Minimum Energy Communication Over a Relay Channel

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Abstract — The paper investigates limits on the energy-per-bit required for reliable communication over AWGN relay channels. Upper and lower bounds on the minimum energy-per-bit that are not tight, but whose ratio is less than 2 are found. Tighter bounds on minimum energy-per-bit are also found for two frequency-division AWGN relay channel models.

I. DEFINITIONS

The AWGN relay channel considered consists of a sender $X_i$, a receiver $Y_i$, a relay receiver $Y_{i1}$, and a relay sender $X_{i1}$. The relationships between the transmitted and received signals at time $i \geq 1$ are given by $Y_{i1} = aX_i + Z_{i1}$ and $Y_i = X_i + bX_{i1} + Z_i$, where $a, b > 0$ are relative path gains, $Z_{i1}$ and $Z_i$ are independent WGN processes each with power $N$, and $X_{i1}$ is a function of $Y_{11}, Y_{12}, \ldots, Y_{1i-1}$. Average power constraints $P$ on the sender and $\gamma P \geq 0$ on the relay sender are assumed. The definitions of channel coding and capacity $C(P, \gamma P)$ follow [1].

To define minimum energy-per-bit, assume a $(2^{nR_n}, n)$ code. Note that here we allow the rate to vary with $n$ in order to define the minimum energy-per-bit in an unrestricted way. The energy for codeword $k$ is given by

$$E_k^{(n)} = \sum_{i=1}^{n} x_{ik}^2,$$

and the maximum relay transmission energy is given by

$$E_r^{(n)} = \max_i \left( \sum_{i=1}^{n} x_{i1i}^2 \right).$$

Thus the energy-per-bit for the code is given by

$$E_n = \frac{1}{nR_n} \left( \max_k E_k^{(n)} + E_r^{(n)} \right).$$

An energy-per-bit $E$ is said to be achievable if there is a sequence of $(2^{nR_n}, n)$ codes with $P_k^{(n)} \to 0$ and $\sup E_n \leq E$. The minimum energy-per-bit $E_b$ is defined as the the infimum of the set of achievable energy-per-bit values.

II. UPPER AND LOWER BOUNDS ON $E_b$

First note that the following general relationship between minimum energy-per-bit and capacity with average power constraint can be proved

$$E_b = \inf_{\gamma \geq 0} \lim_{P \to 0} \frac{(1 + \gamma)P}{C(P, \gamma P)}$$

Using this relationship and bounds on capacity derived from results in [1] the following bounds on the energy-per-bit can be established.

$$\frac{1 + a^2 + b^2}{(1 + a^2)(1 + b^2)} \leq \frac{E_b}{2N \ln 2} \leq \min \left\{ \frac{1}{a^2(1 + b^2)} \right\}.$$  

Note that these bounds are not tight for any $a, b > 0$. However, their ratio is always less than 2 (approaches 2 for $a = 1$ and $b \to \infty$).

We then find an upper bound on $E_b$ using the quantization scheme in [1] and show that it can improve the above upper bound for $a \approx 1$ and large $b$. In fact for any $a$ the upper bound obtained using the quantization scheme approaches the lower bound as $b \to \infty$.

III. FREQUENCY-DIVISION RELAY CHANNELS

Finally, we consider two FD-AWGN relay channel models. In model (A), the received signal at time $i$, $Y_i = \{Y_{Si}, Y_{Ri}\}$, where $Y_{Si} = X_i + Z_{Si}$ is the received signal from the sender and $Y_{Ri} = bX_{i1} + Z_{Ri}$ is the received signal from the relay, $Y_{i1} = aX_i + Z_{i1}$, and $Z_{ii}, Z_{Si},$ and $Z_{Ri}$ are independent WGN processes each with power $N$. For this model we obtain the bounds

$$\min \left\{ 1, \frac{a^2 + b^2}{b^2 (1 + a^2)} \right\} \leq \frac{E_b^A}{2N \ln 2} \leq \min \left\{ \frac{\frac{a^2 + b^2 - 1}{a^2 b^2}, \frac{1}{1}, \frac{a^2 + b^2 + 1}{a^2 b^2}, \frac{1}{1} \right\}, \text{ if } a, b > 1$$

Note that again the ratio of these bounds is always less than 2. More interestingly, for $b \leq 1$, the bounds coincide and the minimum energy-per-bit is $2N \ln 2$.

In the second FD-AWGN relay channel model (B), $X_i = \{X_{Di}, X_{Ri}\}$, $Y_i = X_{Di} + bX_{Ri} + Z_i$, and $Y_{i1} = aX_{Ri} + Z_{i1}$, where $Z_i$ and $Z_{i1}$ are independent WGN processes each with noise power $N$. The power constraint on the sender is $P$. In this case, the capacity can be shown to be equal to the upper bound in [1] and the minimum energy-per-bit is given by

$$\frac{E_b^B}{2N \ln 2} = \min \left\{ 1, \frac{a^2 + b^2 + 1}{a^2 b^2} \right\}.$$  

REFERENCES


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