nodes R and D perform independent random walks on 1-D tori of size $\sqrt{n}$ on their great circles as shown in Figure 6.2 and R and D meet when both are at a particular pair of lattice points. This is represented schematically in Figure 6.3, where R performs a vertical 1-D random walk and D performs a horizontal 1-D random walk. The joint motion of nodes R and D is equivalent to a random walk on a 2-D torus of size $\sqrt{n} \times \sqrt{n}$ and R and D meet when this 2-D random walk is in state $(0, 0)$. The inter-meeting times of nodes R and D correspond to the i.i.d. process $\{\tau_i\}$. Further, let $\alpha_i = \tau_1 + \ldots + \tau_i$ for $i \geq 1$, i.e., α_i is the time-slot in which R and D meet for the ith time. In a typical configuration, we know that the number of other great circles that pass through $\mathcal{C}_{RD}$ is $\Theta(\sqrt{n})$. Allowing for the worst case, based on Lemma 6.1, let there be $c_7\sqrt{n} = m - 2$ other great circles that pass through $\mathcal{C}_{RD}$. These can also be thought of as performing independent random walks on the horizontal 1-D torus. Let nodes R and D be numbered 1 and 2 and the other $c_7\sqrt{n}$ nodes be numbered from 3 to m and let $X(t) = (X_1(t), \ldots, X_m(t))$ denote the position of these m nodes on the $\sqrt{n} \times \sqrt{n}$ discrete torus at time t.

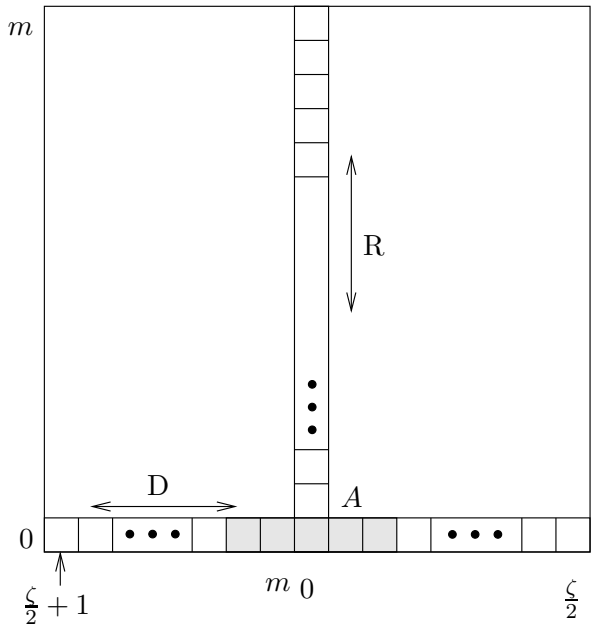


Figure 6.3: Schematic representation of the motion of nodes R and D on their respective great circles with $\zeta = \sqrt{n} - 1$.

A constant number of lattice points of the 1-D torus correspond to $\mathcal{C}_{RD}$ and these are shown by the shaded region in Figure 6.3 and is referred to as set A. Let E_i be the indicator for the event that R chooses D and the transmission is successful in time-slot α_i. That is, E_i is the indicator for the event that α_i is a potential departure instant. Let N_i be the number of other destination nodes in A in time-slot α_i. Then $P\{E_i = 1\}$ depends on N_i only. Now, N_i depends on $X(\alpha_i)$ which depends on the past given by $E^{i-1} = \{E_0, \ldots, E_{i-1}\}$ and $\tau^i = \{\tau_0, \ldots, \tau_i\}$. Thus the potential-departure process is generated by choosing some of the meeting instants of R and D according to a probability modulated by N_i, which is another independent process as shown in Figure 6.4.

Above we described how the process $\{S_i\}$ can be generated using the processes $\{N_i\}$ and $\{\tau_i\}$, which in turn were obtained from $\{X(t)\}$, which corresponds to the independent random walks of all m nodes. Next we shall perturb the process $\{X(t)\}$ to obtain $\{\tilde{X}(t)\}$ and the corresponding $\{\tilde{\tau}_i\}$ and $\{\tilde{N}_i\}$. Let $Z(t)$ be a 1-D horizontal random walk on a torus of size $\sqrt{n}$. Let $\tilde{X}_i(t) = X_i(t) + Z(t)$ be the position of node $i, 1 \leq i \leq m$, where the addition is modulo $\sqrt{n}$. Then the inter-meeting times of any two nodes are the same as before since the position of each node is shifted horizontally by the same amount due to $Z(t)$. As a result the processes τ_i and $\tilde{\tau}_i$ are identical. Under the modified setup, the lattice point at which R and D meet can be any element of the set $B = \{(i, 0) : 0 \leq i \leq \sqrt{n} - 1\}$ instead

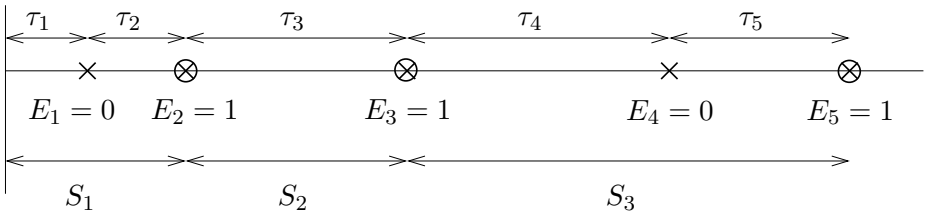


Figure 6.4: The 'x' marks correspond to the times when R and D meet each other. At some of these meeting instants the R-D transmission can be successful. Such points have been circled and correspond to $E_i = 1$. The inter-potential-service times are thus the sum of a few of the inter-meeting times of R and D.

of always being $(0, 0)$. The above perturbation was used to obtain this property which is crucial for the proof technique of Lemma 6.3. Let $\tilde{N}_i$ be the number of other destination nodes in the set $A + Z(t)$. Then, $\{\tilde{N}_i\}$ is identical to $\{N_i\}$. Thus the process $\{S_i\}$ can also be generated (through $\{E_i\}$) using $\{\tilde{N}_i\}$ and $\{\tilde{\tau}_i\}$ instead of $\{N_i\}$ and $\{\tau_i\}$. Therefore we shall use $\tilde{X}_i(t)$ as the position of node i at time t instead of $X_i(t)$. Under this perturbed motion, R can be seen as if it performs a 2-D random walk on the $\sqrt{n} \times \sqrt{n}$ torus while D and the other $m - 2$ nodes perform a 1-D random walk on a 1-D torus of size $\sqrt{n}$ which is subset B of the 2-D torus. Moreover, given $\tilde{X}_3^m(\alpha_i) = (\tilde{X}_3(\alpha_i), \ldots, \tilde{X}_m(\alpha_i))$, $P\{E_i = 1\}$ is independent of everything else.

Lemma 6.3. *There exists a constant (independent of n) $c_8 > 0$ such that*

$$P\left(E_i = 1 | \tau^i, E^{i-1}\right) \geq c_8 > 0.$$

Proof. The initial position of R, $X_1(0)$ has a uniform distribution on the $\sqrt{n} \times \sqrt{n}$ torus. The initial positions of D and nodes 3 to m have independent uniform distributions on subset $B = \{(i, 0) : 0 \leq i \leq \sqrt{n} - 1\}$ of the $\sqrt{n} \times \sqrt{n}$ torus. As a result $X_1(\alpha_1) = X_2(\alpha_1) = I$ where I is a random variable with a uniform distribution over B.

Let $V = (\tilde{X}_3(\alpha_i), \ldots, \tilde{X}_m(\alpha_i))$ be the configuration of the $m-2$ nodes other than R and D. Then the conditional probability of a potential departure given the past can be written

as

$$\begin{aligned}
&P\left(E_i = 1|\tau^i, E^{i-1}\right) \\
=&\sum_V \frac{P\left(E_i = 1|V, \tau^i, E^{i-1}\right) P\left(V, \tau^i, E^{i-1}\right)}{P(\tau^i, E^{i-1})} \\
\geq&\min_V P(E_i = 1|V, \tau^i, E^{i-1}) \left[\sum_V \frac{P(V, \tau^i, E^{i-1})}{P(\tau^i, E^{i-1})}\right] \\
=&\min_V P(E_i = 1|V, \tau^i, E^{i-1}) \\
=&\min_V P\left(E_i = 1|V\right), \qquad (6.6)
\end{aligned}$$

where the last equality holds because E_i is independent of everything else given V.

Given a configuration V, the number of nodes in $A+(i-1,0)$ for $i = 1, \ldots, \sqrt{n}$ torus can be found and this in turn determines the $P\left(E_i = 1|V\right)$. Hence, if V_i denotes the number of nodes other than R and D in the set $A+(i-1,0)$ for $i = 1, \ldots, \sqrt{n}$ then we can equivalently let the configuration be $V = (V_1, \ldots, V_{\sqrt{n}})$.

Now consider a fixed configuration, $V = v = (v_1, \ldots, v_{\sqrt{n}})$, and let Z be a random variable which takes value $v_i, 1 \leq i \leq \sqrt{n}$ with probability $1/\sqrt{n}$. Let A consist of c_9 (some constant) elements. Then

$$E[Z] = \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} v_k = \frac{c_9(m-2)}{\sqrt{n}} = \Theta(1). \qquad (6.7)$$

Recall that $X_1(\alpha_i) = I$, where I is a random variable with uniform distribution on B. Further, from the description of Scheme II, if there are v_k destination nodes other than D in $\mathcal{C}_{RD}$ then $E_i = 1$ if R chooses D out of all destination nodes that are its neighbors and the other v_k nodes do not transmit. Since $\mathcal{C}_{RD}$ contains all neighbors and more, the number of neighbors can be no more than X_i and hence for $k = 1, \ldots, \sqrt{n}$, we obtain

$$P(E_i = 1|V = v, X_1(\alpha_i) = (k-1, 0)) \geq \frac{p_\Delta(1-p_\Delta)^{v_k+1}}{v_k+1}. \qquad (6.8)$$

Define a real valued function $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \frac{p_\Delta(1-p_\Delta)^{x+1}}{x+1}$. It is easy to check that $f(\cdot)$ is a convex function. Hence, by Jensen's inequality,

$$E[f(Z)] \quad \geq \quad f(E[Z]). \qquad (6.9)$$

Using (6.7), (6.8) and (6.9), for any configuration V with corresponding v, we obtain

$$\begin{aligned}
&P(E_i = 1|V = v)\\
=&\sum_{k=1}^{\sqrt{n}} P(E_i = 1|V = v, X_1(\alpha_i) = (k-1, 0))\\
&\quad P(X_1(\alpha_i) = (k-1, 0)|V = v)\\
=&\frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} P(E_i = 1|V = v, X_1(\alpha_i) = (k-1, 0))\\
\geq&\frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} \frac{p_\Delta(1-p_\Delta)^{v_k+1}}{v_k+1}\\
=&E[f(Z)]\\
\geq&f(E[Z])\\
=&f\left(\frac{c_9(m-2)}{\sqrt{n}}\right) \stackrel{\triangle}{=} c_8 > 0. \qquad (6.10)
\end{aligned}$$

Combining (6.6) and (6.10) completes the proof of the lemma. □

Recall that the process $\{S_i\}$ is generated from $\{\tau_i\}$ and $\{E_i\}$. Consider an i.i.d. Bernoulli process $\{\tilde{E}_i\}$ with $P\{\tilde{E}_1 = 1\} = c_8$. Now we can construct a process $\{\tilde{S}_i, i \geq 1\}$ similar to the process $\{S_i\}$ using $\{\tau_i\}$ and $\{\tilde{E}_i\}$ instead of $\{E_i\}$. From Lemma 6.3 and from the construction of the processes $\{S_i\}$ and $\{\tilde{S}_i\}$, it is easy to verify that $\tilde{S}_i|\tilde{S}_{i-1}, \ldots$ stochastically dominates $S_i|X_{i-1}, \ldots$. Now consider queue, $\mathcal{Q}_2$, with the same arrival process as $\mathcal{Q}_1$ but with potential-departure process $\{\tilde{S}_i\}$. Depending on the value of c_8, the value of α can be chosen so that the arrival rate is strictly smaller than the potential departure rate in $\mathcal{Q}_2$ so as to ensure stability. The distribution of $\tilde{S}_1$ is the same as $\tau_1 + \ldots + \tau_G$, where G is an independent Geometric random variable with parameter c_8. As a result, for any $r \in \mathbb{N}$,

$$E[\tilde{S}_1^r] = \Theta(E[\tau_1^r]). \qquad (6.11)$$

In light of (6.11), it is easy to see that the delay scaling of queue $\mathcal{Q}_2$ is the same as the delay scaling of a queue in which an arrival happens each time S and R meet with probability 0.5 and a potential departure occurs each time R and D meet. Since we are interested only in the delay scaling, henceforth we assume that in $\mathcal{Q}_2$, an arrival happens when S and R meet with probability 0.5 and a potential departure occurs whenever R and D meet.

At this stage we have upper bounded the delay in the relay-queue by the delay in $\mathcal{Q}_2$.

The inter-arrival times and the inter-potential departure times in $\mathcal{Q}_2$ are i.i.d. processes. However these two processes are not independent for the following simple reason: if the S-D pair intersects the great circle of R, $k > 0$ vertices apart then R has to travel at least distance k on the discrete torus after an arrival for a potential departure to occur.

The following theorem easily follows by upper bounding the delay in $\mathcal{Q}_2$ as in the proof of Theorem 5.4 in Chapter 5. See [36] for details.

Theorem 6.2. *The delay of Scheme* Π *is* $\Theta(n \log n)$.

6.4 Discussion

In this chapter, we studied the maximum throughput scaling and the corresponding delay scaling in a random mobile network with restricted node mobility. In [10], it was shown that a particular mobility restriction does not affect the throughput scaling. In this chapter, we showed that it does not affect delay scaling either. In particular, we showed that constant throughput scaling with a delay of order $n \log n$ is achievable. This is the same as the delay scaling without any mobility restriction, which was studied in Chapter 5. This is understood to be a consequence of the fact that in spite of an apparent restriction, essentially the node mobility remains unchanged in the sense that (i) each node meets every other node for $\Theta(1/n)$ fraction of the time with only $\Theta(1)$ other neighboring nodes, and (ii) the inter-meeting time of nodes has mean of $\Theta(n)$ and variance of $O(n^2 \log n)$.

Chapter 7

Conclusion

Throughput, delay, and energy consumption are the main metrics of performance in a communication network and the network is characterized by the trade-offs between these metrics. We studied how these trade-offs scale with the number of nodes in an ad hoc wireless network. We used a probabilistic framework in which our results hold with high probability, as the number of nodes increases. Thus our results characterize the performance of large wireless networks.

Our work results in a more complete understanding of the random network framework for static and mobile networks, initiated in [23] and [21], respectively. In addition to throughput, which was studied in previous work, our study includes delay and energy consumption, and the trade-offs between them.

An important implication of our work for static networks is that simple schemes are optimal in terms of the trade-off between throughput, delay, and energy. These simple schemes involve dividing the network into cells, using time division multiplexing between cells to mitigate interference, and routing packets by hops along adjacent cells on the shortest path. Thus cooperation between nodes, by relaying data for each other, results in the best possible performance, although performance inevitably degrades as the number of nodes in the network increase. As shown in Chapter 4, the size of the cells, or equivalently the amount of hopping, depends on the area covered by the network and the trade-off point at which the network is to be operated. Our work also suggests that infrastructure may be necessary for some applications.

An important insight provided by our work for mobile networks is that mobility is a hindrance in the low throughput range, in the sense that it increases the complexity of data communication schemes. Another surprising insight is that attempting to achieve any throughput higher than that achievable in static networks, by using node mobility, incurs

a significantly higher delay.

In order to analytically determine the fundamental trade-offs, we implicitly assumed that centralized knowledge of the location of nodes is available. In static networks, one can have an initialization phase in which the location of nodes, the cellular structure, and the shortest paths are determined. However, in mobile networks, this involves a lot of overhead since nodes are moving. This suggests using algorithms that require only local information. Investigation of such low-overhead algorithms for initialization and maintenance, with performance guarantees, would be very useful.

Our study was based on treating the wireless network as a data communication network. In several applications, data communication, although a critical aspect, is not the main goal. For example, a wireless network of cameras for surveillance would need to process the data from all cameras and detect and flag suspicious events. In several interesting applications, the goal is information processing in networks so that the computing and communication capabilities at all nodes can be effectively used for performing the desired task. This calls for the investigation of fast and energy-efficient algorithms with performance guarantees for information processing in networks. See [41], [39], [25] for representative samples of recent work in this direction.

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