Strong Interference Channel With Delay

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Abstract

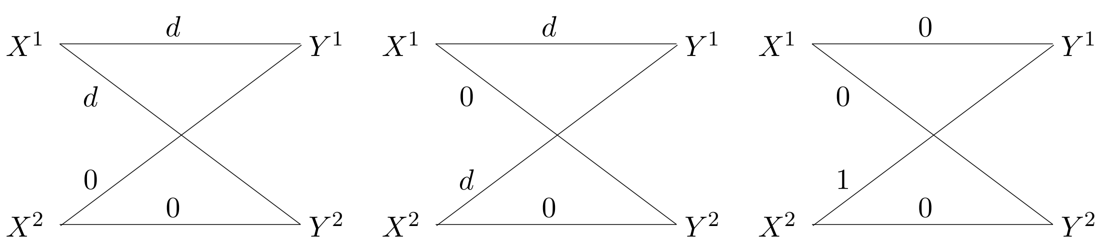
The paper investigates the effect of link delay on the capacity region of the interference channel with strong interference. It is shown that the capacity region of the strong interference channel with unit delay on one of the cross links (SICD) is at least as large as that without delay. This is established by obtaining a "multi-letter" characterization of the capacity region of SICD and showing that it contains the capacity region of the strong interference channel (SIC). For the case of the Gaussian SIC, it is shown that delay does not affect capacity.

I. Introduction

Delay plays an important role in computing, networking, and control systems. How does it affect the information theoretic capacity of multi-user channels? First consider a discrete-memoryless point-to-point channel (DMC) consisting of an input alphabet $\mathcal{X}$, an output alphabet $\mathcal{Y}$ and a probability mass function (pmf) $p(\cdot|x)$ on $\mathcal{Y}$ for each $x \in \mathcal{X}$. Sender 1 transmits the symbol x_t^1 at time t, which results in y_t^1 at the Receiver 1 chosen according to $p(y_t^1|x_t^1)$. If instead there is a transmission delay of $d \in \{1, 2, \dots\}$, then the pmf of the received symbol at time t would be given by $p(\cdot|x_{t-d}^1)$. We refer to this channel as DMC with *link delay* since the output at time t depends on the input at time $t-d$. It is easy to see that the capacity of the delayed channel is unchanged by applying a time shift at the decoder. Similarly it can be shown that link delays do not affect the capacity regions of the multiple access channel or the broadcast channel. In [2], it was shown, however, that link delay can change the capacity of the relay channel. Further, in [3], the effect of link delays on relay network capacity was explored.

In this paper, we investigate the effect of link delay on the capacity of the interference channel (IC). Since there are four links from the inputs to the outputs of a 2-user interference channel, several scenarios can arise depending on the placement of delays as shown in Fig. 1. If there are equal delays on both of the outgoing links of any encoder (see Fig. 1(a)) then it is easy to see that the capacity region remains unchanged by using a shift at that encoder. If there are equal delays on both incoming links of any decoder (see Fig. 1(b)) then again it is clear that the capacity region is unchanged by using a shift at the decoder.

Now consider the case where there is a unit delay on the cross link from X^2 to Y^1 but there are no delays on any of the other links as shown in Fig. 1(c). In this case it is not obvious that the capacity region with delay is the same as that without delay. This is because using a shift at any of the encoders or decoders does not transform the problem to that of IC without delay. The same effect arises if a unit delay is introduced on any on of the four links of the interference channel. We study the case of a unit delay on the cross-link from X^2 to Y^1 for IC as it is representative example of all the above mentioned cases. Since the capacity region for the general interference channel is not known, we restrict ourselves to IC with strong interference [1].



(a) Equal delay d on both outgoing links. (b) Equal delay d on both incoming links. (c) Unit delay on cross link from X^2 to Y^1.

Fig. 1. Three link delay scenarios for the interference channel.

II. Interference Channel with Strong Interference

A discrete-memoryless interference channel $(\mathcal{X}^1, \mathcal{X}^2, p(y^1, y^2|x^1, x^2), \mathcal{Y}^1, \mathcal{Y}^2)$ consists of four finite sets $\mathcal{X}^1$, $\mathcal{X}^2$, $\mathcal{Y}^1$ and $\mathcal{Y}^2$ and a collection of conditional pmfs $p(\cdot, \cdot|x^1, x^2)$ on $\mathcal{Y}_1 \times \mathcal{Y}_2$ for each x^1, x^2 in $\mathcal{X}^1 \times \mathcal{X}^2$.

A $((2^{nR_1}, 2^{nR_2}), n)$ code for the interference channel consists of: (i) Two messages (W_1, W_2) uniformly distributed over $[1, 2^{nR_1}] \times [1, 2^{nR_2}]$, (ii) two encoders, Encoder i, $i = 1, 2$, maps each message $w_i \in [1, 2^{nR_i}]$ into a codeword $x^i_{1:n}(w_i)$, and (iii) two decoders, Decoder i, $i = 1, 2$, maps each received sequence $y^i_{1:n}$ into a message $\hat{w}_i \in [1, 2^{nR_i}]$ or an error message, .

The channel is memoryless in the sense that for each pair of codewords $x^1_{1:n}, x^2_{1:n}$, the pmf of the received sequence is given by

$$p(y^1_{1:n}, y^2_{1:n}|x^1_{1:n}, x^2_{1:n}) = \prod_{t=1}^{n} p(y^1_t, y^2_t|x^1_t, x^2_t).$$

The average probability of error is $P_e^{(n)} = P\{\hat{W}_1(Y_1^n) \neq W_1 \text{ or } \hat{W}_2(Y_2^n) \neq W_2\}$. A rate pair (R_1, R_2) is said to be *achievable* if there exists a sequence of $((2^{nR_1}, 2^{nR_2}), n)$ codes with $P_e^{(n)} \to 0$. The *capacity region* of the interference channel is the closure of the set of achievable rates and is not in general known. Note that it depends only on the marginal transition probabilities $p(y^1|x^1, x^2)$ and $p(y^2|x^1, x^2)$.

The capacity region, however, is known for the *strong interference channel*, which is an interference channel such that

$$I(X^1_t; Y^1_t|X^2_t) \leq I(X^1_t; Y^2_t|X^2_t) \text{ and } I(X^2_t; Y^2_t|X^1_t) \leq I(X^2_t; Y^1_t|X^1_t),$$

for any $p(x^1)p(x^2)$. From now on, we shall refer to this channel as SIC (Strong Interference Channel). The capacity region of SIC was established in [4]. Below, we restate the result in [1].

Theorem 1: The capacity region of the interference channel with strong interference, C^{SIC}, is the union of all rate pairs (R_1, R_2) satisfying

$$0 \leq R_1 \leq I(X^1; Y^1|X^2, Q),\ 0 \leq R_2 \leq I(X^2; Y^2|X^1, Q),$$
$$R_1 + R_2 \leq \min\{I(X^1, X^2; Y^1|Q), I(X^1, X^2; Y^1|Q)\},$$

where the union is over all probability distributions of the form $p(q)p(x^1|q)p(x^2|q)p(y^1, y^2|x^1, x^2)$ and $|Q| \leq 4$.

The Gaussian interference channel is depicted in Fig. 2. The Gaussian interference channel is said to have strong interference [4] if $\alpha^2 N_2 \geq N_1$ and $\beta^2 N_1 \geq N_2$. We shall refer to this channel as GSIC and its capacity region as C^{GSIC}. Sato [4] showed that the capacity region with average power constraint P_i on X^i, $i = 1, 2$, is given by

Theorem 2: Let $\mathcal{C}(x) = 1/2 \log(1 + x)$ then C^{GSIC} is the set of rate pairs (R_1, R_2) such that:

$$0 \leq R_1 \leq \mathcal{C}\left(\frac{P_1}{N_1}\right),\ 0 \leq R_2 \leq \mathcal{C}\left(\frac{P_2}{N_2}\right),$$
$$R_1 + R_2 \leq \min\left\{\mathcal{C}\left(\frac{P_1 + \alpha^2 P_2}{N_1}\right), \mathcal{C}\left(\frac{\beta^2 P_1 + P_2}{N_2}\right)\right\}.$$

III. SIC with Delay

Now we move on to the case with delay on a single cross link. Formally, a discrete-memoryless interference channel with delay $(\mathcal{X}^1, \mathcal{X}^2, p(y^1_t, y^2_t|x^2_{t-1}, x^1_t, x^2_t), \mathcal{Y}^1, \mathcal{Y}^2)$ consists of four finite sets $\mathcal{X}^1$, $\mathcal{X}^2$, $\mathcal{Y}^1$ and $\mathcal{Y}^2$ and a collection of conditional pmfs $p(\cdot, \cdot|x^2_{t-1}, x^1_t, x^2_t)$ on $\mathcal{Y}_1 \times \mathcal{Y}_2$ for each x^2_{t-1}, x^1_t, x^2_t in $\mathcal{X}^2 \times \mathcal{X}^1 \times \mathcal{X}^2$. Due to the memorylessness of the channel, for each pair of codewords $x^1_{1:n}, x^2_{1:n}$ the pmf of the received sequence is given by

$$p(y^1_{1:n}, y^2_{1:n}|x^1_{1:n}, x^2_{1:n}) = \prod_{t=1}^{n} p(y^1_t, y^2_t|x^2_{t-1}, x^1_t, x^2_t).$$

If any of the time indices of the conditioning variables is not positive (that is, not defined), then the corresponding conditional pmf is arbitrary.

The definitions of codes, average probability of error, achievability, and the capacity region of the interference channel with delay are the same as for the interference channel (without delay).

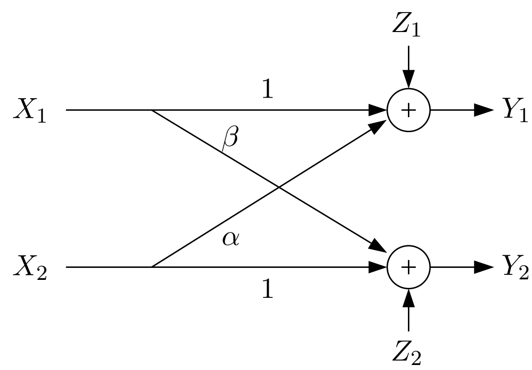


Fig. 2. The Gaussian interference channel.

The interference channel with delay and strong interference has the property that

$$I(X_t^1; Y_t^1 | X_{t-1}^2) \leq I(X_t^1; Y_t^2 | X_t^2),$$
$$I(X_t^2; Y_t^2 | X_t^1) \leq I(X_{t-1}^2; Y_t^1 | X_t^1),$$

for all $p(x_{t-1}^2)p(x_t^1)p(x_t^2)$. We refer to this channel as SICD (Strong Interference Channel with Delay). The Gaussian SICD is defined analogously to the discrete memoryless SICD defined above. We shall refer to it as GSICD and denote its capacity region by C^{GSICD}. The following theorem characterizes the capacity region of SICD.

Theorem 3: The capacity region of the discrete memoryless interference channel with delay and strong interference, C^{SICD}, is the union of all rate pairs (R_1, R_2) satisfying

$$0 \leq R_1 \leq I(X^1; Y^1 | \tilde{X}^2, Q),$$
$$0 \leq R_2 \leq I(X^2; Y^2 | X^1, Q),$$
$$R_1 + R_2 \leq \min\{I(X^1, \tilde{X}^2; Y^1 | Q), I(X^1, X^2; Y^1 | Q)\},$$

where Q is an auxiliary random variable with uniform distribution on $\{1, ..., n\}$ for some $n < \infty$ and the union is over all probability distributions of the form $p(q)p(x^1|q)p(x^2|q)p(\tilde{x}^2|q)p(y^1, y^2|\tilde{x}^2, x^1, x^2)$ such that $p(\tilde{x}^2|q) = p(x^2|q-1)$ for all $q \in \{1, ..., n\}$, where $p(x^2|0) = p(x^2|n)$.
Note that the above result is not a "single-letter" characterization of the capacity region since no bound on the cardinality of Q is provided.

We can show that the capacity region above is not smaller than the capacity region without delay.

Theorem 4: The capacity region of SIC is contained within that of SICD, that is, $C^{SIC} \subset C^{SICD}$.

For the GSICD channel, we can easily show that the capacity region is equal to that of the GSIC.

Theorem 5: The capacity region of GSICD coincides with that of GSIC; that is, $C^{GSIC} = C^{GSICD}$.

References

[1] M. Costa and A. El Gamal, "The Capacity Region of the Discrete Memoryless Interference Channel with Strong Interference," *IEEE Trans. on Inform. Th.*, 27(6), Nov. 1987.
[2] A. El Gamal and N. Hassanpour, "Relay-Without-Delay," *IEEE ISIT*, Adelaide, Sep. 2005.
[3] A. El Gamal and J. Mammen , "Relay Networks With Delay," *UCSD Workshop on Inform. Th. and its Applications*, San Diego, Feb. 2006.
[4] H. Sato, "The Capacity of the Gaussian Interference Channel Under Strong Interference," *IEEE Trans. on Inform. Th.*, 27(6), Nov. 1981.