Optimal Hopping in Ad Hoc Wireless Networks

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***Abstract*— Gupta and Kumar showed that throughput in a static random wireless network increases with the amount of hopping. In a subsequent paper (2004), it was shown that although throughput benefits from a large number of hops, this comes at the expense of higher delay. Separately, several studies have shown that in ad hoc networks, transmission energy decreases as hopping increases. However, when transceiver circuit energy is also taken into account, hopping as much as possible no longer maximizes energy efficiency and the optimal amount of hopping depends on the topology and size of the network.**

This paper attempts to unify these earlier results by establishing the optimal hopping for energy efficiency along with throughput and delay. A random network model with n nodes in area $A(n)$ is considered. The effect of interference is captured by the Physical model and the signal is assumed to decay with distance r as $r^{-\delta}$, $\delta > 1$. Both transmission and transceiver circuit energy are taken into account. Optimal trade-offs between throughput, delay and energy-per-bit scaling for this random network model are established. These results show that the amount of hopping still determines the optimal trade-off and yield the amount of hopping that should be used to achieve any point of the optimal trade-off. In a constant area network, where $A(n) = 1$, $\Theta(1)$ hops result in the best energy and delay scaling of $\Theta(1)$ at the cost of the worst throughput scaling of $\Theta(1/n)$. At the other extreme, in a constant density network, where $A(n) = n$, $\Theta\left(\sqrt{n/\log n}\right)$ hops result in the best throughput scaling of $\Theta\left(1/\sqrt{n \log n}\right)$ and the best energy scaling but at the cost of the worst delay scaling. For intermediate values of $A(n)$, $\sqrt{B(n)} = \Theta\left(\sqrt{\min\{A(n), n/\log n\}}\right)$ hops should be used to obtain the minimum energy-per-bit scaling, which is $\Theta\left(A(n)^{\delta} B(n)^{\frac{1}{2}-\delta}\right)$.

I. INTRODUCTION

Wireless networks of the future are envisioned to be very large and possibly ad hoc, i.e., without fixed infrastructure. To study such large, ad hoc networks, Gupta and Kumar [6] introduced a random network model with n nodes. They showed that the throughput scaling is $O\left(1/\sqrt{n \log n}\right)$ for almost all network realizations. One of the insights provided by their work is that the highest throughput scaling is obtained at the smallest transmission range, or equivalently, maximum amount of hopping is needed to mitigate interference in the network.

Subsequently in [2], the optimal throughput-delay trade-off for a random wireless network was established to be $D(n) = \Theta(nT(n))$, where $T(n)$ and $D(n)$ are the throughput and delay, respectively. This work showed that at an optimal point of the trade-off, delay scales as the number of hops and both throughput and delay increase as the amount of hopping increases. Thus the amount of hopping determines the point of the optimal trade-off at which the network operates; more hops results in higher throughput but also higher delay.

Energy is another important metric and assumes great significance in the case of networks with limited energy resources, particularly sensor networks. With the assumption that signal attenuates with distance r as $r^{-\delta}$, it is easy to see that if relay nodes are placed uniformly between the source and destination, the total transmission energy decreases as the amount of hopping increases. This leads one to believe that hopping as much as possible is good for minimizing energy-per-bit. However, when transceiver circuit energy is also taken into account, this is no longer the case and the optimal number of hops depends on the topology and the size of the network [8]. A large body of work exists on optimizing one of throughput, delay and energy subject to a constraint on one of the others for a given placement of nodes (see [7] for a representative sample). These studies have focused on optimizing the performance metrics for a given realization of the network, which is quite different from the framework of random networks where the goal is to obtain general guidelines for an entire class of networks.

Closer in spirit to the work in this paper is [1], which studied the energy efficiency of random wireless networks. The focus of this work was on energy efficiency without regard to throughput or delay and without considering interference and transceiver circuit energy. In [3], a preliminary study of the trade-off between throughput, delay and transmission energy for constant area random wireless networks was conducted. Both these works reached the conclusion that energy efficiency increases with hopping.

In this paper we determine the optimal hopping for energy efficiency along with throughput and delay using the random network framework, thus extending the work in [2]. We assume that n nodes are randomly placed in area $A(n)$ and are split into $n/2$ source-destination pairs. We allow the density of nodes in the network, $n/A(n)$, to be anywhere between 1, for a constant density network, where $A(n) = n$, and n for a constant area network, where $A(n) = 1$. The reason for characterizing networks according to their node densities, as we shall show later, is that node density determines the amount of optimal hopping in a network. We assume the Physical model for successful transmission [6], which captures the effect of interference in the network due to other simultaneously transmitting users. We consider both Radio-frequency (RF) transmission energy and transceiver circuit energy. Thus the energy used in communicating a bit from a source to a destination has two components – one due to transmission, which depends on the number of hops and the distance and power used at each hop, and the other due to transceiver circuit energy, which is proportional to the number of hops. Note that by increasing the number of hops, the transmission energy component decreases whereas the transceiver energy component increases. We establish the optimal trade-offs between throughput, delay and energy for this random network model. We find that even after the inclusion of energy consumption into the model, which allows for power control, hopping continues to determine the optimal trade-offs between throughput, delay and energy. This happens because the amount of hopping determines the amount of power to use for optimal energy scaling. As a by-product, we obtain the amount of hopping that results in the minimum energy scaling.

We note that the model in this paper differs from that in [3] in three respects. First, in [3], a constant area model was used, which is a special case of the general case of area $A(n)$ considered in this paper. Second, transceiver circuit energy was not taken into account. Third, in [3], a combination of the Protocol model and a rate function depending on the power and distance based on the AWGN channel was used, instead of the more widely used Physical model. These differences yield more general results that are consistent with the work in [2]. New proofs were required due to these differences in the model and the more comprehensive nature of results in this paper.

The outline of the rest of the paper is as follows. Section II presents the random network model, required definitions and a preview of the main results. Section III states Theorem 1, which is the main result of this paper and two corollaries. Section IV presents a cellular scheme that achieves the trade-off that is stated to be optimal in Theorem 1. Section V shows that the trade-off achieved by this scheme is optimal, in that no scheme can outperform it in terms of scaling. Finally in Section VI, we discuss some of the implications of our results on the design of wireless ad hoc networks.

II. Model and Main Results

We first introduce the network model and the definitions of the terms used in the paper.

Definition 1 (Random network model): The random network consists of a torus of area $A(n)$ in which n nodes are distributed uniformly at random. These n nodes are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for packetized transmission. For simplicity, we assume that the time-slots are of unit length.

The network area $A(n)$ is allowed to be a function of the number of nodes n and the density of nodes in the network is $n/A(n)$. We refer to the case when $A(n) = 1$ as the *constant area* model. The other extreme is the *constant density* model in which $A(n) = n$, i.e., n nodes are placed in a torus of area n so that the density of nodes is 1.

Let the distance on the torus between two nodes i and j be denoted by r_{ij}. We assume that signal decays with distance r as $r^{-\delta}$ so that power decays as $r^{-2\delta}$, where $\delta > 1$. We assume the Physical model for successful transmission presented in [6] and also used by several others since.

Definition 2 (Physical model): A transmission from node i to j is successful if

$$SINR = \frac{P r_{ij}^{-2\delta}}{N + \sum_{k\in\Gamma, k\neq i} r_{kj}^{-2\delta}} \geq \beta,$$

where Γ is the set of simultaneously transmitting nodes. When the transmission is successful, communication occurs at a constant rate W.

The basis of the Physical model is the AWGN channel with noise power N where interference from other transmitting users is treated as independent Gaussian noise. Thus when the transmission from node i to node j is successful under the Physical model, the energy-per-bit due to RF transmission is $P/W \geq \beta(N+I)r_{ij}^{2\delta}$, where I is the total interference power at node j due to other simultaneous transmissions.

In order to take the transceiver circuit power into account, we assume a constant amount of energy per bit, c_0, is also dissipated during each transmission/reception [8]. Thus the total energy-per-bit for a successful transmission from node i to j is $P/W + c_0 \geq \beta(N+I)r_{ij}^{2\delta} + c_0$.

Some authors assume the signal to attenuate with distance r as $(1+r)^{-\delta}$ since the $r^{-\delta}$ attenuation is valid only in the far field. This results in transmission energy-per-bit approaching a constant strictly greater than 0 as the distance approaches 0. We note that the results in this paper remain unchanged under this model for the constant density random network. This is because a constant amount of energy is anyway consumed at each hop due to the circuits and further the rate is constant when transmission is successful.

We now define a communication scheme and associated performance metrics.

Definition 3 (Scheme): A scheme Π for a random network is a sequence of communication policies, (Π_n), where policy Π_n determines how communication occurs in a network of n nodes.

Let the maximum power used by any node in a scheme be $P_{max}(n)$ and let the minimum power be $P_{min}(n)$. We impose the further condition that

$$P_{max}(n)/P_{min}(n) < \beta. \tag{1}$$

Thus the power used by a scheme can depend on n but all transmitters have more or less the same power.

Definition 4 (Throughput of a scheme): Let $B_{\Pi_n}(i,t)$ be the number of bits of S-D pair $i, 1 \leq i \leq n/2$, transferred in t time-slots under policy Π_n. Scheme Π is said to have throughput $T_\Pi(n)$ if $\exists$ a sequence of events $A_\Pi(n)$ such that

$$A_\Pi(n) = \left\{ \min_{1\leq i\leq n/2} \liminf_{t\to\infty} \frac{1}{t} B_{\Pi_n}(i,t) \geq T_\Pi(n) \right\},$$

and $P\left(A_\Pi(n)\right) \to 1$ as $n \to \infty$.

We say that an event A_n occurs with high probability (*whp*) if $P\left(A_n\right) \to 1$ as $n \to \infty$.

Definition 5 (Delay of a scheme): The delay of a packet is the time it takes for the packet to reach its destination after it leaves the source. Let $D^i_{\Pi_n}(j)$ denote the delay of packet j of S-D pair i under policy Π_n, then the sample mean of delay (over packets that reach their destinations) for S-D pair i under is

$$\bar{D}^i_{\Pi_n} = \limsup_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} D^i_{\Pi_n}(j).$$

The average delay over all S-D pairs for a particular realization of the random network is then

$$\bar{D}_{\Pi_n} = \frac{2}{n}\sum_{i=1}^{n/2} \bar{D}^i_{\Pi_n}.$$

The delay for a scheme Π is the expectation of the average delay over all S-D pairs, i.e.,

$$D_{\Pi}(n) = E[\bar{D}_{\Pi_n}] = \frac{2}{n}\sum_{i=1}^{n/2} E[\bar{D}^i_{\Pi_n}].$$

Definition 6 (Energy-per-bit): Let $\mathcal{E}^i_{\Pi_n}(j)$ be the energy spent to communicate bit j of S-D pair i, then the sample mean of energy-per-bit for S-D pair i is

$$\bar{\mathcal{E}}^i_{\Pi_n} = \limsup_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} \mathcal{E}^i_{\Pi_n}(j).$$

The average over all S-D pairs of the energy-per-bit for a particular realization is

$$\bar{\mathcal{E}}_{\Pi}(n) = \frac{2}{n}\sum_{i=1}^{n} \bar{\mathcal{E}}^i_{\Pi_n}.$$

Energy-per-bit of scheme Π is the expectation of the above average, i.e.,

$$\mathcal{E}_{\Pi}(n) = E[\bar{\mathcal{E}}_{\Pi}(n)].$$

We will use $\mathcal{E}_T(n)$ and $\mathcal{E}_C(n)$ to denote the components of $\mathcal{E}_{\Pi}(n)$ due to transmission and the circuit respectively.

The number of hops of a scheme Π is denoted by $H_{\Pi}(n)$ and is defined in the same way as the delay of a scheme. Since it is a repeat of the above with obvious modifications, the complete definition is omitted.

The T-D trade-off is defined as in [4]. The D-E and T-E trade-offs are defined similarly.

Definition 7 (T-D-E trade-off): A triple $(T(n), D(n), \mathcal{E}(n))$ is T-D-E optimal if there exists a scheme achieving it and for any scheme Π such that $T_{\Pi}(n) = \Omega(T(n))$, $D_{\Pi}(n) = \Omega(D(n))$ and for any scheme satisfying $T_{\Pi}(n) = \Omega(T(n))$ and $D_{\Pi}(n) = O(D(n))$, $\mathcal{E}_{\Pi}(n) = \Omega(\mathcal{E}(n))$. The T-D-E trade-off consists of all the T-D-E optimal triples.

We define T-E-D, D-T-E, D-E-T, E-T-D and E-D-T trade-offs similarly.

Throughout the paper, we use c_i to denote constants that do not depend on n, the number of nodes. We also use the shorthand notation, $\Omega(f(n)) = T(n) = O(g(n))$ to mean $T(n) = \Omega(f(n))$ and $T(n) = O(g(n))$.

Now that the model and the performance metrics have been defined, we summarize the T-D-E trade-offs that capture the essential elements of our results. The T-D-E and all other trade-offs are stated and proved in detail in subsequent sections.

For any value of $A(n)$ and $\delta > 1$, $D(n) = \Theta(H(n))$ at any optimal trade-off point and the T-D trade-off turns out to be $D(n) = \Theta(nT(n))$, which is the same as in [2]. The trade-offs involving delay and energy-per-bit depend on the value of $A(n)$ and are discussed below.

Constant Area Network (see Corollary 1(i))

The T-D-E trade-off for $A(n) = 1$ is given by

$$D(n) = \Theta(nT(n)) \quad \text{and} \quad \mathcal{E}(n) = \Theta(D(n)),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n\log n}\right)$.

In fact, $\mathcal{E}_T(n) = o(\mathcal{E}_C(n))$ and $\mathcal{E}_C(n) = \Theta(D(n))$. That is, the circuit energy, which is proportional to the number of hops, or equivalently the delay, dominates over the transmission energy as depicted in Figure 1(a). As a result, as long as $\delta > 1$, the exact value of δ does not affect the results.

The best $\mathcal{E}(n)$ that can be achieved in the constant area network is $\Theta(1)$ and is achieved when a constant number of hops (that does not increase with n) is used. The corresponding delay is $\Theta(1)$, which is the best possible. However, the highest energy efficiency and lowest delay come at the cost of the lowest throughput of $\Theta(1/n)$. As the amount of hopping in the network increases by decreasing the transmission range, throughput increases but at the cost of higher delay and higher energy-per-bit.

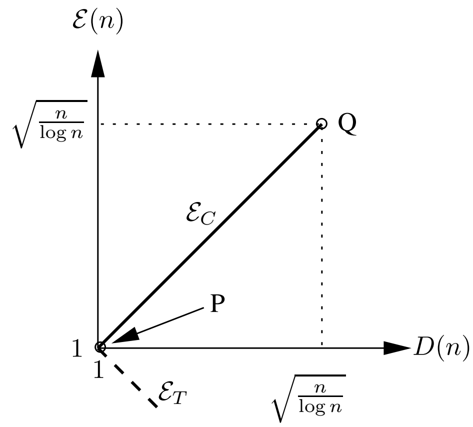
Constant Density Network (see Corollary 2(i))

The T-D-E trade-off for $A(n) = n$ is given by

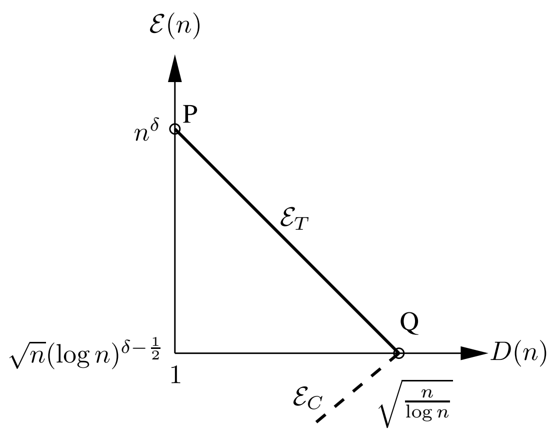
$$D(n) = \Theta(nT(n)) \quad \text{and}$$
$$\mathcal{E}(n) = \Theta\left(n^{\delta} D(n)^{1-2\delta}\right),$$

for $\quad \Omega(1/n) = T(n) = O\left(1/\sqrt{n\log n}\right)$.

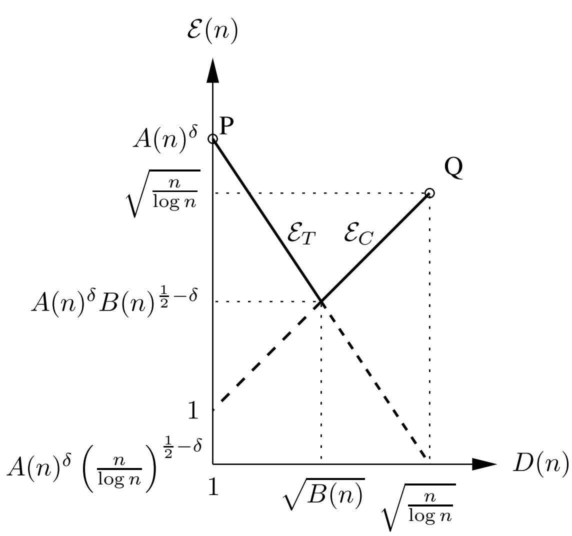
In fact, $\mathcal{E}_C(n) = \Theta(D(n)) = o(\mathcal{E}_T(n))$ and $\mathcal{E}_T(n) = \Theta\left(n^{\delta}(D(n))^{1-2\delta}\right)$ as shown in Figure 1(b). Contrary to the case of the constant area network, here the transmission energy dominates over the circuit energy. Therefore, the value of δ affects the energy consumption. The best $\mathcal{E}(n)$ that can be achieved in the constant density network is $\Theta\left(\sqrt{n}(\log n)^{\delta-\frac{1}{2}}\right)$. Thus even at best, the energy efficiency of the network decreases with increase in the size of the network. The best energy scaling is achieved when the maximum amount of hopping is used. This is because although the circuit energy consumption increases in proportion to the number of hops, the RF transmission energy, which is the dominant component, decreases by using more hops. The corresponding throughput is $\Theta\left(1/\sqrt{n\log n}\right)$, which is the highest possible. However, the highest energy efficiency and the highest throughput come at the cost of the highest delay of $\Theta\left(\sqrt{n/\log n}\right)$. As the amount of hopping in the network decreases, delay decreases but at the cost of lower throughput and higher energy-per-bit.



(a) Constant area random network.



(b) Constant density random network.



(c) Random network with area $A(n)$.

Fig. 1. The D-E trade-off in a random wireless network. The scale of each axis is in terms of the order in n. Point P corresponds to $\Theta(1)$ hops while point Q corresponds to $\Theta\left(\sqrt{n/\log n}\right)$ hops.

Intermediate Density Network (see Theorem 1(i))

For a random network with area $A(n)$, the T-D-E trade-off is given by

$$D(n) = \Theta\left(nT(n)\right), \quad \mathcal{E}_C(n) = \Theta(D(n))$$
$$\text{and} \quad \mathcal{E}_T(n) = \Theta\left(A(n)^\delta D(n)^{1-2\delta}\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n\log n}\right)$.

Thus for general $A(n)$, $\mathcal{E}_T$ dominates at $\Theta(1)$ hops and $\mathcal{E}_C$ dominates as hops increase as shown in Figure 1(c). The minimum energy-per-bit scaling is obtained when $\mathcal{E}_T = \Theta(\mathcal{E}_C)$. This happens at $\sqrt{B(n)}$ hops, where $B(n) = \min\{A(n), n/\log n\}$. The minimum energy-per-bit scaling is $\Theta\left(A(n)^\delta B(n)^{1-2\delta}\right)$. The trade-off between $D(n)$ and $\mathcal{E}(n)$ for general $A(n)$ is thus a combination of that for the extreme cases of $A(n) = 1$ and $A(n) = n$.

III. Optimal Trade-offs

This section states the following main result of the paper, which establishes the optimal trade-offs between throughput, delay and energy-per-bit. The corollaries following it are specializations to the case of the constant area network and the constant density network.

Theorem 1: For the random network model with area $A(n)$, at any optimal trade-off point, $D(n) = \Theta(H(n))$. Further the following statements hold, where $B(n) = \min\{A(n), n/\log n\}$.

(i) The T-D-E and D-T-E trade-off are given by

$$D(n) = \Theta\left(nT(n)\right), \quad \mathcal{E}_C(n) = \Theta(D(n))$$
$$\text{and} \quad \mathcal{E}_T(n) = \Theta\left(A(n)^\delta D(n)^{1-2\delta}\right), \tag{2}$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n\log n}\right)$.

(ii) The D-E-T and E-D-T trade-offs are given by (2) for $\Omega(1/n) = T(n) = O\left(\sqrt{B(n)}/n\right)$.

For $\Omega\left(\sqrt{B(n)}/n\right) = T(n) = O\left(1/\sqrt{n\log n}\right)$, the D-E-T and E-D-T trade-offs are degenerate and are given by $T(n) = \Theta\left(\sqrt{B(n)}/n\right)$, $D(n) = \Theta\left(\sqrt{B(n)}\right)$ and $\mathcal{E}(n) = \Theta\left(\sqrt{B(n)}\right)$.

(iii) The T-E-D and E-T-D trade-offs are given by (2) for $\Omega\left(\sqrt{B(n)}/n\right) = T(n) = O\left(1/\sqrt{n\log n}\right)$.

For $\Omega(1/n) = T(n) = O\left(\sqrt{B(n)}/n\right)$, the T-E-D and E-T-D trade-offs are degenerate and are given by $T(n) = \Theta\left(\sqrt{B(n)}/n\right)$, $D(n) = \Theta\left(\sqrt{B(n)}\right)$ and $\mathcal{E}(n) = \Theta\left(A(n)^\delta B(n)^{\frac{1}{2}-\delta}\right)$.

Substituting $A(n) = 1$ in Theorem 1 yields the following result for the constant area network.

Corollary 1: For the random network with area $A(n) = 1$, at any optimal trade-off point, $D(n) = \Theta(H(n))$. Further the following statements hold.

(i) The T-D-E trade-off is given by

$$D(n) = \Theta\left(nT(n)\right) \quad \text{and} \quad \mathcal{E}(n) = \Theta\left(D(n)\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$. In fact, $\mathcal{E}_T(n) = \Theta\left(D(n)^{1-2\delta}\right) = o(\mathcal{E}_C(n))$ and $\mathcal{E}_C(n) = \Theta(D(n))$.

(ii) The T-E-D, D-T-E and E-T-D trade-offs are identical to the T-D-E trade-off.

(iii) The D-E-T and E-D-T trade-offs are degenerate with $D(n) = \mathcal{E}(n) = \Theta(1)$ and $T(n) = \Theta(1/n)$.

The D-E-T and E-D-T trade-offs are degenerate, since both the lowest delay and lowest energy scaling come together when using a constant number of hops as shown by point P in Figure 1(a). The other trade-offs are identical because as the amount of hopping increases, throughput increases but at the cost of higher energy-per-bit and higher delay.

Substituting $A(n) = n$ in Theorem 1 yields the following result for the constant density network.

Corollary 2: For the constant density random network model, at any optimal trade-off point, $D(n) = \Theta(H(n))$. Further the following statements hold.

(i) The T-D-E trade-off is given by

$$D(n) = \Theta\left(nT(n)\right) \quad \text{and}$$
$$\mathcal{E}(n) = \Theta\left(n^{\delta} D(n)^{1-2\delta}\right),$$

for $\Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right)$. In fact, $\mathcal{E}_C(n) = \Theta(D(n)) = o(\mathcal{E}_T(n))$ and $\mathcal{E}_T(n) = \Theta\left(n^{\delta}(D(n))^{1-2\delta}\right)$.

(ii) The D-T-E, D-E-T and E-D-T trade-offs are identical to the T-D-E trade-off.

(iii) The T-E-D and E-T-D trade-offs are degenerate with $T(n) = \Theta\left(1/\sqrt{n \log n}\right)$, $D(n) = \Theta\left(\sqrt{n/\log n}\right)$ and $\mathcal{E}(n) = \Theta\left(\sqrt{n}(\log n)^{\delta - \frac{1}{2}}\right)$.

The T-E-D and E-T-D trade-offs are degenerate, since both the highest throughput and the lowest energy-per-bit come together when using maximal hopping. The other trade-offs are identical because as the amount of hopping decreases, delay decreases but at the cost of lower throughput and higher energy-per-bit. This is clear from the T-D trade-off and the D-E trade-off shown in Figure 1(b).

IV. A Cellular TDM Scheme

In this section, we present Scheme Π that achieves a trade-off between throughput, delay and energy in a random network of area $A(n)$ with n nodes. We will show that the scaling trade-off provided by this scheme is of the same order as that claimed in Theorem 1. This will establish that the trade-off claimed in Theorem 1 is achievable.

Scheme Π is similar to Scheme 1 in [2], with modifications to account for the Physical model. It is a multi-hop, time-division-multiplexed (TDM), cellular scheme parameterized by $a(n)$, where $a(n) = \Omega(\log n/n)$ and $a(n) \leq 1$. The network area is divided into square cells, each of area $b(n) = a(n)A(n)$ so that the torus of area $A(n)$ consists of $1/a(n)$ cells as shown in Figure 2. The parameter $a(n)$ is the fraction of the total network area that each cell occupies.

Let the straight line joining a source, S, and a its destination, D, be called an S-D line. Scheme Π is described below.

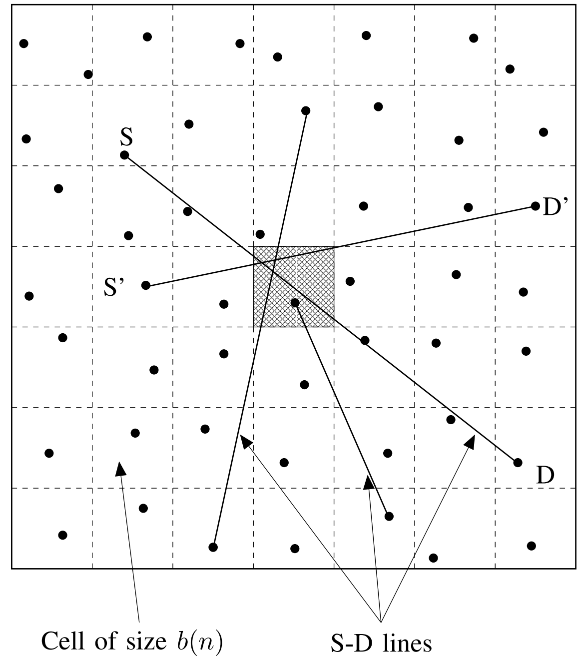


Fig. 2. The torus of area $A(n)$ is divided into cells of area $b(n) = a(n)A(n)$ for Scheme 1. The S-D lines passing through the shaded cell in the center are shown.

Scheme Π

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1) Divide the unit torus using a square grid into square cells, each of area $b(n) = a(n)A(n)$ (see Figure 2). The packet size is $\Theta\left(1/n\sqrt{a(n)}\right)$.
2) Verify whether the following conditions are satisfied for the given realization of the random network.
   - Condition 1: No cell is empty.
   - Condition 2: The number of S-D lines through each cell is at most $c_2 n\sqrt{a(n)}$.
3) If either of the above conditions is not satisfied then use a time-division multiplexing (TDM) policy, where each of the $n/2$ sources transmits directly to its destination in a round-robin fashion.
4) Otherwise, i.e., if both conditions are satisfied, use the following policy Π_n:
   a) The cells are divided into k^2 groups, where k depends only on δ, β, and N, and is independent of n. Figure 4 illustrates this for the case of $k = 3$. All cells belonging to the same group become active simultaneously and each group becomes active at a regular interval of k^2 time-slots. Thus the scheme uses TDM between nearby cells.
   b) A source S transmits data to its destination D by hops along the adjacent cells lying on its S-D line as shown in Figure 2.
   c) When a cell becomes active, it transmits a single packet for each of the S-D lines passing through it. This is again performed using a TDM scheme

that slots each cell time-slot into *packet time-slots* as shown in Figure 3.

d) Each transmitting node transmits with power $P(n) = Pb(n)^\delta$, where P depends only on β, δ and N and not on n.

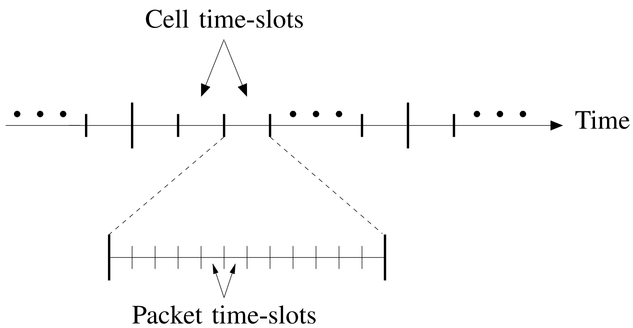

Fig. 3. The TDM transmission schedule of Scheme 1. Each cell becomes active once in $1 + c_1$ time-slots and each active time-slot is divided into several packet-slots.

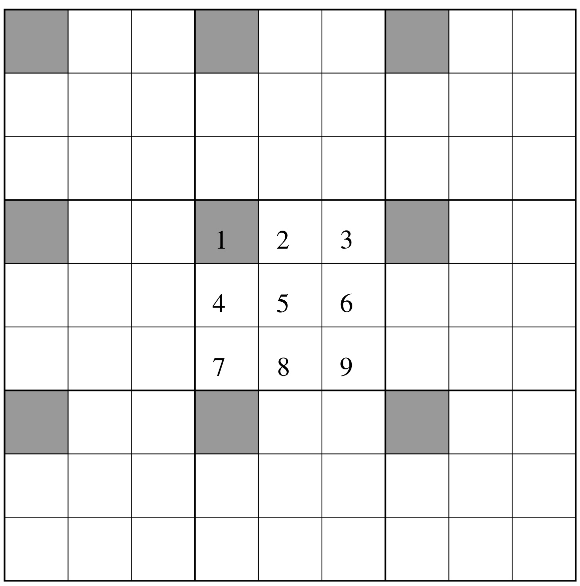

Fig. 4. An illustration of the cells being divided into k^2 groups for the case of $k = 3$, i.e., 9 groups. All the shaded cells which are in group 1 transmit in the same time-slot. In the next time-slot all the cells in group 2 transmit and so on.

The point of trade-off at which Scheme Π operates is determined by the parameter $a(n)$ and this dependence is made precise in the following theorem.

Theorem 2: With $a(n) = \Omega(\log n/n)$, Scheme Π has

$$T(n) = \Omega\left(\frac{1}{n\sqrt{a(n)}}\right), \quad D(n) = \Theta\left(\frac{1}{\sqrt{a(n)}}\right),$$

$$\mathcal{E}(n) = O\left(\frac{1}{\sqrt{a(n)}} + A(n)^\delta a(n)^{\delta - \frac{1}{2}}\right).$$

Thus the trade-off achieved by this scheme is

$$D(n) = \Theta(nT(n)) \quad \text{and}$$

$$\mathcal{E}(n) = \Theta\left(D(n) + A(n)^\delta D(n)^{1-2\delta}\right)$$

$$\text{for} \quad \Omega(1/n) = T(n) = O\left(1/\sqrt{n \log n}\right).$$

Proof: As shown in Lemma 1 of [4], no cell is empty *whp*. Further, it follows from Lemma 3 in [4] that in our setting, the number of S-D lines passing through each cell is $O\left(n\sqrt{a(n)}\right)$ *whp*. This guarantees the existence of a constant c_2 so that Condition 2 is satisfied *whp*. Thus Conditions 1 and 2 are satisfied *whp*.

If the time-division policy with direct transmission is used, then the throughput is $2W/n$ with a delay of 1. But since it happens with a vanishingly low probability, the throughput and delay for Scheme Π are determined by that of policy Π_n. Hence we will only consider policy Π_n for the rest of the proof.

First we will establish that for an appropriate choice of $k = k(\beta, \delta, N)$ and $P = P(\beta, \delta, N)$, the SINR is greater than β at each receiver as required by the Physical model. This will be done by showing that the worst-case interference power $P_I(n)$ at any receiver is bounded above by a constant that decreases monotonically in k and does not depend on n for $\delta > 1$. It is easy to see that the placement of the receiver and the transmitters as shown in Figure 5 results in the the worst case interference at the receiver node. Let $I_{max}(n)$ be the total interference from all other transmitters when each transmitter uses power $Pb(n)^\delta$. This interference can be split into 3 components ($I_1(n)$, $I_2(n)$ and $I_3(n)$) based on the positions of the interfering transmitters as shown in Figure 5, so that

$$I_{max}(n) \leq I_1(n) + I_2(n) + 4I_3(n). \tag{3}$$

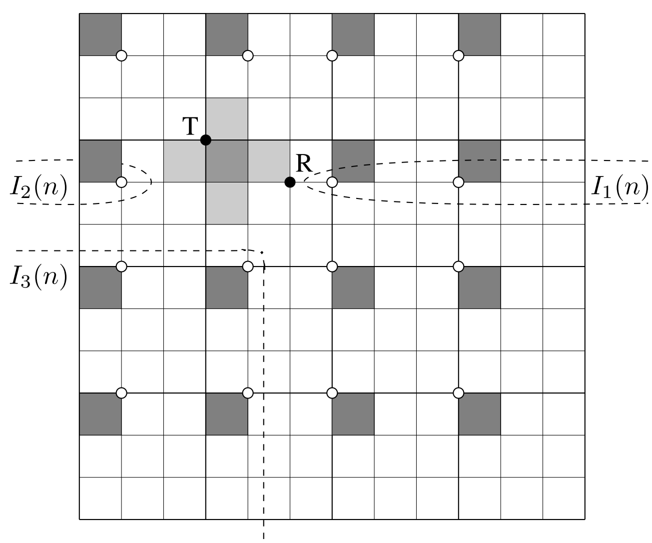

Fig. 5. The node marked T transmits to the node marked R in an adjacent cell. The hollow nodes are the other nodes transmitting simultaneously.

Using $m = 1/a(n)$ to denote the total number of cells, we have,

$$\begin{aligned} I_1(n) &= \sum_{i=1}^{\sqrt{m}/2} \frac{Pb(n)^\delta}{\left((ki-2)\sqrt{b(n)}\right)^{2\delta}} \\ &\leq \frac{P}{(k-2)^{2\delta}} + \frac{P}{(2k-2)^{2\delta}} + \sum_{i=2}^{\sqrt{m}/2} \frac{P}{(ki)^{2\delta}} \\ &\leq \frac{P}{(k-2)^{2\delta}} + \frac{P}{(2k-2)^{2\delta}} \\ &\quad + \frac{P}{k^{2\delta}} \int_1^{\sqrt{m}/2-1} x^{-2\delta} dx \\ &\leq \frac{P}{(k-2)^{2\delta}} + \frac{P}{(2k-2)^{2\delta}} + \frac{P}{(2\delta-1)k^{2\delta}}, \end{aligned}$$

for $\delta > 1/2$. Similarly for $\delta > 1/2$, we obtain

$$\begin{aligned} I_2(n) &= \sum_{i=1}^{\sqrt{m}/2} \frac{Pb(n)^\delta}{\left((ki+1)\sqrt{b(n)}\right)^{2\delta}} \\ &\leq \frac{P}{(k+1)^{2\delta}} + \sum_{i=2}^{\sqrt{m}/2} \frac{P}{(ki)^{2\delta}} \\ &\leq P\left(\frac{1}{(k+1)^{2\delta}} + \frac{1}{(2\delta-1)k^{2\delta}}\right). \end{aligned}$$

Using similar but tedious manipulations, which we do not include here, it can be shown that for $\delta > 1$, $I_3(n)$ is less than P times a decreasing function of k. Thus from (3) it follows that $I_{max}(n) = Pf(k)$ where $f(k)$ is monotonically decreasing in k. Thus in the worst case,

$$\begin{aligned} SINR &\geq \frac{Pb(n)^\delta \left(5b(n)\right)^{-2\delta}}{N + I_{max}(n)} \\ &= \frac{5^{-\delta}}{\frac{N}{P} + f(k)}. \end{aligned}$$

Since $f(k)$ decreases monotonically in k, we can choose k and P depending on β, δ and N so that even in the worst case, $SINR \geq \beta$.

We can use this to analyze the throughput of the trade-off scheme. When policy Π_n is used, since Condition 1 is satisfied, each cell has at least one node. This guarantees that each source can send data to its destination by hops along adjacent cells on its S-D line. In Scheme Π, each cell becomes active once in every k^2 time-slots and moreover the rate at each transmission is W according to the Physical model since at each receiver, $SINR \geq \beta$ as shown above. Hence the cell throughput is $\Theta(1)$. The total traffic through each cell is that due to all the S-D lines passing through the cell, which is $O\left(n\sqrt{a(n)}\right)$ since Condition 2 is also satisfied. This shows that

$$T(n) = \Omega\left(1/n\sqrt{a(n)}\right).$$

Next we compute the average packet delay $D(n)$. The delay of a packet is the time it takes to reach its destination after leaving its source. This is equal to the sum of the amounts of time spent at each hop and so we first bound the average number of hops over all $n/2$ S-D pairs.

Since each hop covers a distance of $\Theta\left(\sqrt{b(n)}\right)$, the number of hops per packet for S-D pair i is $\Theta\left(d_i/\sqrt{b(n)}\right)$, where d_i is the length of S-D line i. Thus the number of hops taken by a packet averaged over all S-D pairs is $\Theta\left(\frac{2}{n}\sum_{i=1}^{n/2} d_i/\sqrt{b(n)}\right)$. Now the expectation of the average distance between S-D pairs, $E[\frac{2}{n}\sum_{i=1}^{n/2} d_i] = \Theta\left(\sqrt{A(n)}\right)$ and so the expectation of the average number of hops is $\Theta\left(1/\sqrt{a(n)}\right)$, since $b(n) = a(n)A(n)$.

Recall that each cell is active once every k^2 time-slots and since Condition 2 is satisfied, each S-D line passing through a cell can have its own packet time-slot within that cell's time-slot. Since we allow the packet size to scale in proportion to the throughput $T(n)$, each packet arriving at a node in the cell departs in the next active time-slot of the cell. Thus the delay is at most k^2 times the number of hops. Since k does not depend on n, we conclude that the delay, $D(n) = \Theta\left(1/\sqrt{a(n)}\right)$.

Now we can compute the energy-per-bit $\mathcal{E}(n)$ for this scheme. Since the throughput is $T(n)$ and there are $n/2$ S-D pairs, $nT(n)L/2$ bits are communicated from the sources to their destinations over a long enough period of L time-slots. The total transmission energy spent in the network during this time is that due to $1/k^2a(n)$ cells in each time slot, which is equal to $Pb(n)^\delta L/k^2a(n)$. The total circuit energy spent is proportional to the number of hops in the network, which is $L/k^2a(n)$. Hence

$$\begin{aligned} \mathcal{E}(n) &= O\left(PA(n)^\delta a(n)^{\delta-1}/nT(n) + 1/na(n)T(n)\right) \\ &= O\left(A(n)^\delta a(n)^{\delta-1/2} + 1/\sqrt{a(n)}\right). \end{aligned}$$

This concludes the proof of Theorem 2. ■

V. Proof of Optimality

In this section, we present a converse to Theorem 2 to show that the trade-off provided by our trade-off scheme is indeed the optimal trade-off as far as scaling is concerned. That is, we show that no scheme can provide a better scaling trade-off than the one achieved by the scheme presented in Section IV. This is the content of Theorem 3 and this along with Theorem 2 proves Theorem 1 thus establishing the optimal scaling trade-off between throughput, delay and energy.

To establish a converse, we need to show that if any scheme has throughput $T(n)$ then its delay scaling is $\Omega(nT(n))$, i.e., its delay scaling can be no better than that achieved by Scheme Π in Section IV. By the definitions of the performance metrics, this means, we need to show that if any scheme has throughput $T(n)$ *whp* over all realizations then its expected delay over all realizations is $\Omega(nT(n))$. We also need to show a similar relationship between delay and energy scaling. Before doing this, we consider a fixed realization of a network and determine how the throughput, delay and energy-per-bit of any scheme depend on the average transmission range. The analysis of the trade-off scheme in Section IV showed that the transmission range of scheme determines the amount of hops used by S-D pairs and this in turn determines the throughput, delay and energy-per-bit. The following lemma shows that the transmission range of a scheme puts a bound on its performance and that our trade-off scheme uses multi-hopping in the best possible way, as far was scaling is concerned.

Lemma 1: Consider any realization of the random network with $2n$ nodes in area $A(n)$. Let d_i be the distance between S-D pair $i, 1 \leq i \leq n$ and let $\bar{L} = \frac{1}{n}\sum_{i=1}^{n} d_i$. With any scheme for this realization, let the throughput be λ, the average transmission range be $\bar{r}$, the average number of hops per bit be $\bar{h}$ and the average energy-per-bit be $\bar{\mathcal{E}}$. Then the following hold:

(a) $\bar{h} \geq \bar{L}/\bar{r}$,

(b) $\lambda \leq c_3 A(n)\bar{h}/\left(n\bar{L}^2\right)$,

(c) $\bar{\mathcal{E}} \geq \beta \bar{L}^{2\delta}\bar{h}^{1-2\delta} + c_0\bar{h}$.

Proof: (a) Consider any realization of the random network with $2n$ nodes. Suppose that a scheme provides throughput λ for this realization. Then given a sufficiently long time interval T, each source communicates λT bits to its destination. The total number of bits is $B = \lambda n T$ since there are n S-D pairs. The most general scheme can transmit bits via multiple hops and paths in the network. Suppose that bit $b, 1 \leq b \leq B$, is communicated to its destination by H_b hops and let $r(b,h), 1 \leq b \leq B, 1 \leq h \leq H_b$ be the length of hop h of bit b. Thus $r(b,h)$ is the transmission range at hop h of bit b.

First note that the average number of hops,

$$\bar{h} = \frac{1}{B}\sum_{b=1}^{B} H_b, \tag{4}$$

and the average transmission range for the scheme is

$$\bar{r} = \frac{1}{\sum_{b=1}^{B} H_b}\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h). \tag{5}$$

Recall that the distance between the source and destination of S-D pair $i, 1 \leq i \leq n$, is d_i. Since each bit belonging to S-D pair i has to travel at least distance d_i, it follows that

$$\lambda T \sum_{i=1}^{n} d_i \leq \sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h). \tag{6}$$

Starting from (4), we obtain the following.

$$\begin{aligned}
\bar{h} &= \frac{1}{\lambda n T}\sum_{b=1}^{B} H_b \\
&= \frac{1}{\lambda T \sum_{i=1}^{n} d_i}\frac{1}{n}\sum_{i=1}^{n} d_i \sum_{b=1}^{B} H_b \\
&\overset{(a)}{\geq} \frac{1}{n}\sum_{i=1}^{n} d_i \, \frac{\sum_{b=1}^{B} H_b}{\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h)} \\
&\overset{(b)}{=} \frac{\bar{L}}{\bar{r}},
\end{aligned}$$

where inequality (a) is due to (6) and (b) is by the definitions of $\bar{r}$ and $\bar{L}$.

(b) The proof of this part of the lemma is essentially the same as that of Theorem 2 in [2] which uses the Protocol model and follows from the equivalence between the Physical model and the Protocol model as shown in [6].

Let the position of node i be denoted by X_i and let $|X_i - X_j|$ denote the distance between nodes i and j. Then as shown in the proof of Theorem 2.1 of [6], the Physical model implies that if node i is transmitting to node j, and k is any other simultaneously transmitting node then

$$|X_k - X_j| \geq (1+\Delta)|X_i - X_j|,$$

where $\Delta = \left(\frac{\beta P_{min}}{P_{max}}\right)^{\frac{1}{2\delta}} - 1$.

As a result of this equivalence with the Protocol model, as shown in [6], in every time-slot t

$$\sum_{(b,h)\in\Gamma_t} r(b,h)^2 \leq c_3 A(n),$$

where Γ_t is the set of (b,h) pairs such that hop h of bit b occurs in time-slot t and c_3 is a constant that depends only on W. This is based on the idea that each transmission consumes an area proportional to the square of the range of transmission and the total area is $A(n)$. Summing over all T time-slots, we obtain

$$\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h)^2 \leq c_3 A(n) T. \tag{7}$$

By convexity,

$$\bar{r}^2 \leq \frac{1}{\sum_{b=1}^{B} H_b}\sum_{b=1}^{B}\sum_{h=1}^{H_b} r(b,h)^2.$$

Combining the above two equations and rearranging, we obtain

$$\sum_{b=1}^{B} H_b \leq \frac{c_3 A(n) T}{\bar{r}^2}. \tag{8}$$

Substituting from (5) into (6) and using (8), we obtain

$$\lambda T \sum_{i=1}^{n} d_i \leq \frac{c_3 A(n) T}{\bar{r}^2}\bar{r}.$$

This can be rewritten as

$$\lambda n \left(\frac{1}{n}\sum_{i=1}^{n} d_i\right) \leq \frac{c_3 A(n)}{\bar{r}}.$$

Now using part (a) of the lemma, we obtain

$$\lambda n \bar{L} \leq \frac{c_3 A(n)\bar{h}}{\bar{L}},$$

which proves part (b) of the lemma.

(c) Consider hop h of bit b and let $P(b,h)$ be the power used for this transmission. Suppose this transmission occurs in time-slot t. Then as per the Protocol model

$$\frac{P(b,h) r(b,h)^{-2\delta}}{N + \sum_{(i,j)\in\Gamma_t} P(i,j) r(i,j)^{-2\delta}} \geq \beta.$$

Thus by ignoring the interference, we obtain

$$P(b,h) \geq \beta N r(b,h)^{2\delta}.$$

Therefore the average transmission energy-per-bit over time T,

$$\begin{aligned}
\bar{\mathcal{E}}_T &= \frac{1}{B}\sum_{b=1}^{B}\sum_{h=1}^{H_b} P(b,h) \\
&\geq \frac{\sum_{b=1}^{B} H_b}{B}\frac{1}{\sum_{b=1}^{B} H_b}\sum_{b=1}^{B}\sum_{h=1}^{H_b} P(b,h) \\
&\overset{(a)}{\geq} \bar{h}\beta\bar{r}^{2\delta} \\
&\overset{(b)}{\geq} \beta\bar{L}^{2\delta}\bar{h}^{1-2\delta}, \qquad (9)
\end{aligned}$$

where inequality (a) is due to convexity and inequality (b) is due to part (a) of the lemma.

As per our energy model, each hop consumes a constant amount of energy, c_0 and hence the average circuit energy, $\bar{\mathcal{E}}_C = c_0\bar{h}$. As a result, the average energy-per-bit

$$\bar{\mathcal{E}} = \bar{\mathcal{E}}_T + \bar{\mathcal{E}}_C \geq \beta\bar{L}^{2\delta}\bar{h}^{1-2\delta} + c_0\bar{h},$$

which proves part (c) of the lemma. ■

We would like to note that (9) in the proof of part (c) of the above lemma is a natural extension of the minimum transmission energy-per-bit for an AWGN channel to the case of a network with multiple hops. For the AWGN channel with noise power N, the rate when using power P to communicate over a distance r is $\frac{1}{2}\log\left(1 + Pr^{-2\delta}/N\right)$, which implies that the minimum transmission energy-per-bit is $Nr^{2\delta}/2$. In our case, if we ignore the interference due to other simultaneous transmissions, the minimum energy-per-bit for a single transmission over distance r would be $\beta N r^{2\delta}/W$, which is the same as that for the AWGN channel except for the constant β/W. Thus the above lemma lower bounds the minimum energy-per-bit in the case of a network by taking into account multiple hops and ignoring the effect of interference due to other simultaneous transmissions. Ignoring interference does not hurt in determining the correct scaling, as long as it does not dominate the noise N and this is what our trade-off scheme does.

Using the above lemma, we prove the converse to Theorem 2 for the cases of the T-D-E and D-T-E trade-offs. This converse establishes that no scheme can provide a better T-D-E or D-T-E scaling trade-off than that provided by our trade-off scheme.

Theorem 3: In a random network with area $A(n)$, if a communication scheme has has throughput, $T(n)$, delay, $D(n)$ and energy-per-bit, $\mathcal{E}(n)$ then

$$D(n) = \Omega\left(nT(n)\right) \quad \text{and}$$

and if the scheme has $D(n) = \Theta(nT(n))$ then

$$\mathcal{E}(n) = \Omega\left(D(n) + A(n)^{\delta}D(n)^{1-2\delta}\right).$$

Proof: From part (b) of Lemma 1, we have

$$\lambda \leq \frac{c_3 A(n)\bar{h}}{n\bar{L}^2}. \qquad (10)$$

Combining this with the obvious fact that $\lambda \leq W$, we obtain

$$\lambda n \leq \min\left\{\frac{c_3 A(n)\bar{h}}{\bar{L}^2},\ nW\right\}. \qquad (11)$$

By the law of large numbers, $\bar{L} = \Theta\left(\sqrt{A(n)}\right)$ *whp*. Moreover the rate of convergence is exponential in n. Let B_n be the set such that $\bar{L} = \Theta(1)$ and let $I(B_n)$ be the indicator of set B_n. Then from (11), we have

$$\begin{aligned}
nE[\lambda] &\leq E\left[\frac{c_3 A(n)\bar{h}}{\bar{L}^2} I(B_n)\right] + E\left[nWI(B_n^c)\right] \\
&\leq c_4 E[\bar{h}] + o(1), \qquad (12)
\end{aligned}$$

where the last term is $o(1)$ since $P(B_n^c)$ converges to 0 exponentially.

By definition, if a scheme has throughput $T(n)$ then there exists a set F_n on which $\lambda \geq T(n)$ and $P(F_n)$ converges to 1. Therefore we have

$$\begin{aligned}
E[\lambda] &= E[\lambda I(F_n)] + E[\lambda I(F_n^c)] \\
&\geq T(n)(1 - o(1)). \qquad (13)
\end{aligned}$$

From (12) and (13), it follows that $nT(n)(1 - o(1)) \leq c_4 E[\bar{h}] + o(1)$, which is the same as $E[\bar{h}] = \Omega(nT(n))$. Now each packet spends at least one time-slot at each hop and hence the delay of each packet is at least as much as the number of hops it takes. As a result, if $D(n)$ is the delay of the scheme under consideration then by definition, $D(n) \geq E[\bar{h}] = H(n)$. Thus we have shown that for any scheme,

$$D(n) = \Omega(H(n)) = \Omega(nT(n)). \qquad (14)$$

This establishes the desired relationship between throughput and delay. Next we deal with energy-per-bit. Taking expectation on both sides of part (c) of Lemma 1, we obtain

$$\begin{aligned}
\mathcal{E}(n) &= E\left[\bar{\mathcal{E}}\right] \\
&\geq \beta E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}\right] + c_0 E\left[\bar{h}\right] \\
&= E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}\right] + c_0 H(n). \qquad (15)
\end{aligned}$$

Now $\bar{L} = \Theta\left(\sqrt{A(n)}\right)$ *whp* and moreover the rate of convergence in the law of large numbers is exponential in n. Let A be the event that $\bar{L} = \Theta\left(\sqrt{A(n)}\right)$ then

$$E[\bar{h}^{1-2\delta}] = E[\bar{h}^{1-2\delta}|A]P(A) + E[[\bar{h}^{1-2\delta}|A^c](1 - P(A)). \qquad (16)$$

Now $\bar{h}^{1-2\delta}$ is a polynomial in n and since $1 - P(A)$ decays exponentially fast to 0, it follows that

$$E\left[\bar{h}^{1-2\delta}\right] = E\left[\bar{h}^{1-2\delta}|A\right]P(A)(1 - o(1)).$$

Using the above, we obtain

$$\begin{aligned} E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}\right] &\geq E\left[\bar{L}^{2\delta}\bar{h}^{1-2\delta}|A\right]P(A) \\ &\geq c_4 A(n)^{\delta} E\left[\bar{h}^{1-2\delta}|A\right]P(A) \\ &= c_4 A(n)^{\delta} E\left[\bar{h}^{1-2\delta}\right](1-o(1)) \\ &\overset{(a)}{\geq} c_4 A(n)^{\delta} E\left[\bar{h}\right]^{1-2\delta}(1-o(1)) \\ &= c_4 A(n)^{\delta} H(n)^{1-2\delta}(1-o(1)), \end{aligned}$$

where inequality (a) is due to Jensen's inequality.

Using the above equation, (15) can be rewritten as

$$\mathcal{E}(n) = \Omega\left(A(n)^{\delta}H(n)^{1-2\delta} + H(n)\right).$$

Now assume that the optimal trade-off between throughput and delay is achieved by a scheme, i.e., $D(n) = \Theta(nT(n))$. Then it follows from (14) that $D(n) = \Theta(H(n))$. As a result, we have

$$\mathcal{E}(n) = \Omega\left(A(n)^{\delta}D(n)^{1-2\delta} + D(n)\right).$$

■

The above converse shows that the trade-off obtained by Scheme Π in Section IV is optimal in terms of scaling and this establishes the optimal T-D-E and D-T-E trade-offs for the random network. This also proves part (i) of Theorem 1. Converses for the remaining two parts can be proved similarly. The additional variable $B(n)$ arises in Theorem 1 because, to ensure connectivity in the network, the number of hops must be $O\left(\sqrt{n/\log n}\right)$.

VI. Conclusion

Using a random network model to study large, ad hoc wireless networks, previous work established the optimal throughput-delay trade-off. The optimal number of hops to minimize energy consumption for a given placement of nodes has been studied in a separate body of work. In this paper, we used a random network model to unify these results by establishing the optimal trade-offs between throughput, delay and energy-per-bit. In so doing, we also determined the amount of hopping needed to operate at an optimal point of the trade-off between these performance metrics. We also showed that at any optimal trade-off point, delay scales as the number of hops and the amount of hopping determines the trade-off point at which the network operates depending on the node density. This is a consequence of the interference-limited nature of communication, due to which the amount of hopping determines the optimal power to use for transmissions.

Maximum hopping is required for the highest throughput but results in the worst delay. Whether it is energy efficient or not depends on the network node density. Hopping is expensive in terms of energy in a constant area network with node density n and the minimum energy-per-bit is obtained using $\Theta(1)$ hops. On the other hand, in a constant density network, hopping is necessary for energy efficiency. But even with maximum hopping that provides the highest energy efficiency, the energy-per-bit increases as the network grows in size.

Our results suggest some general guidelines for building ad hoc wireless networks.

1) In high density networks consisting of low data-rate nodes, where the main concerns are energy and delay, our results suggest the use of minimal hopping. This also saves the energy and delay overheads of implementing complex multi-hopping protocols.
2) In high density networks consisting of high data-rate nodes with limited energy, maximum hopping is needed to accommodate the high throughput requirement. As our results show, this requires the nodes to operate at far from minimum energy consumption in addition to the overhead of high protocol complexity. In this case it may be necessary to have an infrastructure of relay nodes with unconstrained access to energy.
3) In applications where the network extends over a large geographical area, in spite of maximal hopping being optimal, energy efficiency can be low. Again this problem can be mitigated by adding wired or wireless infrastructure.
4) We showed that a cellular, TDM network architecture with equal node transmission powers determined by the cell size achieves the optimal scaling. Such an architecture is attractive due to its simplicity.

Finally, we note that the results in this paper used a "fluid" model, which means that the size of packets is allowed to be arbitrarily small. Scheme Π presented in Section IV required the fluid nature of packets in order to achieve the optimal trade-off since the size of packets scaled down with n. The fluid model was used to avoid the problem of scheduling packets in the network. However, from recent work in [5], it is clear that these results hold even if the size of packets is kept constant.

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