Minimum Energy Communication Over a Relay Channel

Abbas El Gamal, and Sina Zahedi[1]
Department of Electrical Engineering
Stanford University, Stanford, CA 94305, USA.
abbas@ee.stanford.edu, szahedi@stanford.edu

Abstract — **The paper investigates limits on the energy-per-bit required for reliable communication over AWGN relay channels. Upper and lower bounds on the minimum energy-per-bit that are not tight, but whose ratio is less than 2 are found. Tighter bounds on minimum energy-per-bit are also found for two frequency-division AWGN relay channel models.**

I. Definitions

The AWGN relay channel considered consists of a sender X, a receiver Y, a relay receiver Y_1, and a relay sender X_1. The relationships between the transmitted and received signals at time $i \geq 1$ are given by $Y_{1i} = aX_i + Z_{1i}$ and $Y_i = X_i + bX_{1i} + Z_i$, where $a, b > 0$ are relative path gains, Z_{1i} and Z_i are independent WGN processes each with power N, and X_{1i} is a function of $Y_{11}, Y_{12}, \ldots, Y_{1i-1}$. Average power constraints P on the sender and $\gamma P \geq 0$ on the relay sender are assumed. The definitions of channel coding and capacity $C(P, \gamma P)$ follow [1].

To define minimum energy-per-bit, assume a $(2^{nR_n}, n)$ code. Note that here we allow the rate to vary with n in order to define the minimum energy-per-bit in an unrestricted way. The energy for codeword k is given by

$$\mathcal{E}_k^{(n)} = \sum_{i=1}^{n} x_{ik}^2,$$

and the maximum relay transmission energy is given by

$$\mathcal{E}_r^{(n)} = \max_{y_1} \left(\sum_{i=1}^{n} x_{1i}^2 \right).$$

Thus the energy-per-bit for the code is given by

$$\mathcal{E}_n = \frac{1}{nR_n} \left(\max_k \mathcal{E}_k^{(n)} + \mathcal{E}_r^{(n)} \right).$$

An energy-per-bit $\mathcal{E}$ is said to be achievable if there is a sequence of $(2^{nR_n}, n)$ codes with $\mathrm{P}_e^{(n)} \to 0$ and $\sup \mathcal{E}_n \leq \mathcal{E}$. The minimum energy-per-bit $\mathcal{E}_b$ is defines as the the infimum of the set of achievable energy-per-bit values.

II. Upper and Lower Bounds on $\mathcal{E}_b$

First note that the following general relationship between minimum energy-per-bit and capacity with average power constraint can be proved

$$\mathcal{E}_b = \inf_{\gamma \geq 0} \lim_{P \to 0} \frac{(1+\gamma)P}{C(P, \gamma P)}.$$

Using this relationship and bounds on capacity derived from results in [1] the following bounds on the energy-per-bit can be established.

$$\frac{1 + a^2 + b^2}{(1+a^2)(1+b^2)} \leq \frac{\mathcal{E}_b}{2N \ln 2} \leq \min \left\{ 1, \frac{a^2 + b^2}{a^2(1+b^2)} \right\}.$$

Note that these bounds are not tight for any $a, b > 0$. However, their ratio is always less than 2 (approaches 2 for $a = 1$ and $b \to \infty$).

We then find an upper bound on $\mathcal{E}_b$ using the quantization scheme in [1] and show that it can improve the above upper bound for $a \approx 1$ and large b. In fact for any a the upper bound obtained using the quantization scheme approaches the lower bound as $b \to \infty$.

III. Frequency-Division Relay Channels

Finally, we consider two FD-AWGN relay channel models. In model (A), the received signal at time i, $Y_i = \{Y_{Si}, Y_{Ri}\}$, where $Y_{Si} = X_i + Z_{Si}$ is the received signal from the sender and $Y_{Ri} = bX_{1i} + Z_{Ri}$ is the received signal from the relay, $Y_{1i} = aX_i + Z_{1i}$, and Z_{1i}, Z_{Si}, and Z_{Ri} are independent WGN processes each with power N. For this model we obtain the bounds

$$\min \left\{ 1, \frac{a^2 + b^2}{b^2(1+a^2)} \right\} \leq \frac{\mathcal{E}_b^{\mathrm{A}}}{2N \ln 2} \leq \begin{cases} \frac{a^2+b^2-1}{a^2 b^2}, & \text{if } a, b > 1 \\ 1, & \text{otherwise} \end{cases}$$

Note that again the ratio of these bounds is always less than 2. More interestingly, for $b \leq 1$, the bounds coincide and the minimum energy-per-bit is $2N \ln 2$.

In the second FD-AWGN relay channel model (B), $X_i = \{X_{Di}, X_{Ri}\}$, $Y_i = X_{Di} + bX_{1i} + Z_i$, and $Y_{1i} = aX_{Ri} + Z_{1i}$, where Z_i and Z_{1i} are independent WGN processes each with noise power N. The power constraint on the sender is P. In this case, the capacity can be shown to be equal to the upper bound in [1] and the minimum energy-per-bit is given by

$$\frac{\mathcal{E}_b^{\mathrm{B}}}{2N \ln 2} = \min \left\{ 1, \frac{a^2 + b^2 + 1}{a^2(1+b^2)} \right\}.$$

References

[1] T. M. Cover, A. El Gamal, "Capacity Theorems for the Relay Channel," *IEEE Transcations on Information Theory*, Vol. 25, No. 5, pp. 572-584, September 1979.

[1]This work was supported by Stanford Network Research Center (SNRC).