Throughput and Delay in Random Wireless Networks with Restricted Mobility

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Abstract

Grossglauser and Tse (2001) introduced a mobile random network model where each node moves independently on a unit disk according to a stationary uniform distribution and showed that a throughput of $\Theta(1)$ is achievable. El Gamal, Mammen, Prabhakar and Shah (2004) showed that the delay associated with this throughput scales as $\Theta\left(n \log n\right)$, when each node moves according to an independent random walk. In a later work, Diggavi, Grossglauser and Tse (2002) considered a random network on a sphere with a restricted mobility model, where each node moves along a randomly chosen great circle on the unit sphere. They showed that even with this one-dimensional restriction on mobility, constant throughput scaling is achievable. Thus, this particular mobility restriction does not affect the throughput scaling. This raises the question whether this mobility restriction affects the delay scaling.

This paper studies the delay scaling at $\Theta(1)$ throughput for a random network with restricted mobility. First, a variant of the scheme presented by Diggavi, Grossglauser and Tse (2002) is presented and it is shown to achieve $\Theta(1)$ throughput using different (and perhaps simpler) techniques. The exact order of delay scaling for this scheme is determined, somewhat surprisingly, to be of $\Theta(n \log n)$, which is the same as that without the mobility restriction. Thus, this particular mobility restriction *does not* affect either the maximal throughput scaling or the corresponding delay scaling of the network. This happens because under this 1-D restriction, each node is in the proximity of every other node in essentially the same manner as without this restriction.

Index Terms

Random wireless networks, scaling laws, constant throughput scaling, delay, 1-D mobility.

I. Introduction

Gupta and Kumar [7] introduced a random network model for studying throughput scaling in a fixed wireless network (that is, when the nodes do not move). They defined a random network to consist of n nodes where each node is distributed uniformly and independently on the unit sphere in $\mathbb{R}^3$. The network has $n/2$ distinct source-destination pairs formed at random. Each node can transmit at W bits-per-second provided that the interference is sufficiently small. They showed that in such a random network the throughput scales as $\Theta(1/\sqrt{n \log n})$ per source-destination (S-D) pair.

Grossglauser and Tse [8] showed that by allowing the nodes to move, the throughput scaling changes dramatically. Indeed, if node motion is independent across nodes and has a uniform stationary distribution, a constant throughput scaling ($\Theta(1)$) per S-D pair is feasible. This raised the question: what kind of mobility is necessary for achieving constant throughput scaling? Diggavi, Grossglauser and Tse [3] considered a restricted mobility model where each node is allowed to move along a randomly chosen great circle on the unit sphere with a uniform stationary distribution along the great circle. They showed that a constant throughput per S-D pair is feasible even with this restricted mobility model. Thus they established that node motion with a stationary distribution on the entire network area is not necessary for achieving constant throughput scaling.

El Gamal, Mammen, Prabhakar and Shah [5] (see [6] for complete details) determined the throughput-delay trade-off for both fixed and mobile wireless networks. In particular, it was shown that for mobile networks at throughput of $\Theta(1)$, the delay is $\Theta(n \log n)$. For mobile networks, the mobility model consisted of each node moving independently according to a symmetric random walk on a $\sqrt{n} \times \sqrt{n}$ grid on the unit torus.

The constant throughput scaling result of [3] for a network with restricted mobility raises the question whether the high throughput in spite of restricted mobility is at the expense of increased delay. Motivated by this question, we study the delay scaling for constant throughput scaling in a network with restricted mobility. Somewhat surprisingly,

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we find that delay scaling is not affected by this mobility restriction either. That is, delay scales as $\Theta(n \log n)$, which is the same as the delay scaling when mobility is not restricted. This paper is a consolidation of the preliminary work presented in [10].

This seemingly surprising result can be explained as follows. Since there are n nodes in a network of constant area, the neighborhood of each node is $\Theta(1/n)$. Based on this, let us say that two nodes *meet* or are *neighbors* when they are within a distance of $\Theta\left(1/\sqrt{n}\right)$. The following condition ensures constant throughput scaling in the mobile network models presented in [8], [3] and this paper: *for* $\Theta(1/n)$ *fraction of the time, each node is a neighbor of every other node with only* $\Theta(1)$ *other nodes in its neighborhood*. This ensures that the total network throughput is $\Theta(n)$ and that it is distributed evenly among the $n/2$ S-D pairs, so that the throughput is $\Theta(1)$. Delay is determined by the first and second moments of the inter-meeting time of the nodes. In the case of unrestricted mobility, the inter-meeting time of any two nodes is equivalent to the inter-visit time to state $(0,0)$ for a 2-D random walk on a $\sqrt{n} \times \sqrt{n}$ grid. In the restricted mobility case also the inter-meeting time turns out to be equivalent to the inter-visit time to state $(0,0)$ for a slightly different random walk. However the first two moments are still of the same order and hence the queueing delay is the same, leading to the same delay scaling. As a result, even with this particular mobility restriction, the maximal throughput scaling and the corresponding delay scaling remain unchanged.

The rest of the paper is organized as follows. In Section II, we introduce the random mobile network model, some definitions and notation. In Section III, we present a scheme using random relaying and show that it achieves constant throughput scaling. In Section IV, we show that the delay for this scheme is $\Theta(n \log n)$ using results which are proved in Section V. The proof of delay of $\Theta(n \log n)$ consists of analyzing a queue at a relay node in two parts. The first part presented in Section IV identifies an i.i.d. component that is embedded in the arrival and service processes of the queue. The second part breaks the dependence between the arrival and departure processes by introducing a virtual Bernoulli server. The queueing analysis that follows is carried out in Section V.

II. Models and Definitions

In this section, we present the network model, and the definitions of the performance metrics – throughput and delay. We begin by reminding the reader of the order notation: (i) $f(n) = O(g(n))$ means that there exists a constant c and integer N such that $f(n) \le cg(n)$ for $n > N$. (ii) $f(n) = o(g(n))$ means that $\lim_{n\to\infty} f(n)/g(n) = 0$. (iii) $f(n) = \Omega(g(n))$ means that $g(n) = O(f(n))$, (iv) $f(n) = \omega(g(n))$ means that $g(n) = o(f(n))$. (v) $f(n) = \Theta(f(n))$ means that $f(n) = O(g(n));\ g(n) = O(f(n))$.

Now let us recall what is meant by the uniform distribution of great circles on a sphere. Let S^2 denote the surface of a sphere in $\mathbb{R}^3$ with unit area. For $x \in S^2$, let $x' \in S^2$ be the diametrically opposite point of x. Let $G(x)$ denote the great circle obtained by the intersection of S^2 with the plane passing through the center of S^2 and perpendicular to the line xx'. Let x be called the pole of $G(x)$. If the pole of a great circle is chosen according to a uniform distribution on S^2 then the great circle is said to have a uniform distribution.

Definition 1 (Natural random walk): A natural random walk on a discrete torus of size m is the process $S(t) \in \{0,\dots,m-1\},\ t = 0,1,\dots$, such that $S(0)$ is uniformly distributed over $\{0,\dots,m-1\}$ and $S(t+1)$ is equally likely to be any element of $\{S(t), S(t)-1 \mod m, S(t)+1 \mod m\}$.

This differs from a simple random walk, where $S(t+1)$ is equally likely to be any element of $\{S(t)-1 \mod m, S(t)+1 \mod m\}$. Since we are interested only in scaling results, we use the terms (simple) random walk and natural random walk interchangeably.

Definition 2 (Random network): The random network consists of n nodes that are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for transmission. Associated with each node is a great circle of S^2 chosen independently according to a uniform distribution.

The great circle of each node has $\sqrt{n}$ equidistant lattice points numbered from 0 to $\sqrt{n}-1$ placed on it arbitrarily resulting in a one-dimensional discrete torus of size $\sqrt{n}$. Each node moves according to a natural random walk on these lattice points on its great circle. Figure 1 shows a realization of the random network model. Note that since the sphere has unit area, its radius is $1/2\sqrt{\pi}$. Hence each great circle has perimeter $\sqrt{\pi}$ because of which the distance between two adjacent lattice points is $\sqrt{\pi/n}$.

Let the distance on the sphere between nodes i and j be denoted by $d(i,j)$. We assume the Relaxed Protocol model [5] similar to the Protocol model in [7] for successful transmission.

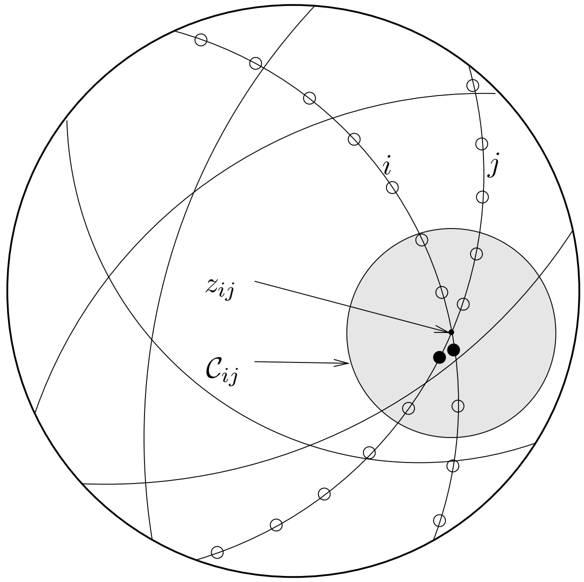

Fig. 1. A realization of the random network model. Only the lattice points on the great circles of nodes i and i are shown. The intersection of their great circles is z_{ij}. The shaded circle is $\mathcal{C}_{ij}$ and i and j become neighbors when they are at the two dark lattice points.

Definition 3 (Relaxed Protocol Model): A transmission from node i to node j is successful if for any other simultaneously transmitting node k,

$$d(k,j) \geq (1+\Delta)d(i,j)$$

for some $\Delta > 0$. If a transmission is successful then communication occurs at a constant rate of W bits-per-second. For simplicity, we assume that time-slots are of unit length so that when a successful transmission occurs a packet of size W is communicated.

In the other commonly used model (e.g., [8], [7], [3]), known as the *Physical* model, a transmission is successful if the Signal to Interference and Noise Ratio (SINR) is greater than some constant. It is well known [7] that the *Protocol* model is equivalent to the *Physical* model when each transmitter uses the same power.

The differences between this model and the model in [3] are: (i) the Relaxed Protocol model is used instead of the Physical model, and (ii) each node is assumed to move according to a natural random walk instead of just a stationary, ergodic motion with uniform stationary distribution on the great circle. However, this model has the same 1-D mobility restriction. Further, the proofs clearly show that the assumption of mobility according to a natural random walk is not necessary for achieving constant throughput scaling and is used only for computing delay.

Definition 4 (Scheme): A scheme Π for a random network is a sequence of communication policies, (Π_n), where policy Π_n determines how communication occurs in a network of n nodes.

Definition 5 (Throughput of a scheme): Let $B_{\Pi_n}(i,t)$ be the number of bits of S-D pair $i, 1 \leq i \leq n/2$, transferred in t time-slots under policy Π_n. Note that this could be a random quantity for a given realization of the network. Scheme Π is said to have throughput $T_\Pi(n)$ if $\exists$ a sequence of sets $A_\Pi(n)$ such that

$$A_\Pi(n) = \left\{ \omega : \min_{1\leq i \leq n/2} \liminf_{t\to\infty} \frac{1}{t} B_{\Pi_n}(i,t) \geq T_\Pi(n) \right\}$$

and $P\left(A_\Pi(n)\right) \to 1$ as $n \to \infty$.
We allow randomness in policies. Hence, $P\left(A_\Pi(n)\right)$ denotes the probability of $A_\Pi(n)$ over the joint probability space that captures randomness in the policy as well as the random network instance. We say that event A occurs with high probability (*whp*) if $P(A) \to 1$ as $n \to \infty$.

Definition 6 (Delay of a scheme): The delay of a packet is the time it takes for the packet to reach its destination after it leaves the source. Let $D^i_{\Pi_n}(j)$ denote the delay of packet j of S-D pair i under policy Π_n, then the sample mean of delay for S-D pair i under Π_n is

$$\bar{D}^i_{\Pi_n} = \limsup_{k\to\infty} \frac{1}{k} \sum_{j=1}^{k} D^i_{\Pi_n}(j).$$

The average delay over all S-D pairs for a particular realization of the random network is then

$$\bar{D}_{\Pi_n} = \frac{2}{n} \sum_{i=1}^{n/2} \bar{D}^i_{\Pi_n}.$$

The delay for a scheme Π is the expectation of the average delay over all S-D pairs, i.e.,

$$D_{\Pi}(n) = E[\bar{D}_{\Pi_n}] = \frac{2}{n} \sum_{i=1}^{n/2} E[\bar{D}^i_{\Pi_n}].$$

Now observe that some realizations of the random network may result in the configuration of nodes being such that it is not possible to achieve constant throughput scaling. Hence we first define a typical configuration which captures the fact that the distribution of great circles is sufficiently uniform everywhere on the sphere. We need some notation to introduce this definition.

Let G_i denote the great circle of node $i \in \{1, \ldots, n\}$. For any two nodes $i \neq j$, G_i and G_j are not identical with probability 1 under the random network model. Two distinct great circles must intersect in exactly two points. For each pair $i \neq j$, select one of the two distinct intersection points of G_i and G_j uniformly at random and call it z_{ij}. Let $\mathcal{C}_{ij}$ denote the disk on the sphere centered at z_{ij} with radius $(2+\Delta)\sqrt{\pi/n}$. See Figure 1 for an illustration.

Definition 7 (Typical configuration): A configuration (i.e., realization of the random network) is said to be *typical* if the number of great circles passing through each $\mathcal{C}_{ij}$ is $\Theta\left(\sqrt{n}\right)$.

Definition 8 (Neighbor): We say that nodes i and j are *neighbors* at time t if both nodes i and j are at the lattice points of their respective great circles that are closest to z_{ij}.

In Figure 1, the lattice points for nodes i and j that are closest to z_{ij} have been darkened. Under the random walk model, it is possible that in some time-slot, a node may not have any neighbors.

III. Scheme with Constant Throughput Scaling

In this section we present Scheme Π and show that it achieves constant throughput scaling. In the next section its delay scaling will be analyzed. Before presenting the scheme, we prove a property of the random network model which makes the scheme feasible.

Lemma 1: Configurations are typical *whp*.

Proof: Consider any two nodes i and j. First note that the probability that G_i and G_j coincide is zero. Also any two distinct great circles necessarily intersect at exactly two points. By definition, $\mathcal{C}_{ij}$ has area c_1/n since it has radius $(2+\Delta)\sqrt{\pi/n}$.

Let $I_k,\ k = 1, \ldots, n,\ k \neq i, j$, be an indicator random variable for the event that the great circle of node k, G_k, passes through $\mathcal{C}_{ij}$. By definition, I_k are i.i.d. Bernoulli random variables with parameter p, where $p = c_2/\sqrt{n}$ where c_2 is a positive constant. This is because a great circle passes through a disk of radius R if and only if its pole lies in an equatorial band of width $2R$. The probability of this event is $\Theta(R)$ as the position of pole is uniformly distributed over the sphere.

Thus, the total number of great circles passing through $\mathcal{C}_{ij}$ is given by a random variable $X = \sum_k I_k$ with $E[X] \approx 0.5 c_1 \sqrt{n} = \Theta\left(\sqrt{n}\right)$. An application of the well-known Chernoff bound for the sum of i.i.d. Bernoulli random variable (e.g., see [11]), yields

$$\begin{aligned} P\{|X - E[X]| \geq \delta E[X]\} &\leq 2\exp\left(-\delta^2 E[X]/2\right) \\ &= \frac{1}{n^3}, \quad \text{for } \delta = \sqrt{\frac{2(\log 2 + 3\log n)}{E[X]}}. \end{aligned} \tag{1}$$

The choice of δ in (1) shows that $X \leq c_1\sqrt{n}$ or $X = \Theta\left(\sqrt{n}\right)$ with probability at least $1 - 1/n^3$. Hence by the union bound over all $n(n-1)/2$ possible $\mathcal{C}_{ij}$ for $i, j = 1, \ldots, n$, we obtain that with probability at least $1 - 1/n$, the number of great circles passing through each $\mathcal{C}_{ij}$ is $\Theta\left(\sqrt{n}\right)$. ■

A. The Scheme

The operation of Scheme Π depends on whether the configuration is typical or not. If the configuration is not typical, direct transmission is used between the S-D pairs along with time-division multiplexing. That is, the sources transmit to their destinations once in $2/n$ time-slots in a round-robin fashion. If the configuration is typical then Policy Σ_n as described below is used. Policy Σ_n is a variant of the policies presented in [8], [3].

Policy Σ_n:

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1) Each time-slot is divided into two sub-slots – A and B.
2) Sub-slot A
   (a) Each source node independently becomes *active* with probability $p_\Delta > 0$.
   (b) If an active node has one or more neighbors then with probability $0 < \alpha < 1$, it chooses one at random and a packet intended for its destination is transmitted to this randomly chosen neighbor, which acts as a relay node.
3) Sub-slot B
   a) Each node independently becomes *active* with probability $p_\Delta > 0$.
   b) If an active node has one or more neighbors that are destination nodes, it chooses one at random. The active node, which acts as a relay, transmits a packet intended for this destination node, if it has any, in FIFO order.

In policy Σ_n, each node acts as a relay for all the other $n/2 - 1$ S-D pairs. A packet reaches from its source to its destination as shown in Figure 2. A source node, S, transmits its packet to a random relay node, R, which may also happen to be the destination itself. The random relay node then moves around carrying the packet. Finally, when it becomes a neighbor of the destination, D, the packet is transmitted to D. A relay node may receive several packets from a source before it gets a chance to transmit to the destination. To handle this, each relay node maintains a separate queue for each of the other $n/2 - 1$ S-D pairs.

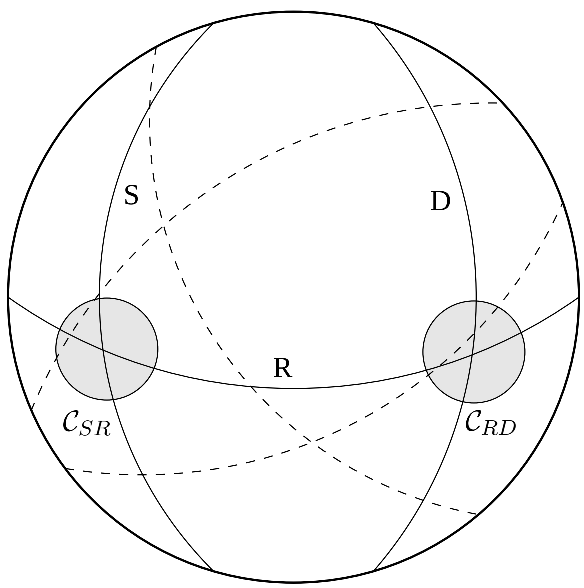

Fig. 2. Source node, S, transmits its packet to a random relay node, R. The packet is carried by R, until its transmission to the destination node, D, when R and D become neighbors. The dotted great circles correspond to other nodes which can act as relays.

The actual mechanism is slightly more complicated. Since each node decides to transmit at random, it is possible that two nearby nodes transmit simultaneously so that transmission is not successful under the Protocol model. In order to analyze the throughput of Scheme Π, we first state a result about the probability of successful transmission between two nodes when they are neighbors under policy Σ_n.

Lemma 2: Under policy Σ_n, the following hold in a typical configuration.

(a) In sub-slot A, if nodes S and R are neighbors of each other, S transmits a packet to R successfully with a strictly positive probability, independent of n.
(b) In sub-slot B, if nodes R and D are neighbors of each other, R transmits a packet to D successfully with a strictly positive probability, independent of n.

Proof: We shall only prove for the case of sub-slot A since the proof for the other part is similar. Consider a sub-slot A in which S and R are neighbors. Let E_1 be the event that S becomes active and E_2 be the event that S chooses R as a random relay and no other source node in $\mathcal{C}_{SR}$ becomes active. If both events E_1 and E_2 occur, S transmits to R and the transmission is successful under the Relaxed Protocol model. Thus,

$$\begin{aligned} P(\text{ S transmits to R successfully}) &= P(E_1 \cap E_2) \\ &= P(E_1)P(E_2|E_1). \end{aligned} \tag{2}$$

From the description of Policy Σ_n it is clear that $P(E_1) = \alpha p_\Delta$, which is a strictly positive constant. Next we compute $P(E_2|E_1)$ and show that it is lower bounded by a strictly positive constant, independent of n, which will imply the statement of the lemma.

Given that S is active, the probability of successful transmission to R depends on how many other nodes are present in $\mathcal{C}_{SR}$ since these nodes could interfere, i.e., transmit simultaneously so that the transmission from S to R is not successful under the Relaxed Protocol model.

Since we have a typical configuration, $\Theta(\sqrt{n})$ distinct great circles or source nodes intersect $\mathcal{C}_{SR}$. Moreover each great circle has $\Theta(1)$ lattice points that are in $\mathcal{C}_{SR}$. For a natural random walk on a discrete torus of size $\sqrt{n}$, the probability of being at any particular position is $1/\sqrt{n}$. Hence the probability that any of the $\Theta\left(\sqrt{n}\right)$ source nodes whose great circles intersect $\mathcal{C}_{SR}$ is present in $\mathcal{C}_{SR}$ with probability $\Theta(1/\sqrt{n})$. Due to the independent movement of all nodes, we obtain that for a typical configuration, the probability of k nodes being present in the $\mathcal{C}_{SR}$ is

$$q(k) = \binom{c_1\sqrt{n}}{k}\left(\frac{c_3}{\sqrt{n}}\right)^k\left(1-\frac{c_4}{\sqrt{n}}\right)^{c_2\sqrt{n}-k} \approx \frac{(c_1c_3)^k\exp(-c_2c_4)}{k!},$$

for large enough n. If $\mathcal{C}_{SR}$ has k nodes not including S and R then S certainly has no more than $k+1$ neighbors. In this situation, R is chosen by S with probability at least $1/(k+1)$. Further there are at most k other source nodes and the probability that no other node in $\mathcal{C}_{SR}$ becomes active is at least $(1-p_\Delta)^k$. Thus,

$$\begin{aligned} P(E_2|E_1) &\geq \sum_{k=0}^{n-2}\frac{(c_1c_3)^k\exp(-c_2c_4)}{k!}\frac{1}{k+1}(1-p_\Delta)^k \\ &\geq \exp(-c_2c_4)\sum_{k=0}^{n-2}\frac{(c_1c_3(1-p_\Delta))^k}{(k+1)!}. \end{aligned}$$

It is easy to see that for $0 < p_\Delta < 1$, the term on the right hand side is a lower bounded by a strictly positive constant. Hence, $P(E_2|E_1)$ is strictly positive. This completes the proof of the lemma. ■

Theorem 1: Scheme Π achieves $T(n) = \Theta(1)$.

Proof: Consider a typical configuration so that policy Σ_n is used. Fix a source node S and a relay node R. Let $A(t)$ be the number of bits transmitted from S to R in sub-slot A of time-slot t. If S transmits to R successfully in sub-slot A of time-slot t, $A(t) = W/2$ otherwise $A(t) = 0$.

First we determine $E[A(t)]$. Let F_1 be the event that S and R are neighbors and F_2 be the event that S transmits to R successfully. Then

$$E[A(t)] = \frac{W}{2}P\{F_1 \cap F_2\} = \frac{W}{2}P\{F_1\}P\{F_2|F_1\}. \tag{3}$$

From Lemma 2(a), $P\{F_2|F_1\} \geq c_5 > 0$. Due to the independent motion of nodes S and R according to natural random walks, the joint description of their positions is a two-dimensional random walk on a discrete torus of size $\sqrt{n} \times \sqrt{n}$. It is easy to see that the stationary distribution for this process is the uniform distribution on n joint positions. Since S and R become neighbors when they are in one particular joint position out of these n joint positions, it follows that the probability of S and R being neighbors is $1/n$, i.e., $P(F_1) = 1/n$. Hence from (3) it follows that $E[A(t)] = \Theta(1/n)$.

Now the positions of nodes S and R form an irreducible, finite state Markov chain and $A(t)$ is a bounded, non-negative function of the state of this Markov chain at time t. Therefore by the ergodicity of such a Markov chain, the long-term throughput between S and R is

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}A(t)=E[A(t)]=\Theta(1/n).$$

Thus the throughput between a source node S and any other node in sub-slot A is $\Theta(1/n)$. Similarly, it can be shown that the throughput between any node and a destination node D in sub-slot B is also $\Theta(1/n)$. The value of $0<\alpha<1$ guarantees that the arrival rate of packets belonging to every S-D pair at any relay node is strictly less than the service rate. This ensures the stability of the queues formed at the relay nodes, which in turn implies that the throughput between each S-D pair is simply the sum of the throughputs between S and the other $n-1$ nodes in sub-slot A. Hence the throughput of each S-D pair is $\Theta(1)$.

We have shown that in a typical configuration, Scheme Π provides $\Theta(1)$ throughput between all S-D pairs. From Lemma 1, configurations are typical *whp*. Hence it follows that Scheme Π has throughput $T(n)=\Theta(1)$. ■

Note that for the unrestricted mobility models in [8] and [6], it is possible to prove a stronger result that each S-D pair has $\Theta(1)$ throuhgput for any n with probability 1 instead of probability approaching 1 as n tends to infinity, as in the present case.

IV. Delay of Scheme Π

Under Scheme Π, if the configuration is not typical, direct transmission is used, in which case the delay for each packet is 1. Since the delay of a scheme is defined to be the expectation over all configurations of the average delay, the delay for Scheme Π is determined by the expected delay over typical configurations. So we shall assume that the configuration is typical.

Consider a particular S-D pair. Packets from S reach D either directly by a single hop in sub-slot A or through any of the other $n-2$ nodes, which act as relays. Since the nodes perform independent random walks, only $\Theta(1/n)$ of the packets belonging to any S-D pair reach their destination in a single hop. Thus, most of the packets reach their destination via a relay node, in which case the delay is two time-slots for two hops plus the mobile-delay, which is the time spent by the packet at the relay node.

Each relay node maintains a separate queue for each of the S-D pairs. Fix a relay node, R, and consider the queue for the S-D pair under consideration. The mobile-delay mentioned above is the delay at this relay-queue.To compute the average delay for this relay-queue, we need to study the characteristics of its arrival and potential departure processes.

First we obtain a lower bound on the delay at the relay-queue. Each node performs a random walk on a 1-D torus of size $\sqrt{n}$ on its great circle. We say that an S-D pair intersects node R's great circle k vertices apart if the lattice points where R can become neighbors of S and D are k lattice points (vertices) apart on the 1-D discrete torus of R.

Fix an S-D pair and consider a particular relay node R. When a packet is transmitted successfully from S to R, D is equally likely to be in any of its $\sqrt{n}$ lattice points since it performs an independent random walk. Let T_{ij} be the random time it takes for a random walk on a $\sqrt{n}\times\sqrt{n}$ torus to hit $(0,0)$ starting from (i,j). If the S-D pair intersects the great circle of R i vertices apart then the expected delay for packets of this S-D pair relayed through R is lower bounded by $\sum_{j=0}^{\sqrt{n}-1}T_{ij}$.

Using the Chernoff bound for the sum of i.i.d. Bernoulli random variable (e.g., see [11]), it can be shown that $\Theta(\sqrt{n})$ S-D pairs intersect the great circle of each node i points apart for $0\le i\le\sqrt{n}-1$ *whp*. Hence the delay of Scheme Π, which is the expected delay over all packets is

$$D(n)=\Omega\left(E\left[\frac{1}{n}\sum_{i,j=1}^{\sqrt{n}-1}T_{ij}\right]\right).$$

As shown in [1], $E\left[\frac{1}{n}\sum_{i,j=1}^{\sqrt{n}-1}T_{ij}\right]=\Theta(n\log n)$. Therefore,

$$D(n)=\Omega(n\log n). \tag{4}$$

The rest of this section derives an upper bound which is of the same order as the lower bound. It is hard to obtain an upper bound on the delay in the relay-queue since the arrival and service processes are complicated and dependent. We progressively obtain queues that are simpler to analyze and upper bound the delay of the previous queue as follows. We first upper bound the delay in the relay-queue by that in another queue, $\mathcal{Q}_1$, in which the arrival process is simpler. The delay of $\mathcal{Q}_1$ is upper bounded by that in $\mathcal{Q}_2$, which has a relatively simpler service process. However, the arrival and service process are not independent. The final part consists of introducing a virtual server with i.i.d. Geometric service times to break this dependence. With this overview, we proceed to the details.

Recall that a packet arrives at the relay-queue when (i) S and R are neighbors, (ii) S becomes active (which happens with probability αp_Δ), (iii) S chooses R as a random relay, and (iv) the transmission from S to R is successful. Similarly, a packet can depart from the queue when (i') R and D are neighbors, (ii') R becomes active (which happens with probability p_Δ), (iii') R chooses D as the destination node, and (iv') the transmission is successful. We call such a time-slot a potential departure instant and the sequence of inter-potential-departure times is called the potential-departure process. Let the potential-departure process of the relay-queue be called $\{S_i\}$. The qualifier potential is used since a departure can occur only if R has a packet for D.

Consider a queue $\mathcal{Q}_1$ in which arrivals happen whenever (i), (ii) and (iii) above are satisfied, irrespective of whether (iv) is satisfied or not. The potential departure process for $\mathcal{Q}_1$ is the same as that for the relay-queue. Then it is clear that the expected delay in $\mathcal{Q}_1$ provides an upper bound on that in the relay-queue.

Recall that the motion of each node is an independent 1-D random walk on a discrete torus of size $\sqrt{n}$. We will say that two nodes *meet* when they become neighbors. Since nodes move independently the joint position of nodes R and D is a random walk on a $\sqrt{n} \times \sqrt{n}$ discrete torus and R and D become neighbors when the 2-D random walk is in state $(0, 0)$, without loss of generality. Therefore, the inter-meeting time of R and D is distributed like the inter-visit time of state $(0, 0)$ of a 2-D random walk. Since this is a Markov chain with n states having a uniform stationary distribution, we know that the sequence of inter-meeting times of nodes R and D, denoted by $\{\tau_i, i \geq 0\}$, is an i.i.d. process. Further, if τ is a random variable with the common distribution then

$$E[\tau] = n. \tag{5}$$

However a potential departure instant does not occur each time R and D meet. A potential departure instant occurs only if R also becomes active, chooses D as the random destination and the transmission is successful. If R and D are not chosen in spite of being in the same cell, it increases the likelihood of there being many more nodes in the same cell. Due to the random walk model of the node mobility, if there is a crowding of nodes in some part of the network then it remains crowded for some time in the future. Hence due to the Markovian nature of node mobility, the inter-potential-departure times are not independent.

We want to obtain an upper bound on the delay of $\mathcal{Q}_1$ which has potential-departure process $\{S_i\}$. To do this we will consider a queue, $\mathcal{Q}_2$, which has the same arrival process as $\mathcal{Q}_1$ but a different departure process $\{\tilde{S}_i\}$ such that $S_i \leq \tilde{S}_i$. Then the expected delay in $\mathcal{Q}_2$ would provide an upper bound on the the expected delay in the relay-queue.

Nodes R and D perform independent random walks on 1-D tori of size $\sqrt{n}$ on their great circles as shown in Figure 2 and R and D meet when both are at a particular pair of lattice points. This is represented schematically in Figure 3, where R performs a vertical 1-D random walk and D performs a horizontal 1-D random walk. The joint motion of nodes R and D is equivalent to a random walk on a 2-D torus of size $\sqrt{n} \times \sqrt{n}$ and R and D meet when this 2-D random walk is in state $(0, 0)$. The inter-meeting times of nodes R and D correspond to the i.i.d. process $\{\tau_i\}$. Further, let $\alpha_i = \tau_1 + \ldots + \tau_i$ for $i \geq 1$, i.e., α_i is the time-slot in which R and D meet for the ith time. In a typical configuration, we know that the number of other great circles that pass through $\mathcal{C}_{RD}$ is $\Theta\left(\sqrt{n}\right)$. Allowing for the worst case, based on Lemma 1, let there be $c_1\sqrt{n} = m - 2$ other great circles that pass through $\mathcal{C}_{RD}$. These can also be thought of as performing independent random walks on the horizontal 1-D torus. Let nodes R and D be numbered 1 and 2 and the other $c_1\sqrt{n}$ nodes be numbered from 3 to m and let $X(t) = (X_1(t), \ldots, X_m(t))$ denote the position of these m nodes on the $\sqrt{n} \times \sqrt{n}$ discrete torus at time t.

A constant number of lattice points of the 1-D torus correspond to $\mathcal{C}_{RD}$ and these are shown by the shaded region in Figure 3 and is referred to as set A. Let E_i be the indicator for the event that R chooses D and the transmission is successful in time-slot α_i. That is, E_i is the indicator for the event that α_i is a potential departure instant. Let N_i be the number of other destination nodes in A in time-slot α_i. Then $P\{E_i = 1\}$ depends on N_i only. Now, N_i depends on $X(\alpha_i)$ which depends on the past given by $E^{i-1} = \{E_0, \ldots, E_{i-1}\}$ and $\tau^i = \{\tau_0, \ldots, \tau_i\}$. Thus

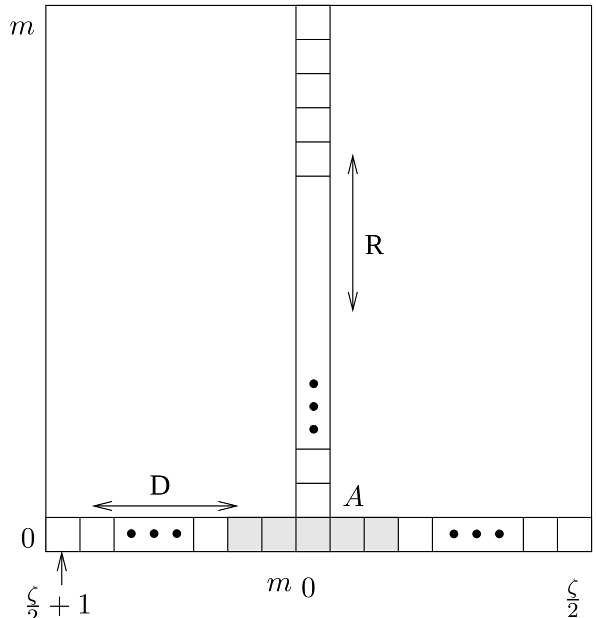

Fig. 3. Schematic representation of the motion of nodes R and D on their respective great circles with $\zeta = \sqrt{n} - 1$.

the potential-departure process is generated by choosing some of the meeting instants of R and D according to a probability modulated by N_i, which is another independent process as shown in Figure 4.

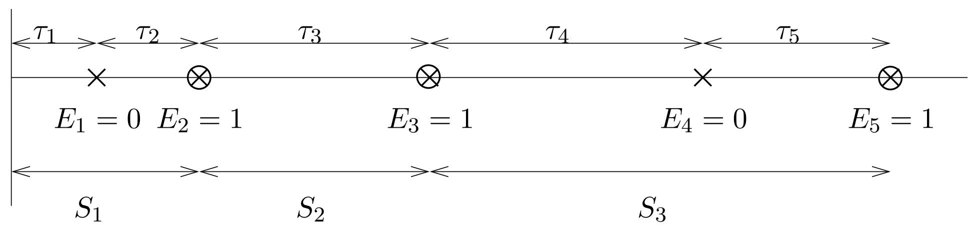

Fig. 4. The 'x' marks correspond to the times when R and D meet each other. At some of these meeting instants the R-D transmission can be successful. Such points have been circled and correspond to $E_i = 1$. The inter-potential-service times are thus the sum of a few of the inter-meeting times of R and D.

Above we described how the process $\{S_i\}$ can be generated using the processes $\{N_i\}$ and $\{\tau_i\}$, which in turn were obtained from $\{X(t)\}$, which corresponds to the independent random walks of all m nodes. Next we shall perturb the process $\{X(t)\}$ to obtain $\{\tilde{X}(t)\}$ and the corresponding $\{\tilde{\tau}_i\}$ and $\{\tilde{N}_i\}$. Let $Z(t)$ be a 1-D horizontal random walk on a torus of size $\sqrt{n}$. Let $\tilde{X}_i(t) = X_i(t) + Z(t)$ be the position of node $i, 1 \leq i \leq m$, where the addition is modulo $\sqrt{n}$. Then the inter-meeting times of any two nodes are the same as before since the position of each node is shifted horizontally by the same amount due to $Z(t)$. As a result the processes τ_i and $\tilde{\tau}_i$ are identical. Under the modified setup, the lattice point at which R and D meet can be any element of the set $B = \{(i, 0) : 0 \leq i \leq \sqrt{n} - 1\}$ instead of always being $(0, 0)$. Similarly, let $\tilde{N}_i$ be the number of other destination nodes in the set $A + Z(t)$. Then, $\{\tilde{N}_i\}$ is identical to $\{N_i\}$. Thus the process $\{S_i\}$ can also be generated (through $\{E_i\}$) using $\{\tilde{N}_i\}$ and $\{\tilde{\tau}_i\}$ instead of $\{N_i\}$ and $\{\tau_i\}$. Therefore we shall use $\tilde{X}_i(t)$ as the position of node i at time t instead of $X_i(t)$. Under this perturbed motion, R can be seen as if it performs a 2-D random walk on the $\sqrt{n} \times \sqrt{n}$ torus while D and the other $m - 2$ nodes perform a 1-D random walk on a 1-D torus of size $\sqrt{n}$ which is subset B of the 2-D torus. Moreover, given $\tilde{X}_3^m(\alpha_i) = (\tilde{X}_3(\alpha_i), \ldots, \tilde{X}_m(\alpha_i))$, $P\{E_i = 1\}$ is independent of everything else.

Lemma 3: There exists a constant (independent of n) $c_6 > 0$ such that

$$P\left(E_i = 1 | \tau^i, E^{i-1}\right) \geq c_6 > 0.$$

Proof: The initial position of R, $X_1(0)$ has a uniform distribution of the $\sqrt{n} \times \sqrt{n}$ torus. The initial positions of D and nodes 3 to m have independent uniform distributions on subset $B = \{(i,0) : 0 \le i \le \sqrt{n}-1\}$ of the $\sqrt{n} \times \sqrt{n}$ torus. As a result $X_1(\alpha_1) = X_2(\alpha_1) = I$ where I is a random variable with a uniform distribution over B.

Let $V = (\tilde{X}_3(\alpha_i), \ldots, \tilde{X}_m(\alpha_i))$ be the configuration of the $m-2$ nodes other than R and D. Then the conditional probability of a potential departure given the past can be written as

$$\begin{aligned} P\left(E_i = 1 | \tau^i, E^{i-1}\right) &= \sum_V \frac{P\left(E_i = 1 | V, \tau^i, E^{i-1}\right) P\left(V, \tau^i, E^{i-1}\right)}{P(\tau^i, E^{i-1})} \\ &\ge \min_V P(E_i = 1 | V, \tau^i, E^{i-1}) \left[\sum_V \frac{P(V, \tau^i, E^{i-1})}{P(\tau^i, E^{i-1})}\right] \\ &= \min_V P(E_i = 1 | V, \tau^i, E^{i-1}) \\ &= \min_V P\left(E_i = 1 | V\right), \end{aligned} \tag{6}$$

where the last equality holds because E_i is independent of everything else given V.

Given a congfiguration V, the number of nodes in $A + (i-1,0)$ for $i = 1, \ldots, \sqrt{n}$ torus can be found and this in turn determines the $P\left(E_i = 1 | V\right)$. Hence, if V_i denotes the number of nodes other than R and D in the set $A + (i-1,0)$ for $i = 1, \ldots, \sqrt{n}$ then we can equivalently let the configuration be $V = (V_1, \ldots, V_{\sqrt{n}})$.

Now consider a fixed configuration, $V = v = (v_1, \ldots, v_{\sqrt{n}})$, and let Z be a random variable which takes value $v_i, 1 \le i \le \sqrt{n}$ with probability $1/\sqrt{n}$. Let A consists of c_2 (some constant) elements. Then

$$E[Z] = \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} v_k = \frac{c_2(m-2)}{\sqrt{n}} = \Theta(1). \tag{7}$$

Recall that $X_1(\alpha_i) = I$, where I is a random variable with uniform distribution on B. Further, from the description of Scheme Π, if there are v_k destination nodes other than D in $\mathcal{C}_{RD}$ then $E_i = 1$ if R chooses D out of all destination nodes that are its neighbors and the other v_k nodes do not transmit. Since $\mathcal{C}_{RD}$ contains all neighbors and more, the number of neighbors can be no more than X_i and hence for $k = 1, \ldots, \sqrt{n}$, we obtain

$$P(E_i = 1 | V = v, X_1(\alpha_i) = (k-1, 0)) \ge \frac{p_\Delta (1-p_\Delta)^{v_k+1}}{v_k+1}. \tag{8}$$

Define a real valued function $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \frac{p_\Delta(1-p_\Delta)^{x+1}}{x+1}$. It is easy to check that $f(\cdot)$ is a convex function. Hence, by Jensen's inequality,

$$E[f(Z)] \ \ \ge \ \ f(E[Z]). \tag{9}$$

Using (7), (8) and (9), for any configuration V with corresponding v, we obtain

$$\begin{aligned} P(E_i = 1 | V = v) &= \sum_{k=1}^{\sqrt{n}} P(E_i = 1 | V = v, X_1(\alpha_i) = (k-1,0)) P(X_1(\alpha_i) = (k-1,0) | V = v) \\ &= \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} P(E_i = 1 | V = v, X_1(\alpha_i) = (k-1,0)) \\ &\ge \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} \frac{p_\Delta (1-p_\Delta)^{v_k+1}}{v_k+1} \\ &= E[f(Z)] \ \ge \ f(E[Z]) \\ &= f\left(\frac{c_2(m-2)}{\sqrt{n}}\right) \stackrel{\triangle}{=} c_6 > 0. \end{aligned} \tag{10}$$

Combining (6) and (10) completes the proof of the lemma. ■

Recall that the process $\{S_i\}$ is generated from $\{\tau_i\}$ and $\{E_i\}$. Consider an i.i.d. Bernoulli process $\{\tilde{E}_i\}$ with $P\{\tilde{E}_1 = 1\} = c_6$. Now we can construct a process $\{\tilde{S}_i, i \geq 1\}$ similar to the process $\{S_i\}$ using $\{\tau_i\}$ and $\{\tilde{E}_i\}$ instead of $\{E_i\}$. Lemma 3 shows that the processes $\{S_i\}$ and $\{\tilde{S}_i\}$ are coupled such that $\tilde{S}_i \geq S_i$ (the inequality corresponds to standard stochastic dominance). Now consider queue, $\mathcal{Q}_2$, with the same arrival process as $\mathcal{Q}_1$ but with potential-departure process $\{\tilde{S}_i\}$. Depending on the value of c_6, the value of α can be chosen so that the arrival rate is strictly smaller than the potential departure rate in $\mathcal{Q}_2$ so as to ensure stability. The distribution of $\tilde{S}_1$ is the same as $\tau_1 + \ldots + \tau_G$, where G is an independent Geometric random variable with parameter c_6. As a result, for any $r \in \mathbb{N}$,

$$E[\tilde{S}_1^r] = \Theta(E[\tau_1^r]). \tag{11}$$

In light of (11), it is easy to see that the delay scaling of queue $\mathcal{Q}_2$ is the same as the delay scaling of a queue in which an arrival happens each time S and R meet with probability 0.5 and a potential departure occurs each time R and D meet. Since we are interested only in the delay scaling, henceforth we assume that in $\mathcal{Q}_2$, an arrival happens when S and R meet with probability 0.5 and a potential departure occurs whenever R and D meet.

At this stage we have upper bounded the delay in the relay-queue by the delay in $\mathcal{Q}_2$. The inter-arrival times and the inter-potential departure times in $\mathcal{Q}_2$ are i.i.d. processes. However these two processes are not independent for the following simple reason: if the S-D pair intersects the great circle of R, $k > 0$ vertices apart then R has to travel at least distance k on the discrete torus after an arrival for a potential departure to occur.

Next, we will bound the delay in $\mathcal{Q}_2$ by the sum of the delays through two *virtual* queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, in tandem. Both $\mathcal{Q}_3$ and $\mathcal{Q}_4$ will be shown to have delay of $O(n \log n)$. This will imply that the delay of $\mathcal{Q}_2$ is $O(n \log n)$. Queues $\mathcal{Q}_3$ and $\mathcal{Q}_4$ are constructed as follows. The arrival process of $\mathcal{Q}_3$ is the same as that of $\mathcal{Q}_2$. The potential-departure process of $\mathcal{Q}_3$ is an i.i.d. Bernoulli process with parameter $2/3n$ (or potential departure rate $\frac{2}{3n}$). An arrival occurs at $\mathcal{Q}_4$ whenever there is a potential-departure at $\mathcal{Q}_3$. If $\mathcal{Q}_3$ is non-empty, then the arrival to $\mathcal{Q}_4$ is the head-of-line packet transferred from $\mathcal{Q}_3$ to $\mathcal{Q}_4$ or else a *dummy* packet is fed to $\mathcal{Q}_4$. Thus the arrival process at $\mathcal{Q}_4$ is the same as the potential-service process at $\mathcal{Q}_3$. By construction, the delay of a packet through this tandem of queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, upper bounds the delay experienced by a packet through $\mathcal{Q}_2$. Now, from Lemmas 5 and 6 stated in the next section, the expected delay through $\mathcal{Q}_3$ and $\mathcal{Q}_4$ is $O(n \log n)$. Thus the expected delay of the packets of each S-D pair relayed through each relay R in a typical configuration is $O(n \log n)$. The delay of scheme is the expectation of the packet delay averaged over all S-D pairs and all relay nodes. Hence it follows that the delay of the scheme is $O(n \log n)$. Combining this with (4), we have proved the following.

Theorem 2: The delay of Scheme Π is $\Theta(n \log n)$.

V. Remaining Proofs

In this section, we prove Lemmas 5 and 6, which were used to prove that Scheme Π has delay of $O(n \log n)$. Before proving these, we present Lemma 4 which will be useful for both these proofs.

Recall that each node performs an independent random walk on a 1-D discrete torus of size $\sqrt{n}$ on its great circle. Let Z be a random variable which is distributed as the inter-meeting time of two distinct nodes. The following lemma provides the first two moments of Z.

Lemma 4:

$$E[Z] = n, \quad E[Z^2] = \Theta(n^2 \log n).$$

Proof: As nodes perform independent random walks, the joint position of two nodes is a 2-D random walk on the $\sqrt{n} \times \sqrt{n}$ discrete torus. Thus the inter-meeting time of any two nodes is equivalent to the first return time to state $(0,0)$ for this random walk on a $\sqrt{n} \times \sqrt{n}$ torus. Since we are interested only in determining the exact order of the moments, we will consider a simple random walk.

Let $X(t) = (X_1(t), X_2(t)) \in \{0, \ldots, \sqrt{n} - 1\}^2$ be a simple random walk on the $\sqrt{n} \times \sqrt{n}$ torus. Then the first return time to state $(0,0)$ is

$$T = \inf\{t \geq 1 : X(t) = (0,0),\ X(0) = (0,0)\}.$$

Note that $X(t)$ is a finite-state Markov chain with a uniform equilibrium on the n states. For any finite-state Markov chain, the expectation of the first return time to any state is the reciprocal of the equilibrium probability of the Markov chain being in that state. Hence, $E[T] = n$.

Define, $T_0 = \inf\{t \geq 1 : X(t) = (0,0)\}$. Observe that T_0 differs from T in that T is conditioned on starting at $X(0) = (0,0)$. Let $E_{(i,j)} T_{(k,l)}$ denote the expected time to hit state (k,l) for the first time starting from state

(i,j). Let $E_\pi[T_0]$ denote the expectation of T_0 given that $X(0)$ is distributed according to the uniform stationary probability distribution π. Then,

$$E_\pi[T_0] = \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)} T_{(0,0)} = \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)} T_{(k,l)} \tag{12}$$

$$= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \frac{1}{n} \pi(i,j) E_{(i,j)} T_{(k,l)} = \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \pi(i,j)\pi(k,l) E_{(i,j)} T_{(k,l)}$$

$$= n \log n, \tag{13}$$

where (12) holds because $\sum_{ij} E_{(i,j)} T_{(0,0)} = \sum_{ij} E_{(i,j)} T_{(k,l)}$ for any $0 \le k, l \le \sqrt{n} - 1$ due to symmetry of states corresponding to cells on the torus. For the validity of (13), see page 11 of Chapter 5 in [1].

Using Kac's formula (see Corollary 24 in Chapter 2 of [1]) and (13), we obtain

$$E[T^2] = \frac{2E_\pi[T_0] + 1}{\pi(0,0)} = 2n^2 \log n + n. \tag{14}$$

Therefore, we obtain $E[Z] = n$ and $E[Z^2] = \Theta(n^2 \log n)$. ■

Lemma 5: Let D_3 denote the delay of a packet through queue, $\mathcal{Q}_3$, as defined above. Then,

$$E[D_3] = O(n \log n).$$

Proof: An arrival occurs to $\mathcal{Q}_3$ when S and R meet with probability 0.5. Let $\{X_i\}$ be the sequence of inter-arrival times to this queue. Then, X_i are i.i.d. with $E[X_1] = 2E[Z] = 2n$ and $E[X_1^2] = \Theta(E[Z^2]) = \Theta(n^2 \log n)$ from Lemma 4. The potential-departure process is an i.i.d. Bernoulli process with parameter $1/1.5n$. Let $\{Y_i\}$ be the sequence of service times then Y_i is a Geometric random variable with mean $1.5n$. Hence $E[Y_1] = 1.5n$ and $E[Y_1^2] = \Theta(n^2)$. By construction, the service process is independent of the arrival process and hence $\mathcal{Q}_3$ is a GI/GI/1 FCFS queue. Then, by Kingman's upper bound [12] on the expected delay for a GI/GI/1 – FCFS queue, the expected delay of $\mathcal{Q}_3$ is upper bounded as

$$E[D_3] = O\left(\frac{E[X_1^2] + E[Y_1^2]}{E[X_1]}\right) = O\left(\frac{n^2 \log n + n^2}{n}\right) = O\left(n \log n\right). \tag{15}$$

■

Lemma 6: Let D_4 denote the delay of a packet through queue, $\mathcal{Q}_4$, as defined above. Then,

$$E[D_4] = O(n \log n).$$

Proof: Consider the service process of $\mathcal{Q}_4$, which is 1 at a potential departure instant and 0 otherwise. This is a stationary, ergodic process since the inter-potential-departure times are i.i.d. with mean n. The Bernoulli arrival process to $\mathcal{Q}_4$ is independent of the service process with mean inter-arrival time $1.5n$. Since the arrival and service processes form a jointly stationary and ergodic process with mean service time strictly less than mean inter-arrival time, the queue has a stationary, ergodic distribution with finite expectation as shown by [9]. Thus $\mathcal{Q}_4$ is stable.

Let $\tilde{Q}_t$ be the number of packets in the queue in time-slot t and let Q_i be the number of packets in the queue at potential departure instant i. Thus the process $\{Q_i\}$ is obtained by sampling $\{\tilde{Q}_t\}$ at potential departure instants. Let A_{i+1} be the number of arrivals between potential departure instants i and $i+1$. Then the evolution of Q_i is given by

$$Q_{i+1} = Q_i - \mathbf{1}_{\{Q_i > 0\}} + A_{i+1}. \tag{16}$$

Comparing the evolution of the process $\{Q_i\}$ with that of $\{\tilde{Q}_t\}$ shows that $\{Q_i\}$ also has a stationary, ergodic distribution. Let Z be the inter-meeting time of any two nodes as defined in the beginning of this section. Then since the arrival process is Bernoulli and the inter-potential departure times are i.i.d. with common distribution that of Z, it is clear the $\{A_i\}$ is a stationary process. Let $\tilde{Q}$, Q and A be random variables with the common stationary marginals of $\{\tilde{Q}_t\}$, $\{Q_i\}$ and $\{A_i\}$ respectively. Then taking expection in (16) under the stationary distribution, we obtain

$$P(Q > 0) = E[A]. \tag{17}$$

The arrival proces is i.i.d. Bernoulli and hence conditioned on Z, the distribution of A is Binomial $(Z, 2/3n)$. Since $E[Z] = n$ from Lemma 4, we obtain

$$E[A] \;=\; E[E[A|Z]] \;=\; E\left[\frac{Z}{1.5n}\right] \;=\; 2/3. \tag{18}$$

Squaring (16), taking expecation, using the independence of Q_i and A_{i+1} and then rearranging terms, we obtain

$$2(1-E[A])E[Q] = P(Q>0) + E[A^2] - 2E[A]P(Q>0). \tag{19}$$

Using (17) and (18) in the above, we obtain

$$E[Q] \;=\; \frac{E[A]+E[A^2]-2E[A]^2}{1(1-E[A])} \;=\; \frac{3}{2}\left(E[A^2]-\frac{2}{9}\right). \tag{20}$$

Recall that conditioned on Z the distribution of A is Binomial $(Z, 2/3n)$ and hence

$$\begin{aligned} E[A^2] \;&=\; E[E[A^2]Z]] \;=\; \frac{2E[Z]}{3n} + \frac{4}{9n^2}\left(E[Z^2]-E[Z]\right) \\ &=\; \left(\frac{2}{3}-\frac{4}{9n}\right) + \frac{4}{9n^2}\Theta(n^2\log n) \;=\; \Theta(\log n), \end{aligned} \tag{21}$$

where we used Lemma 4. As a result it follows from (20) that

$$E[Q] = \Theta(\log n). \tag{22}$$

Next, we will bound $E[\tilde{Q}]$ using $E[Q]$. To this end, consider a time-slot t and let the number of potential departures before time-slot t be $I(t)$. Thus time-slot t is flanked by potential departures $I(t)$ and $I(t)+1$. Then $\tilde{Q}_t \le Q_{I(t)} + A_{I(t)+1}$. Also using the fact that $\{\tilde{Q}_t\}$ is ergodic, with probability 1, we have

$$\begin{aligned} E[\tilde{Q}] \;&=\; \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}\tilde{Q}_k \\ &\le\; \lim_{T\to\infty}\frac{1}{T}\sum_{j=1}^{I(T)+1}\left(Q_jZ_{j+1}+A_{j+1}Z_{j+1}\right) \\ &=\; \lim_{T\to\infty}\frac{I(T)+1}{T}\frac{1}{I(T)+1}\sum_{j=1}^{I(T)+1}\left(Q_jZ_{j+1}+A_{j+1}Z_{j+1}\right) \\ &=\; \frac{1}{E[Z]}\left(E[Q_1Z_2]+E[A_1Z_1]\right) && (23) \\ &=\; \frac{1}{n}\left(E[Q]E[Z]+\frac{2}{3n}E[Z^2]\right) && (24) \\ &=\; O(\log n). && (25) \end{aligned}$$

We used the fact that $I(T)/T \to 1/E[Z]$ by the elementary renewal theorem [12] in (23) and the independence of Q_j and Z_{j+1} in (24). Now using Little's formula, since the arrival rate is $2/3n$, we conclude that

$$E[D_4] = E[A]E[\tilde{Q}] = \frac{3n}{2}O(\log n) = O(n\log n).$$

■

VI. Conclusion

In this paper, we studied the maximal throughput scaling and the corresponding delay scaling in a random mobile network with restricted node mobility. In [3], it was shown that a particular mobility restriction does not affect the throughput scaling. In this paper, we showed that it does not affect delay scaling either. In particular, we show that delay scales as $D(n) = \Theta(n\log n)$ for a network of n nodes, which is the same as the delay scaling without any mobility restriction. This was understood to be a consequence of the fact that in spite of an apparent restriction, essentially the node mobility remaining unchanged in the sense that (i) each node meets every other node for $\Theta(1/n)$ fraction of the time with only $\Theta(1)$ other neighboring nodes, and (ii) the inter-meeting time of nodes has mean of $\Theta(n)$ and variance of $O(n^2\log n)$.

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